

67. Let C = circumference, r = radius, S = surface area, and V = volume.

(a) Since $C = 2\pi r$, we have $\frac{dC}{dr} = 2\pi$ and so $dC = 2\pi dr$.

Therefore, $\left|\frac{dC}{C}\right| = \left|\frac{2\pi dr}{2\pi r}\right| = \left|\frac{dr}{r}\right| < \frac{0.4 \text{ cm}}{10 \text{ cm}} = 0.04$. The calculated radius will be within approximately 4% of the correct radius.

(b) Since $S = 4\pi r^2$, we have $\frac{dS}{dr} = 8\pi r$ and so

$$dS = 8\pi r dr. \text{ Therefore, } \left|\frac{dS}{S}\right| = \left|\frac{8\pi r dr}{4\pi r^2}\right|$$

$$= \left|\frac{2 dr}{r}\right| \leq 2(0.04) = 0.08. \text{ The calculated surface area will be within approximately 8\% of the correct surface area.}$$

(c) Since $V = \frac{4}{3}\pi r^3$, we have $\frac{dV}{dr} = 4\pi r^2$ and so

$$dV = 4\pi r^2 dr. \text{ Therefore } \left|\frac{dV}{V}\right| = \left|\frac{4\pi r^2 dr}{\frac{4}{3}\pi r^3}\right| = \left|\frac{3 dr}{r}\right| \leq 3(0.04) = 0.12.$$

The calculated volume will be within approximately 12% of the correct volume.

68. By similar triangles, we have $\frac{a}{6} = \frac{a+20}{h}$, which gives $ah = 6a + 120$, or $h = 6 + 120a^{-1}$. The height of the lamp post is approximately $6 + 120(15)^{-1} = 14$ ft. The estimated error in measuring a was $|da| \leq 1$ in. $= \frac{1}{12}$ ft. Since $\frac{dh}{da} = -120a^{-2}$, we have $|dh| = |-120a^{-2} da| \leq 120(15)^{-2} \left(\frac{1}{12}\right) = \frac{2}{45}$ ft, so the estimated possible error is $\pm \frac{2}{45}$ ft or $\pm \frac{8}{15}$ in.

69. $\frac{dy}{dx} = 2 \sin x \cos x - 3$. Since $\sin x$ and $\cos x$ are both between 1 and -1 , the value of $2 \sin x \cos x$ is never greater than 2.

Therefore, $\frac{dy}{dx} \leq 2 - 3 = -1$ for all values of x .

Since $\frac{dy}{dx}$ is always negative, the function decreases on every interval.

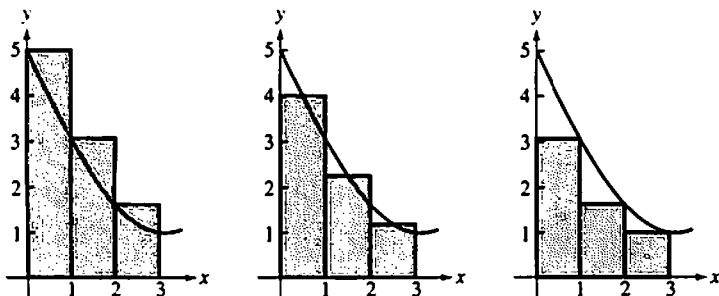
Chapter 5

The Definite Integral

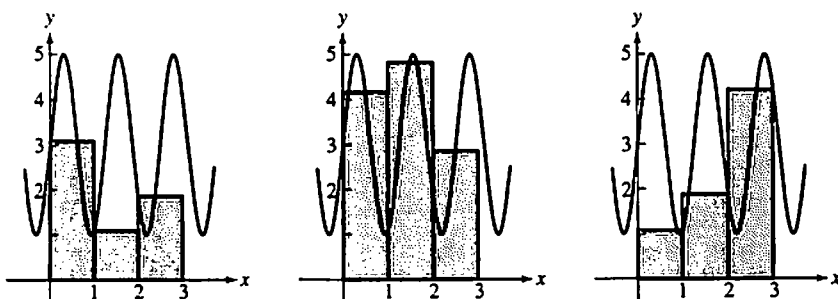
Section 5.1 Estimating with Finite Sums

(pp. 247–257)

Exploration 1 Which Riemann Sum is the Biggest?



1. LRAM > MRAM > RRAM



2. MRAM > RRAM > LRAM

3. RRAM > MRAM > LRAM, because the heights of the rectangles increase as you move toward the right under an increasing function.

4. LRAM > MRAM > RRAM, because the heights of the rectangles decrease as you move toward the right under a decreasing function.

Quick Review 5.1

1. $80 \text{ mph} \cdot 5 \text{ hr} = 400 \text{ mi}$

2. $48 \text{ mph} \cdot 3 \text{ hr} = 144 \text{ mi}$

3. $10 \text{ ft/sec}^2 \cdot 10 \text{ sec} = 100 \text{ ft/sec}$

$$100 \text{ ft/sec} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \cdot \frac{3600 \text{ sec}}{1 \text{ h}} \approx 68.18 \text{ mph}$$

4. $300,000 \text{ km/sec} \cdot \frac{3600 \text{ sec}}{1 \text{ hr}} \cdot \frac{24 \text{ hr}}{1 \text{ day}} \cdot \frac{365 \text{ days}}{1 \text{ yr}} \cdot 1 \text{ yr}$
 $\approx 9.46 \times 10^{12} \text{ km}$

5. $(6 \text{ mph})(3 \text{ h}) + (5 \text{ mph})(2 \text{ h}) = 18 \text{ mi} + 10 \text{ mi} = 28 \text{ mi}$

6. $20 \text{ gal/min} \cdot 1 \text{ h} \cdot \frac{60 \text{ min}}{1 \text{ h}} = 1200 \text{ gal}$

7. $(-1^\circ\text{C/h})(12 \text{ h}) + (1.5^\circ\text{C})(6 \text{ h}) = -3^\circ\text{C}$

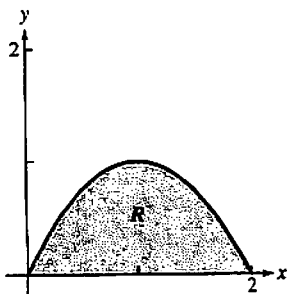
8. $300 \text{ ft}^3/\text{sec} \cdot \frac{3600 \text{ sec}}{1 \text{ h}} \cdot \frac{24 \text{ h}}{1 \text{ day}} \cdot 1 \text{ day} = 25,920,000 \text{ ft}^3$

9. $350 \text{ people/mi}^2 \cdot 50 \text{ mi}^2 = 17,500 \text{ people}$

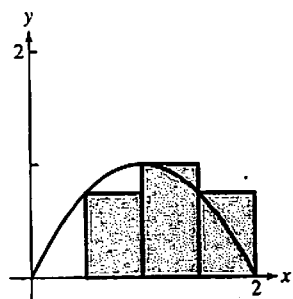
10. $70 \text{ times/sec} \cdot \frac{3600 \text{ sec}}{1 \text{ h}} \cdot 1 \text{ h} \cdot 0.7 = 176,400 \text{ times}$

