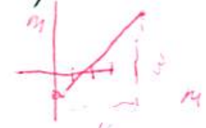
Units of Measure. $\int^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

1. $y = \frac{3}{4}x - 1$, where x and y are in meters. Find the length of the curve using the arc length formula, from $(0,-1)$ to $(4,2)$

$\frac{dy}{dx} = \frac{3}{4}$ $\quad \int_0^4 \sqrt{1+\left(\frac{3}{4}\right)^2}\,dx \quad \int \sqrt{\frac{25}{16}} \quad \int_0^4 \frac{5}{4}\,dx \quad \frac{5}{4}x \Big|_0^4 = 5$



2. $\vec{r} = (4t)\hat{i} + (3t-1)\hat{j}$, where $t \in$ seconds and x and y are in meters. Sketch the position of the particle $t \in [0,1]$ and find the distance traveled during that time interval by inspection of your graph.

t	(x,y)
0	0, −1
1	4, 2

3. The position of a particle is described by
$x = 4t$
$y = -1 + 3t$
where $t \in$ seconds and $(x,y) \in$ meters.

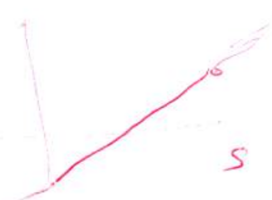
Find the distance traveled using the formula $d = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

between the locations $(0,-1)$ and $(4,2)$

$\frac{dx}{dt} = 4 \quad \frac{dy}{dt} = 3 \quad d = \int_0^1 \sqrt{4^2 + 3^2}\,dt = \int_0^1 5\,dt = 5t\Big|_0^1 = 5$

4. $s(t) = \frac{3}{4}t - 1$, $s \in$ meters and $t \in$ seconds.

Evaluate $\int_0^4 s(t)\,dt$ and explain the meaning of this value using units. $\int_0^4 \left(\frac{3}{4}t - 1\right)dt$



$\frac{3}{4}\frac{t^2}{2} - t \Big|_0^4$

$\frac{3}{8}t^2 - t \Big|_0^4 = \frac{3}{8}(4)^2 - 4$

$\frac{3}{8} \cdot 16 - 4 = 6 - 4 = 2$ m·s

5. $v(t)$ is a velocity of a particle with location $s(t)$, where $s \in$ meters and $t \in$ seconds

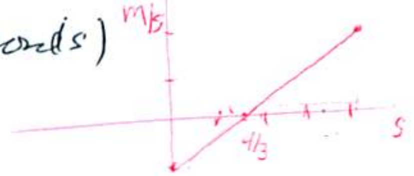
Find $s(4) - s(0)$ if $v(t) = \frac{3}{4}t - 1$

$$\int_0^4 \left(\tfrac{3}{4}t - 1\right)dt = \frac{3t^2}{8} - t \Big|_0^4 = \left(\frac{3(4)^2}{8} - 4\right) - \left(\frac{3(0)^2}{8} - 0\right)$$

$$6 - 4 = 2\ m$$

6. $\frac{ds}{dt} = \frac{3}{4}t - 1$ and $s(0) = 0$. Sketch $s(t)$ carefully, $t \in [0,4]$ and label units (meters and seconds)

$v(t) = s'(t)$



$0 = \frac{3}{4}t - 1$

$\frac{4}{3} = t$

$\frac{12}{3}$

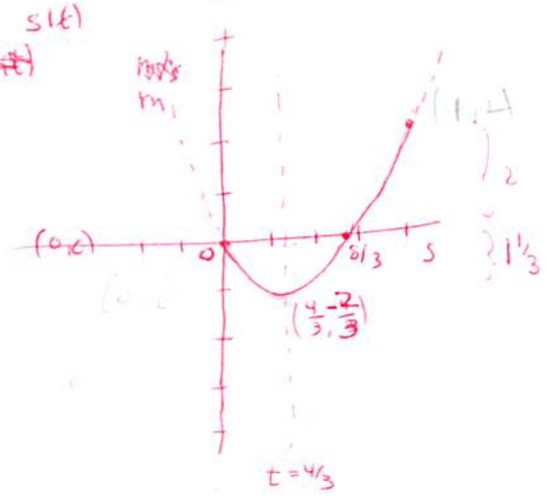
$$\int \frac{ds}{dt} = \int \tfrac{3}{4}t - 1$$

$$\int ds = \int \left(\tfrac{3}{4}t - 1\right)dt$$

$$s = \frac{3t^2}{8} - t + C$$

$s(0) = 0 \quad C = 0$

~~s(t) = v(t)~~



$\frac{4}{3}\left(\frac{3}{8}\left(\frac{4}{3}\right) - 1\right)$

$\frac{4}{3}\left(\frac{1}{2} - 1\right)$

$\frac{4}{3}\left(-\frac{1}{2}\right)$

$$s = t\left(\tfrac{3}{8}t - 1\right)$$

$$t = 0, \ t = \frac{8}{3}$$

7. $v(t) = \frac{3}{4}t - 1$, $v \in m/s$, $t \in$ seconds. Find the total distance the partical travels $t \in [0,4]$

$$\int_0^4 |v(t)|\,dt$$

$$-\int_0^{4/3} \left(\tfrac{3}{4}t - 1\right)dt + \int_{4/3}^4 \left(\tfrac{3}{4}t - 1\right)dt$$