Section 5.1 Exercises

1. (a)



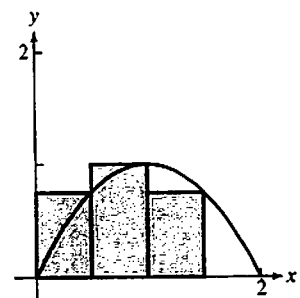
(b)



$$\Delta x = \frac{1}{2}$$

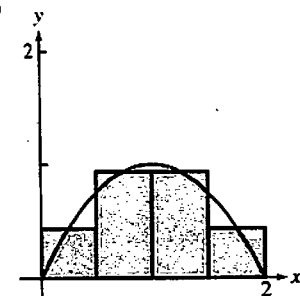
$$\text{LRAM: } [2(0) - (0)^2]\left(\frac{1}{2}\right) + \left[2\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^2\right]\left(\frac{1}{2}\right) + [2(1) - (1)^2]\left(\frac{1}{2}\right) + \left[2\left(\frac{3}{2}\right) - \left(\frac{3}{2}\right)^2\right]\left(\frac{1}{2}\right) = \frac{5}{4} = 1.25$$

2. (a)



$$\text{RRAM: } \left[2\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^2\right]\left(\frac{1}{2}\right) + [2(1) - (1)^2]\left(\frac{1}{2}\right) + \left[2\left(\frac{3}{2}\right) - \left(\frac{3}{2}\right)^2\right]\left(\frac{1}{2}\right) + [2(2) - (2)^2]\left(\frac{1}{2}\right) = \frac{5}{4} = 1.25$$

(b)



$$\text{MRAM: } \left[2\left(\frac{1}{4}\right) - \left(\frac{1}{4}\right)^2\right]\left(\frac{1}{2}\right) + \left[2\left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2\right]\left(\frac{1}{2}\right) + \left[2\left(\frac{5}{4}\right) - \left(\frac{5}{4}\right)^2\right]\left(\frac{1}{2}\right) + \left[2\left(\frac{7}{4}\right) - \left(\frac{7}{4}\right)^2\right]\left(\frac{1}{2}\right) = \frac{11}{8} = 1.375$$

3.

n	LRAM_n	MRAM_n	RRAM_n
10	1.32	1.34	1.32
50	1.3328	1.3336	1.3328
100	1.3332	1.3334	1.3332
500	1.333328	1.333336	1.333328

 4. The area is $1.\overline{333} = \frac{4}{3}$.

5.

n	LRAM $_n$	MRAM $_n$	RRAM $_n$
10	12.645	13.4775	14.445
50	13.3218	13.4991	13.6818
100	13.41045	13.499775	13.59045
500	13.482018	13.499991	13.518018

Estimate the area to be 13.5.

6.

n	LRAM $_n$	MRAM $_n$	RRAM $_n$
10	1.16823	1.09714	1.03490
50	1.11206	1.09855	1.08540
100	1.10531	1.09860	1.09198
500	1.09995	1.09861	1.09728
1000	1.09928	1.09861	1.09795

Estimate the area to be 1.0986.

7.

n	LRAM $_n$	MRAM $_n$	RRAM $_n$
10	0.98001	0.88220	0.78367
50	0.90171	0.88209	0.86244
100	0.89190	0.88208	0.87226
500	0.88404	0.88208	0.88012
1000	0.88306	0.88208	0.88110

Estimate the area to be 0.8821.

8.

n	LRAM $_n$	MRAM $_n$	RRAM $_n$
10	1.98352	2.00825	1.98352
50	1.99934	2.00033	1.99934
100	1.99984	2.00008	1.99984
500	1.99999	2.00000	1.99999

Estimate the area to be 2.

9. LRAM:

$$\begin{aligned} \text{Area} &\approx f(2) \cdot 2 + f(4) \cdot 2 + f(6) \cdot 2 + \dots + f(22) \cdot 2 \\ &= 2 \cdot (0 + 0.6 + 1.4 + \dots + 0.5) \\ &= 44.8 \text{ (mg/L)} \cdot \text{sec} \end{aligned}$$

RRAM:

$$\begin{aligned} \text{Area} &\approx f(4) \cdot 2 + f(6) \cdot 2 + f(8) \cdot 2 + \dots + f(24) \cdot 2 \\ &= 2(0.6 + 1.4 + 2.7 + \dots + 0) \\ &= 44.8 \text{ (mg/L)} \cdot \text{sec} \end{aligned}$$

Patient's cardiac output:

$$\frac{5 \text{ mg}}{44.8 \text{ (mg/L)} \cdot \text{sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} \approx 6.7 \text{ L/min}$$

Note that estimates for the area may vary.

10. (a) LRAM: $1 \cdot (0 + 12 + 22 + 10 + 5 + 13 + 11 + 6 + 2 + 6) = 87 \text{ in.} = 7.25 \text{ ft}$

(b) RRAM: $1 \cdot (12 + 22 + 10 + 5 + 13 + 11 + 6 + 2 + 6 + 0) = 87 \text{ in.} = 7.25 \text{ ft}$

11. 5 min = 300 sec

(a) LRAM: $300 \cdot (1 + 1.2 + 1.7 + \dots + 1.2) = 5220 \text{ m}$

(b) RRAM: $300 \cdot (1.2 + 1.7 + 2.0 + \dots + 0) = 4920 \text{ m}$

12. LRAM: $10 \cdot (0 + 44 + 15 + \dots + 30) = 3490 \text{ ft}$

RRAM: $10 \cdot (44 + 15 + 35 + \dots + 35) = 3840 \text{ ft}$

$$\text{Average} = \frac{3490 \text{ ft} + 3840 \text{ ft}}{2} = 3665 \text{ ft}$$

13. (a) LRAM: $0.001(0 + 40 + 62 + \dots + 137) = 0.898 \text{ mi}$

RRAM: $0.001(40 + 62 + 82 + \dots + 142) = 1.04 \text{ mi}$

Average = 0.969 mi

(b) The halfway point is 0.4845 mi. The average of LRAM and RRAM is 0.4460 at 0.006 h and 0.5665 at 0.007 h. Estimate that it took 0.006 h = 21.6 sec. The car was going 116 mph.

14. Use $f(x) = \sqrt{25 - x^2}$ and approximate the volume using $\pi r^2 h = \pi(\sqrt{25 - n_i^2})^2 \Delta x$, so for the MRAM program, use $\pi(25 - x^2)$ on the interval $[-5, 5]$.

n	MRAM
10	526.21677
20	524.25327
40	523.76240
80	523.63968
160	523.60900

15. $V = \frac{4}{3}\pi(5)^3 = \frac{500\pi}{3} \approx 523.59878$

n	error	% error
10	2.61799	0.5
20	0.65450	0.125
40	0.16362	0.0312
80	0.04091	0.0078
160	0.01023	0.0020

16. (a) Use LRAM with $\pi(16 - x^2)$.

$$S_8 \approx 146.08406$$

S_8 is an overestimate because each rectangle is above the curve.

(b) $\frac{|V - S_8|}{V} \approx 0.09 = 9\%$

17. (a) Use RRAM with $\pi(16 - x^2)$.

$$S_8 \approx 120.95132$$

S_8 is an underestimate because each rectangle is below the curve.

(b) $\frac{|V - S_8|}{V} \approx 0.10 = 10\%$

18. (a) Use LRAM with $\pi(64 - x^2)$ on the interval $[4, 8]$, $n = 8$.

$$S \approx 372.27873 \text{ m}^3$$

(b) $\frac{|V - S_8|}{V} \approx 0.11 = 11\%$

19. (a) $(5)(6.0 + 8.2 + 9.1 + \dots + 12.7)(30) \approx 15,465 \text{ ft}^3$

(b) $(5)(8.2 + 9.1 + 9.9 + \dots + 13.0)(30) \approx 16,515 \text{ ft}^3$

20. Use LRAM with πx on the interval $[0, 5]$, $n = 5$.

$$1(0 + \pi + 2\pi + 3\pi + 4\pi) = 10\pi \approx 31.41593$$

21. Use MRAM with πx on the interval $[0, 5]$, $n = 5$.

$$1\left(\frac{1}{2}\pi + \frac{3}{2}\pi + \frac{5}{2}\pi + \frac{7}{2}\pi + \frac{9}{2}\pi\right) = \frac{25}{2}\pi \approx 39.26991$$

22. (a) LRAM₅:

$$32.00 + 19.41 + 11.77 + 7.14 + 4.33 = 74.65 \text{ ft/sec}$$

- (b) RRAM₅:

$$19.41 + 11.77 + 7.14 + 4.33 + 2.63 = 45.28 \text{ ft/sec}$$

- (c) The upper estimates for speed are 32.00 ft/sec for the first sec, $32.00 + 19.41 = 51.41$ ft/sec for the second sec, and $32.00 + 19.41 + 11.77 = 63.18$ ft/sec for the third sec. Therefore, an upper estimate for the distance fallen is $32.00 + 51.41 + 63.18 = 146.59$ ft.

23. (a) $400 \text{ ft/sec} - (5 \text{ sec})(32 \text{ ft/sec}^2) = 240 \text{ ft/sec}$

- (b) Use RRAM with $400 - 32x$ on $[0, 5]$, $n = 5$.

$$368 + 336 + 304 + 272 + 240 = 1520 \text{ ft}$$

24. (a) Upper = $70 + 97 + 136 + 190 + 265 = 758$ gal

$$\text{Lower} = 50 + 70 + 97 + 136 + 190 = 543 \text{ gal}$$

- (b) Upper = $70 + 97 + 136 + 190 + 265 + 369 + 516 + 720 = 2363$ gal

$$\text{Lower} = 50 + 70 + 97 + 136 + 190 + 265 + 369 + 516 = 1693 \text{ gal}$$

- (c) $25,000 - 2363 = 22,637$ gal

$$\frac{22,637}{720} \approx 31.44 \text{ h (worst case)}$$

$$25,000 - 1693 = 23,307 \text{ gal}$$

$$\frac{23,307}{720} \approx 32.37 \text{ h (best case)}$$

25. (a) Since the release rate of pollutants is increasing, an upper estimate is given by using the data for the end of each month (right rectangles), assuming that new scrubbers were installed before the beginning of January. Upper estimate:

$$30(0.20 + 0.25 + 0.27 + 0.34 + 0.45 + 0.52)$$

$$\approx 60.9 \text{ tons of pollutants}$$

A lower estimate is given by using the data for the end of the previous month (left rectangles). We have no data for the beginning of January, but we know that pollutants were released at the new-scrubber rate of 0.05 ton/day, so we may use this value.

Lower estimate:

$$30(0.05 + 0.20 + 0.25 + 0.27 + 0.34 + 0.45)$$

$$\approx 46.8 \text{ tons of pollutants}$$

- (b) Using left rectangles, the amount of pollutants released by the end of October is

$$30(0.05 + 0.20 + 0.25 + 0.27 + 0.34 + 0.45$$

$$+ 0.52 + 0.63 + 0.70 + 0.81) \approx 126.6 \text{ tons.}$$

Therefore, a total of 125 tons will have been released into the atmosphere by the end of October.

26. The area of the region is the total number of units sold, in millions, over the 10-year period. The area units are (millions of units per year)(years) = (millions of units).

27. (a) The diagonal of the square has length 2, so the side length is $\sqrt{2}$. Area = $(\sqrt{2})^2 = 2$

- (b) Think of the octagon as a collection of 16 right

triangles with a hypotenuse of length 1 and an acute

angle measuring $\frac{2\pi}{16} = \frac{\pi}{8}$.

$$\text{Area} = 16\left(\frac{1}{2}\right)\left(\sin \frac{\pi}{8}\right)\left(\cos \frac{\pi}{8}\right)$$

$$= 4 \sin \frac{\pi}{4}$$

$$= 2\sqrt{2} \approx 2.828$$

- (c) Think of the 16-gon as a collection of 32 right triangles

with a hypotenuse of length 1 and an acute angle

measuring $\frac{2\pi}{32} = \frac{\pi}{16}$.

$$\text{Area} = 32\left(\frac{1}{2}\right)\left(\sin \frac{\pi}{16}\right)\left(\cos \frac{\pi}{16}\right)$$

$$= 8 \sin \frac{\pi}{8} \approx 3.061$$

- (d) Each area is less than the area of the circle, π . As n increases, the area approaches π .

28. The statement is false. We disprove it by presenting a

counterexample, the function $f(x) = x^2$ over the interval

$$0 \leq x \leq 1, \text{ with } n = 1. \text{ MRAM}_1 = 1f(0.5) = 0.25$$

$$\frac{\text{LRAM}_1 + \text{RRAM}_1}{2} = \frac{1f(0) + 1f(1)}{2}$$

$$= \frac{0 + 1}{2} = 0.5 \neq \text{MRAM}_1$$

29. $\text{RRAM}_n f = (\Delta x)[f(x_1) + f(x_2) + \cdots + f(x_{n-1}) + f(x_n)]$
 $= (\Delta x)[f(x_0) + f(x_1) + f(x_2) + \cdots + f(x_{n-1})]$
 $+ (\Delta x)[f(x_n) - f(x_0)]$
 $= \text{LRAM}_n f + (\Delta x)[f(x_n) - f(x_0)]$

But $f(a) = f(b)$ by symmetry, so $f(x_n) - f(x_0) = 0$.

Therefore, $\text{RRAM}_n f = \text{LRAM}_n f$.