$\frac{1}{2}(1)\left(\frac{4}{3}\right) + \frac{1}{2}(2)\ \frac{8}{3}$

$\frac{2}{3} + \frac{8}{3}$

$\frac{10}{3}$ m

$\frac{12}{3} - \frac{4}{3}$

$\frac{8}{3}$

Parametric / Vector BC. All answers need units

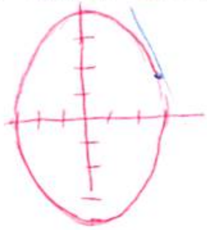
$\frac{x}{3} = \cos t \quad \frac{y}{4} = \sin t$

1. $x = 3\cos t$, $t \in$ seconds, x and $y \in$ meters
$y = 4\sin t \qquad t \geq 0$

$\cos^2 t + \sin^2 t = 1$
$(\frac{x}{3})^2 + (\frac{y}{4})^2 = 1$
$\frac{x^2}{9} + \frac{y^2}{16} = 1$

a. fill in table and sketch

t	(x,y)
0	$3,0$
$\pi/4$	$\frac{3}{\sqrt{2}}, \frac{4}{\sqrt{2}}$
$\pi/2$	$0,4$



b. Find $\frac{dy}{dx}$ and equation (rectangular form) of tangent line at $t = \pi/4$

$\frac{dy}{dt} = 4\cos t$

$\frac{dx}{dt} = -3\sin t$

$\frac{dy}{dx} = \frac{4\cos t}{-3\sin t} = -\frac{4}{3}\cot t$

$\frac{dy}{dx}(\pi/4) = -\frac{4}{3}\cot \pi/4 = -4/3$

$y - \frac{4}{\sqrt{2}} = -\frac{4}{3}(x - \frac{3}{\sqrt{2}})$

c. Find $\frac{d^2y}{dx^2}$ and $\frac{d^2y}{dx^2}$ $t = \pi/4$, and relate that value to curve in a.)

$\frac{\frac{d}{dt}\left(-\frac{4}{3}\cot t\right)}{-3\sin t} = \frac{\frac{4}{3}\csc^2 t}{-3\sin t} = \frac{-4}{9\sin^3 t}$ at $t = \pi/4$ $\frac{-4}{9(\frac{1}{\sqrt{2}})^3} < 0$

$\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

$x = 3\cos t$
$y = 4\sin t$ in xy plane concave down @ $t = \pi/4$

2. $\vec{r} = (3\cos t)\,\hat{i} + (4\sin t)\,\hat{j}$ $\quad t \in$ seconds, $\vec{r} \in$ meters

a.) Find $\vec{v}(t)$, $\vec{a}(t)$, speed in terms of t.

$\vec{v} = (-3\sin t)\,\hat{i} + (4\cos t)\,\hat{j}$ $\qquad \vec{a} = (-3\cos t, -4\sin t)$

$|\vec{v}| = \sqrt{9\sin^2 t + 16\cos^2 t}$

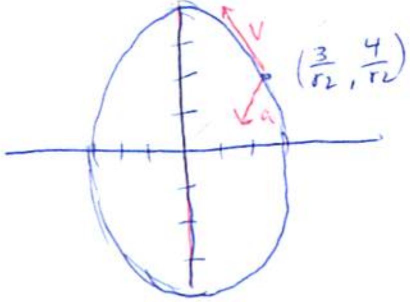
b.) Find the velocity vector, acceleration vector and speed at $t = \pi/4$

$\vec{v}(\pi/4) = \left(\frac{-3}{\sqrt{2}}, \frac{4}{\sqrt{2}}\right) \qquad \vec{a} = \left(\frac{-3}{\sqrt{2}}, \frac{-4}{\sqrt{2}}\right) \qquad \text{speed}(\pi/4) = \sqrt{\left(\frac{-3}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2}$

$\sqrt{\frac{25}{2}} = \frac{5}{\sqrt{2}}$

$\frac{9+16}{2}$

c.) Sketch $\vec{r}$ for $0 \leq t \leq 2\pi$ and sketch $\vec{v}(\pi/4)$ and $\vec{a}(\pi/4)$ at $\left(\frac{3}{\sqrt{2}}, \frac{4}{\sqrt{2}}\right)$



3. $x = 3\cos t$ $t \in$ seconds, $x, y \in$ meters

$y = 4\sin t$

a.) Find arclength of the curve from $t \in [0, \pi/4]$

$\frac{dx}{dt} = -3\sin t$ $\frac{dy}{dt} = 4\cos t$

$$\int_0^{\pi/4} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \cdot \int_0^{\pi/4} \sqrt{9\sin^2 t + 16\cos^2 t}\ dt \qquad 3.012\text{ m}$$

b.) Using 2a.) find the total distance traveled by using the formula

$$\int_a^b |v(t)\, dt| \quad \text{for } t \in [0, \pi/4]$$

$$\int_0^{\pi/4} \left|(-3\sin t)\,i + (4\cos t)\,j\right|\, dt$$

$$\int_0^{\pi/4} \sqrt{(-3\sin t)^2 + (4\cos^2 t)}\ dt$$

3.012 m