30. (a) Each of the isosceles triangles is made up of two right

triangles having hypotenuse 1 and an acute angle

measuring $\frac{2\pi}{2n} = \frac{\pi}{n}$. The area of each isosceles triangle

$$\text{is } A_T = 2\left(\frac{1}{2}\right)\left(\sin \frac{\pi}{n}\right)\left(\cos \frac{\pi}{n}\right) = \frac{1}{2} \sin \frac{2\pi}{n}.$$

- (b) The area of the polygon is

$$A_P = nA_T = \frac{n}{2} \sin \frac{2\pi}{n}, \text{ so}$$

$$\lim_{n \rightarrow \infty} A_P = \lim_{n \rightarrow \infty} \frac{n}{2} \sin \frac{2\pi}{n} = \pi$$

- (c) Multiply each area by r^2 :

$$A_T = \frac{1}{2}r^2 \sin \frac{2\pi}{n}$$

$$A_P = \frac{n}{2}r^2 \sin \frac{2\pi}{n}$$

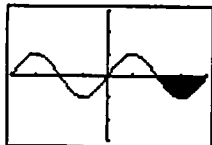
$$\lim_{n \rightarrow \infty} A_P = \pi r^2$$

Section 5.2 Definite Integrals

(pp. 258–268)

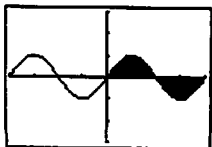
Exploration 1 Finding Integrals by Signed Areas

1. -2 . (This is the same area as $\int_0^\pi \sin x \, dx$, but below the x -axis.)



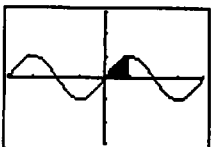
$[-2\pi, 2\pi]$ by $[-3, 3]$

2. 0 . (The equal areas above and below the x -axis sum to zero.)



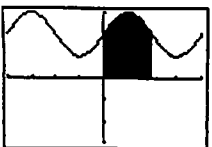
$[-2\pi, 2\pi]$ by $[-3, 3]$

3. 1 . (This is half the area of $\int_0^\pi \sin x \, dx$.)



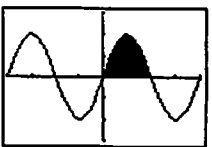
$[-2\pi, 2\pi]$ by $[-3, 3]$

4. $2\pi + 2$. (The same area as $\int_0^\pi \sin x \, dx$ sits above a rectangle of area $\pi \times 2$.)



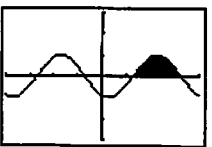
$[-2\pi, 2\pi]$ by $[-3, 3]$

5. 4 . (Each rectangle in a typical Riemann sum is twice as tall as in $\int_0^\pi \sin x \, dx$.)



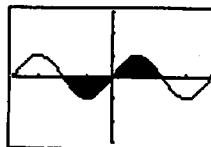
$[-2\pi, 2\pi]$ by $[-3, 3]$

6. 2 . (This is the same region as in $\int_0^\pi \sin x \, dx$, translated 2 units to the right.)



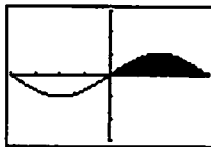
$[-2\pi, 2\pi]$ by $[-3, 3]$

7. 0 . (The equal areas above and below the x -axis sum to zero.)



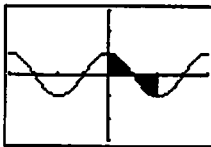
$[-2\pi, 2\pi]$ by $[-3, 3]$

8. 4 . (Each rectangle in a typical Riemann sum is twice as wide as in $\int_0^\pi \sin x \, dx$.)



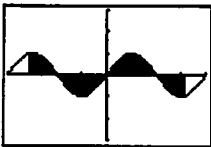
$[-2\pi, 2\pi]$ by $[-3, 3]$

9. 0 . (The equal areas above and below the x -axis sum to zero.)



$[-2\pi, 2\pi]$ by $[-3, 3]$

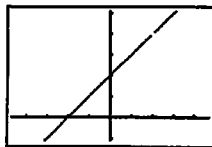
10. 0 . (The equal areas above and below the x -axis sum to zero, since $\sin x$ is an odd function.)



$[-2\pi, 2\pi]$ by $[-3, 3]$

Exploration 2 More Discontinuous Integrand

1. The function has a removable discontinuity at $x = 2$.



$[-4.7, 4.7]$ by $[-1.1, 5.1]$

2. The thin strip above $x = 2$ has zero area, so the area under the curve is the same as $\int_0^3 (x + 2) \, dx$, which is 10.5.



$[-4.7, 4.7]$ by $[-1.1, 5.1]$

3. The graph has jump discontinuities at all integer values, but the Riemann sums tend to the area of the shaded region shown. The area is the sum of the areas of 5 rectangles (one of them with height 0):

$$\int_0^5 \text{int}(x) dx = 0 + 1 + 2 + 3 + 4 = 10.$$



$f(x) = \text{int}(x)$
 $[-2.7, 6.7]$ by $[-1.1, 5.1]$

Quick Review 5.2

- $\sum_{n=1}^5 n^2 = (1)^2 + (2)^2 + (3)^2 + (4)^2 + (5)^2 = 55$
- $\sum_{k=0}^4 (3k - 2) = [3(0) - 2] + [3(1) - 2] + [3(2) - 2] + [3(3) - 2] + [3(4) - 2] = 20$
- $\sum_{j=0}^4 100(j + 1)^2 = 100[(1)^2 + (2)^2 + (3)^2 + (4)^2 + (5)^2] = 5500$
- $\sum_{k=1}^{99} k$
- $\sum_{k=0}^{25} 2k$
- $\sum_{k=1}^{500} 3k^2$
- $2 \sum_{x=1}^{50} x^2 + 3 \sum_{x=1}^{50} x = \sum_{x=1}^{50} (2x^2 + 3x)$
- $\sum_{k=0}^8 x^k + \sum_{k=9}^{20} x^k = \sum_{k=0}^{20} x^k$
- $\sum_{k=0}^n (-1)^k = 0$ if n is odd.
- $\sum_{k=0}^n (-1)^k = 1$ if n is even.

Section 5.2 Exercises

- $\lim_{\|P\| \rightarrow 0} \sum_{k=1}^n c_k^2 \Delta x_k = \int_0^2 x^2 dx$ where P is any partition of $[0, 2]$.
- $\lim_{\|P\| \rightarrow 0} \sum_{k=1}^n (c_k^2 - 3c_k) \Delta x_k = \int_{-7}^5 (x^2 - 3x) dx$ where P is any partition of $[-7, 5]$.
- $\lim_{\|P\| \rightarrow 0} \sum_{k=1}^n \frac{1}{c_k} \Delta x_k = \int_1^4 \frac{1}{x} dx$ where P is any partition of $[1, 4]$.
- $\lim_{\|P\| \rightarrow 0} \sum_{k=1}^n \frac{1}{1 - c_k} \Delta x_k = \int_2^3 \frac{1}{1 - x} dx$ where P is any partition of $[2, 3]$.

5. $\lim_{\|P\| \rightarrow 0} \sum_{k=1}^n \sqrt{4 - c_k^2} \Delta x_k = \int_0^1 \sqrt{4 - x^2} dx$ where P is any partition of $[0, 1]$.

6. $\lim_{\|P\| \rightarrow 0} \sum_{k=1}^n (\sin^3 c_k) \Delta x_k = \int_{-\pi}^{\pi} \sin^3 x dx$ where P is any partition of $[-\pi, \pi]$.

7. $\int_{-2}^1 5 dx = 5[1 - (-2)] = 15$

8. $\int_3^7 (-20) dx = (-20)(7 - 3) = -80$

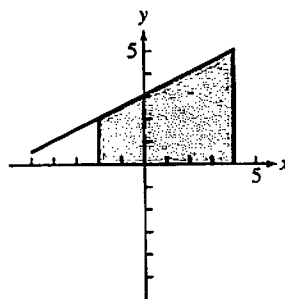
9. $\int_0^3 (-160) dt = (-160)(3 - 0) = -480$

10. $\int_{-4}^{-1} \frac{\pi}{2} d\theta = \frac{\pi}{2}[-1 - (-4)] = \frac{3\pi}{2}$

11. $\int_{-2.1}^{3.4} 0.5 ds = 0.5[3.4 - (-2.1)] = 2.75$

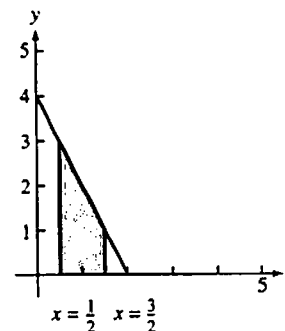
12. $\int_{\sqrt{2}}^{\sqrt{18}} \sqrt{2} dr = \sqrt{2}(\sqrt{18} - \sqrt{2}) = 4$

13. Graph the region under $y = \frac{x}{2} + 3$ for $-2 \leq x \leq 4$.



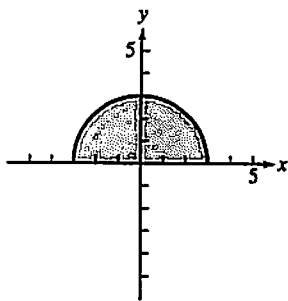
$$\int_{-2}^4 \left(\frac{x}{2} + 3\right) dx = \frac{1}{2}(6)(2 + 5) = 21$$

14. Graph the region under $y = -2x + 4$ for $\frac{1}{2} \leq x \leq \frac{3}{2}$.



$$\int_{1/2}^{3/2} (-2x + 4) dx = \frac{1}{2}(1)(3 + 1) = 2$$

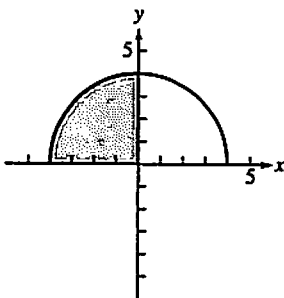
15. Graph the region under
- $y = \sqrt{9 - x^2}$
- for
- $-3 \leq x \leq 3$
- .



This region is half of a circle of radius 3.

$$\int_{-3}^3 \sqrt{9 - x^2} dx = \frac{1}{2}\pi(3)^2 = \frac{9\pi}{2}$$

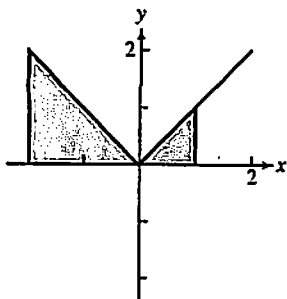
16. Graph the region under
- $y = \sqrt{16 - x^2}$
- for
- $-4 \leq x \leq 0$
- .



The region is one quarter of a circle of radius 4.

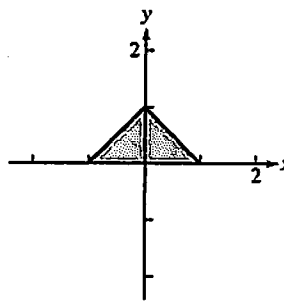
$$\int_{-4}^0 \sqrt{16 - x^2} dx = \frac{1}{4}\pi(4)^2 = 4\pi$$

17. Graph the region under
- $y = |x|$
- for
- $-2 \leq x \leq 1$
- .



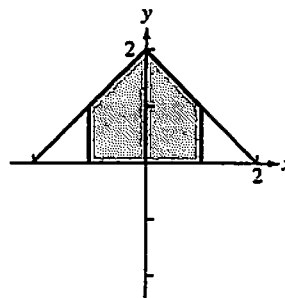
$$\int_{-2}^1 |x| dx = \frac{1}{2}(2)(2) + \frac{1}{2}(1)(1) = \frac{5}{2}$$

18. Graph the region under
- $y = 1 - |x|$
- for
- $-1 \leq x \leq 1$
- .



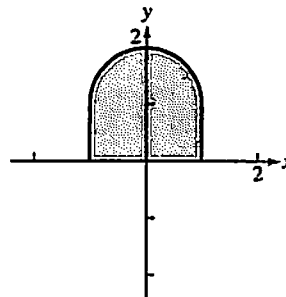
$$\int_{-1}^1 (1 - |x|) dx = \frac{1}{2}(2)(1) = 1$$

19. Graph the region under
- $y = 2 - |x|$
- for
- $-1 \leq x \leq 1$
- .



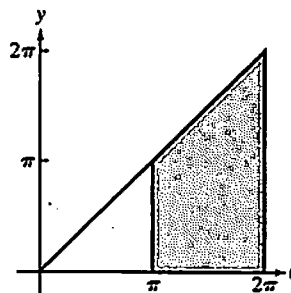
$$\int_{-1}^1 (2 - |x|) dx = \frac{1}{2}(1)(1 + 2) + \frac{1}{2}(1)(1 + 2) = 3$$

20. Graph the region under
- $y = 1 + \sqrt{1 - x^2}$
- for
- $-1 \leq x \leq 1$
- .



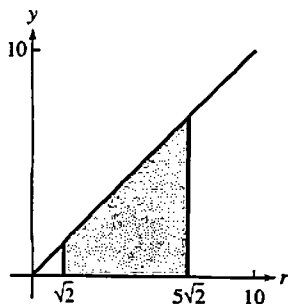
$$\int_{-1}^1 (1 + \sqrt{1 - x^2}) dx = (2)(1) + \frac{1}{2}\pi(1)^2 = 2 + \frac{\pi}{2}$$

21. Graph the region under
- $y = \theta$
- for
- $\pi \leq \theta \leq 2\pi$
- .



$$\int_{\pi}^{2\pi} \theta d\theta = \frac{1}{2}(2\pi - \pi)(2\pi + \pi) = \frac{3\pi^2}{2}$$

22. Graph the region under $y = r$ for $\sqrt{2} \leq r \leq 5\sqrt{2}$.



$$\int_{\sqrt{2}}^{5\sqrt{2}} r \, dr = \frac{1}{2}(5\sqrt{2} - \sqrt{2})(\sqrt{2} + 5\sqrt{2}) = 24$$

23. $\int_0^b x \, dx = \frac{1}{2}(b)(b) = \frac{1}{2}b^2$

24. $\int_0^b 4x \, dx = \frac{1}{2}(b)(4b) = 2b^2$

25. $\int_a^b 2s \, ds = \frac{1}{2}(b-a)(2b+2a) = b^2 - a^2$

26. $\int_a^b 3t \, dt = \frac{1}{2}(b-a)(3b+3a) = \frac{3}{2}(b^2 - a^2)$

27. $\int_a^{2a} x \, dx = \frac{1}{2}(2a-a)(2a+a) = \frac{3a^2}{2}$

28. $\int_a^{\sqrt{3}a} x \, dx = \frac{1}{2}(\sqrt{3}a-a)(\sqrt{3}a+a) = \frac{1}{2}(3a^2 - a^2) = a^2$

29. Observe that the graph of $f(x) = x^3$ is symmetric with respect to the origin. Hence the area above and below the x -axis is equal for $-1 \leq x \leq 1$.

$$\int_{-1}^1 x^3 \, dx = -(\text{area below } x\text{-axis}) + (\text{area above } x\text{-axis}) = 0$$

30. The graph of $f(x) = x^3 + 3$ is three units higher than the graph of $g(x) = x^3$. The extra area is $(3)(1) = 3$.

$$\int_0^1 (x^3 + 3) \, dx = \frac{1}{4} + 3 = \frac{13}{4}$$

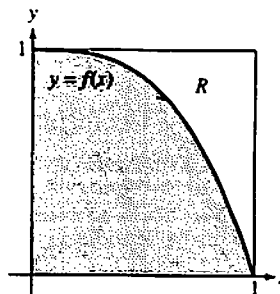
31. Observe that the region under the graph of $f(x) = (x-2)^3$ for $2 \leq x \leq 3$ is just the region under the graph of $g(x) = x^3$ for $0 \leq x \leq 1$ translated two units to the right.

$$\int_2^3 (x-2)^3 \, dx = \int_0^1 x^3 \, dx = \frac{1}{4}$$

32. Observe that the graph of $f(x) = |x|^3$ is symmetric with respect to the y -axis and the right half is the graph of $g(x) = x^3$.

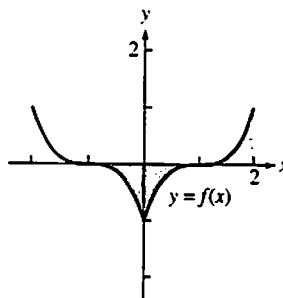
$$\int_{-1}^1 |x|^3 \, dx = 2 \int_0^1 x^3 \, dx = \frac{1}{2}$$

33. Observe from the graph below that the region under the graph of $f(x) = 1 - x^3$ for $0 \leq x \leq 1$ cuts out a region R from the square identical to the region under the graph of $g(x) = x^3$ for $0 \leq x \leq 1$.



$$\int_0^1 (1 - x^3) \, dx = 1 - \int_0^1 x^3 \, dx = 1 - \frac{1}{4} = \frac{3}{4}$$

34. Observe from the graph of $f(x) = (|x| - 1)^3$ for $-1 \leq x \leq 2$ that there are two regions below the x -axis and one region above the axis, each of whose area is equal to the area of the region under the graph of $g(x) = x^3$ for $0 \leq x \leq 1$.



$$\int_{-1}^2 (|x| - 1)^3 \, dx = 2\left(-\frac{1}{4}\right) + \left(\frac{1}{4}\right) = -\frac{1}{4}$$

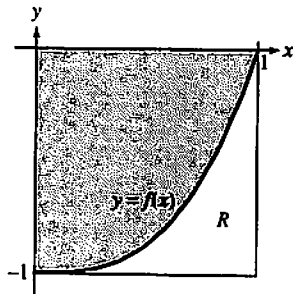
35. Observe that the graph of $f(x) = \left(\frac{x}{2}\right)^3$ for $0 \leq x \leq 2$ is just a horizontal stretch of the graph of $g(x) = x^3$ for $0 \leq x \leq 1$ by a factor of 2. Thus the area under $f(x) = \left(\frac{x}{2}\right)^3$ for $0 \leq x \leq 2$ is twice the area under the graph of $g(x) = x^3$ for $0 \leq x \leq 1$.

$$\int_0^2 \left(\frac{x}{2}\right)^3 \, dx = 2 \int_0^1 x^3 \, dx = \frac{1}{2}$$

36. Observe that the graph of $f(x) = x^3$ is symmetric with respect to the origin. Hence the area above and below the x -axis is equal for $-8 \leq x \leq 8$.

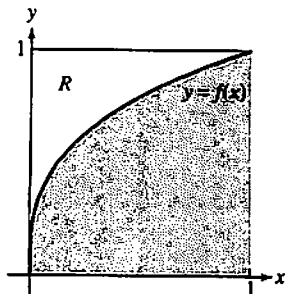
$$\int_{-8}^8 x^3 \, dx = -(\text{area below } x\text{-axis}) + (\text{area above } x\text{-axis}) = 0$$

37. Observe from the graph below that the region between the graph of $f(x) = x^3 - 1$ and the x -axis for $0 \leq x \leq 1$ cuts out a region R from the square identical to the region under the graph of $g(x) = x^3$ for $0 \leq x \leq 1$.



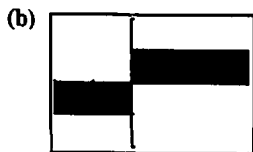
$$\int_0^1 (x^3 - 1) dx = -1 + \frac{1}{4} = -\frac{3}{4}$$

38. Observe from the graph below that the region between the graph of $f(x) = \sqrt[3]{x}$ and the x -axis for $0 \leq x \leq 1$ cuts out a region R from the square identical to the region under the graph of $g(x) = x^3$ for $0 \leq x \leq 1$.



$$\int_0^1 \sqrt[3]{x} dx = 1 - \frac{1}{4} = \frac{3}{4}$$

39. $\text{NINT}\left(\frac{x}{x^2 + 4}, x, 0, 5\right) \approx 0.9905$
 40. $3 + 2 \cdot \text{NINT}\left(\tan x, x, 0, \frac{\pi}{3}\right) \approx 4.3863$
 41. $\text{NINT}(4 - x^2, x, -2, 2) \approx 10.6667$
 42. $\text{NINT}(x^2 e^{-x}, x, -1, 3) \approx 1.8719$
 43. (a) The function has a discontinuity at $x = 0$.



$[-2, 3]$ by $[-2, 2]$

$$\int_{-2}^3 \frac{x}{|x|} dx = -2 + 3 = 1$$

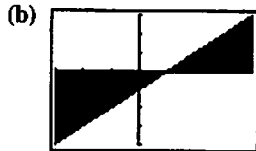
44. (a) The function has discontinuities at $x = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$.



$[-6, 5]$ by $[-18, 4]$

$$\int_{-6}^5 2 \text{int}(x - 3) dx = (-18) + (-16) + (-14) + (-12) + (-10) + (-8) + (-6) + (-4) + (-2) + 0 + 2 = -88$$

45. (a) The function has a discontinuity at $x = -1$.



$[-3, 4]$ by $[-4, 3]$

$$\int_{-3}^4 \frac{x^2 - 1}{x + 1} dx = -\frac{1}{2}(4)(4) + \frac{1}{2}(3)(3) = -\frac{7}{2}$$

46. (a) The function has a discontinuity at $x = 3$.



$[-5, 6]$ by $[-9, 2]$

$$\int_{-5}^6 \frac{9 - x^2}{5x - 3} dx = \frac{1}{2}(2)(2) - \frac{1}{2}(9)(9) = -\frac{77}{2}$$

47. (a) As x approaches 0 from the right, $f(x)$ goes to ∞ .

- (b) Using right endpoints we have

$$\begin{aligned} \int_0^1 \frac{1}{x^2} dx &= \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{1}{\left(\frac{k}{n}\right)^2} \right) \left(\frac{1}{n} \right) \\ &= \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n}{k^2} \\ &= \lim_{n \rightarrow \infty} n \left(1 + \frac{1}{2^2} + \dots + \frac{1}{n^2} \right). \end{aligned}$$

Note that $n\left(1 + \frac{1}{2^2} + \dots + \frac{1}{n^2}\right) > n$ and $n \rightarrow \infty$, so

$$n\left(1 + \frac{1}{2^2} + \dots + \frac{1}{n^2}\right) \rightarrow \infty.$$

48. (a) $\Delta x = \frac{1}{n}, x_k = \frac{k}{n}$
 $\text{RRAM} = \left(\frac{1}{n}\right)^2 \cdot \frac{1}{n} + \left(\frac{2}{n}\right)^2 \cdot \frac{1}{n} + \dots + \left(\frac{n}{n}\right)^2 \cdot \frac{1}{n}$
 $= \sum_{k=1}^n \left(\left(\frac{k}{n}\right)^2 \cdot \frac{1}{n} \right)$

(b) $\sum_{k=1}^n \left(\left(\frac{k}{n}\right)^2 \cdot \frac{1}{n} \right) = \sum_{k=1}^n \left(\frac{k^2}{n^3} \right) = \frac{1}{n^3} \sum_{k=1}^n k^2$

(c) $\frac{1}{n^3} \sum_{k=1}^n k^2 = \frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} = \frac{n(n+1)(2n+1)}{6n^3}$

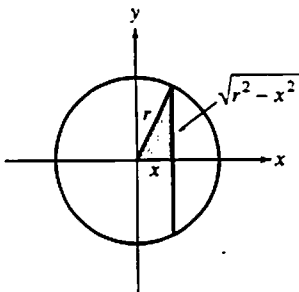
$$\begin{aligned}
 \text{(d)} \quad \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\left(\frac{k}{n} \right)^2 \cdot \frac{1}{n} \right) &= \lim_{n \rightarrow \infty} \frac{n(n+1)(2n+1)}{6n^3} \\
 &= \lim_{n \rightarrow \infty} \frac{2n^3 + 3n^2 + n}{6n^3} \\
 &= \frac{2}{6} = \frac{1}{3}
 \end{aligned}$$

(e) Since $\int_0^1 x^2 dx$ equals the limit of any Riemann sum over the interval $[0, 1]$ as n approaches ∞ , part (d) proves that $\int_0^1 x^2 dx = \frac{1}{3}$.

Section 5.3 Definite Integrals and Antiderivatives (pp. 268–276)

Exploration 1 How Long is the Average Chord of a Circle?

1. The chord is twice as long as the leg of the right triangle in the first quadrant, which has length $\sqrt{r^2 - x^2}$ by the Pythagorean Theorem.



2. Average value = $\frac{1}{r - (-r)} \int_{-r}^r 2\sqrt{r^2 - x^2} dx$.

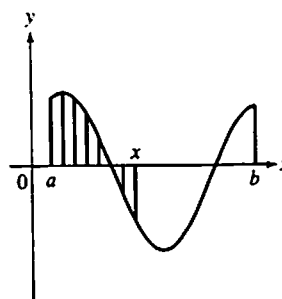
3. Average value = $\frac{2}{2r} \int_{-r}^r \sqrt{r^2 - x^2} dx$
 $= \frac{1}{r} \cdot (\text{area of semicircle of radius } r)$
 $= \frac{1}{r} \cdot \frac{\pi r^2}{2}$
 $= \frac{\pi r}{2}$

4. Although we only computed the average length of chords perpendicular to a particular diameter, the same computation applies to any diameter. The average length of a chord of a circle of radius r is $\frac{\pi r}{2}$.

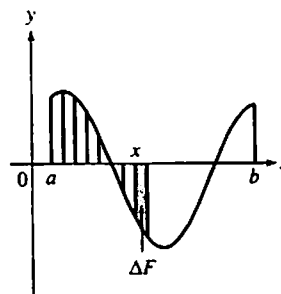
5. The function $y = 2\sqrt{r^2 - x^2}$ is continuous on $[-r, r]$, so the Mean Value Theorem applies and there is a c in $[a, b]$ so that $y(c)$ is the average value $\frac{\pi r}{2}$.

Exploration 2 Finding the Derivative of an Integral

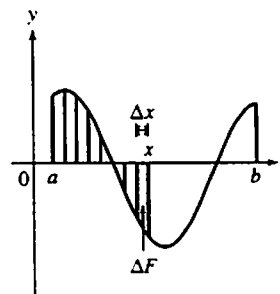
Pictures will vary according to the value of x chosen. (Indeed, this is the point of the exploration.) We show a typical solution here.



1. We have chosen an arbitrary x between a and b .
2. We have shaded the region using vertical line segments.
3. The shaded region can be written as $\int_a^x f(t) dt$ using the definition of the definite integral in Section 5.2. We use t as a dummy variable because x cannot vary between a and itself.
4. The area of the shaded region is our value of $F(x)$.



5. We have drawn one more vertical shading segment to represent ΔF .
6. We have moved x a distance of Δx so that it rests above the new shading segment.



7. Now the (signed) height of the newly-added vertical segment is $f(x)$.
8. The (signed) area of the segment is $\Delta F = \Delta x \cdot f(x)$,

$$\text{so } F'(x) = \lim_{\Delta x \rightarrow 0} \frac{\Delta F}{\Delta x} = f(x)$$

Quick Review 5.3

1. $\frac{dy}{dx} = \sin x$

2. $\frac{dy}{dx} = \cos x$

3. $\frac{dy}{dx} = \frac{\sec x \tan x}{\sec x} = \tan x$

4. $\frac{dy}{dx} = \frac{\cos x}{\sin x} = \cot x$

5. $\frac{dy}{dx} = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} = \sec x$

6. $\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \ln x - 1 = \ln x$

7. $\frac{dy}{dx} = \frac{(n+1)x^n}{n+1} = x^n$

8. $\frac{dy}{dx} = -\frac{1}{(2^x+1)^2} \cdot (\ln 2)2^x = -\frac{2^x \ln 2}{(2^x+1)^2}$

9. $\frac{dy}{dx} = xe^x + e^x$

10. $\frac{dy}{dx} = \frac{1}{x^2+1}$

Section 5.3 Exercises

1. (a) $\int_2^2 g(x) dx = 0$

(b) $\int_5^1 g(x) dx = -\int_1^5 g(x) dx = -8$

(c) $\int_1^2 3f(x) dx = 3\int_1^2 f(x) dx = 3(-4) = -12$

(d) $\int_2^5 f(x) dx = \int_2^1 f(x) dx + \int_1^5 f(x) dx$
 $= -\int_1^2 f(x) dx + \int_1^5 f(x) dx$
 $= 4 + 6 = 10$

(e) $\int_1^5 [f(x) - g(x)] dx = \int_1^5 f(x) dx - \int_1^5 g(x) dx$
 $= 6 - 8 = -2$

(f) $\int_1^5 [4f(x) - g(x)] dx = \int_1^5 4f(x) dx - \int_1^5 g(x) dx$
 $= 4\int_1^5 f(x) dx - \int_1^5 g(x) dx$
 $= 4(6) - 8 = 16$

2. (a) $\int_1^9 -2f(x) dx = -2\int_1^9 f(x) dx = -2(-1) = 2$

(b) $\int_7^9 [f(x) + h(x)] dx = \int_7^9 f(x) dx + \int_7^9 h(x) dx$
 $= 5 + 4 = 9$

(c) $\int_7^9 [2f(x) - 3h(x)] dx = \int_7^9 2f(x) dx - \int_7^9 3h(x) dx$
 $= 2\int_7^9 f(x) dx - 3\int_7^9 h(x) dx$
 $= 2(5) - 3(4) = -2$

(d) $\int_9^1 f(x) dx = -\int_1^9 f(x) dx = 1$

(e) $\int_1^7 f(x) dx = \int_1^9 f(x) dx + \int_9^7 f(x) dx$
 $= \int_1^9 f(x) dx - \int_7^9 f(x) dx$
 $= -1 - 5 = -6$

(f) $\int_9^7 [h(x) - f(x)] dx = \int_9^7 h(x) dx - \int_9^7 f(x) dx$
 $= -\int_7^9 h(x) dx + \int_7^9 f(x) dx$
 $= -4 + 5 = 1$

3. (a) $\int_1^2 f(u) du = 5$

(b) $\int_1^2 \sqrt{3}f(z) dz = \sqrt{3}\int_1^2 f(z) dz = 5\sqrt{3}$

(c) $\int_2^1 f(t) dt = -\int_1^2 f(t) dt = -5$

(d) $\int_1^2 [-f(x)] dx = -\int_1^2 f(x) dx = -5$

4. (a) $\int_0^{-3} g(t) dt = -\int_{-3}^0 g(t) dt = -\sqrt{2}$

(b) $\int_{-3}^0 g(u) du = \sqrt{2}$

(c) $\int_{-3}^0 [-g(x)] dx = -\int_{-3}^0 g(x) dx = -\sqrt{2}$

(d) $\int_{-3}^0 \frac{g(r)}{\sqrt{2}} dr = \frac{1}{\sqrt{2}}\int_{-3}^0 g(r) dr = 1$

5. (a) $\int_3^4 f(z) dz = \int_3^0 f(z) dz + \int_0^4 f(z) dz$
 $= -\int_0^3 f(z) dz + \int_0^4 f(z) dz$
 $= -3 + 7 = 4$

(b) $\int_4^3 f(t) dt = \int_4^0 f(t) dt + \int_0^3 f(t) dt$
 $= -\int_0^4 f(t) dt + \int_0^3 f(t) dt$
 $= -7 + 3 = -4$

6. (a) $\int_1^3 h(r) dr = \int_1^{-1} h(r) dr + \int_{-1}^3 h(r) dr$
 $= -\int_{-1}^1 h(r) dr + \int_{-1}^3 h(r) dr = 6$

(b) $-\int_3^1 h(u) du = -\int_3^{-1} h(u) du - \int_{-1}^1 h(u) du$
 $= \int_{-1}^3 h(u) du - \int_{-1}^1 h(u) du = 6$

7. An antiderivative of 7 is
- $F(x) = 7x$
- .

$$\int_3^1 7 dx = F(1) - F(3) = 7 - 21 = -14$$

8. An antiderivative of
- $5x$
- is
- $F(x) = \frac{5}{2}x^2$

$$\int_0^2 5x dx = F(2) - F(0) = 10 - 0 = 10$$

9. An antiderivative of
- $\frac{x}{8}$
- is
- $F(x) = \frac{x^2}{16}$
- .

$$\int_3^5 \frac{x}{8} dx = F(5) - F(3) = \frac{25}{16} - \frac{9}{16} = 1$$

10. An antiderivative of
- $2t - 3$
- is
- $F(t) = t^2 - 3t$
- .

$$\int_0^2 (2t - 3) dt = F(2) - F(0) = -2 - 0 = -2$$

11. An antiderivative of
- $t - \sqrt{2}$
- is
- $F(t) = \frac{1}{2}t^2 - t\sqrt{2}$
- .

$$\int_0^{\sqrt{2}} (t - \sqrt{2}) dt = F(\sqrt{2}) - F(0) = -1 - 0 = -1$$

12. An antiderivative of
- $1 + \frac{z}{2}$
- is
- $F(z) = z + \frac{z^2}{4}$
- .

$$\int_2^1 \left(1 + \frac{z}{2}\right) dz = F(1) - F(2) = \frac{5}{4} - 3 = -\frac{7}{4}$$

13. An antiderivative of
- $\frac{1}{1+x^2}$
- is
- $F(x) = \tan^{-1} x$
- .

$$\int_{-1}^1 \frac{1}{1+x^2} dx = F(1) - F(-1) = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$$

14. An antiderivative of
- $\frac{1}{\sqrt{1-x^2}}$
- is
- $F(x) = \sin^{-1} x$
- .

$$\int_{-1/2}^{1/2} \frac{dx}{\sqrt{1-x^2}} = F\left(\frac{1}{2}\right) - F\left(-\frac{1}{2}\right) = \frac{\pi}{6} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{3}$$

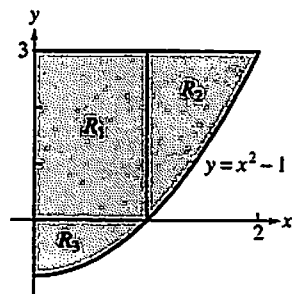
15. An antiderivative of
- e^x
- is
- $F(x) = e^x$
- .

$$\int_0^2 e^x dx = F(2) - F(0) = e^2 - 1 \approx 6.389$$

16. An antiderivative of
- $\frac{3}{x+1}$
- is
- $F(x) = 3 \ln |x+1|$
- .

$$\begin{aligned} \int_0^3 \frac{3 dx}{x+1} &= F(3) - F(0) \\ &= 3 \ln 4 - 0 \\ &= 3 \ln 4 \approx 4.159 \end{aligned}$$

17. Divide the shaded area as follows.



Note that an antiderivative of $x^2 - 1$ is $F(x) = \frac{1}{3}x^3 - x$.

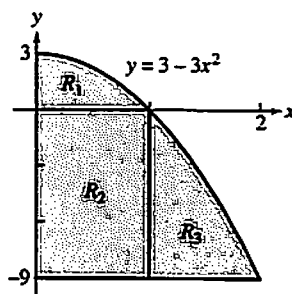
$$\text{Area of } R_1 = 3(1) = 3$$

$$\begin{aligned} \text{Area of } R_2 &= (3)(1) - \int_1^2 (x^2 - 1) dx \\ &= 3 - [F(2) - F(1)] \\ &= 3 - \left[\left(\frac{2}{3}\right) - \left(-\frac{2}{3}\right)\right] = \frac{5}{3} \end{aligned}$$

$$\begin{aligned} \text{Area of } R_3 &= -\int_0^1 (x^2 - 1) dx \\ &= -[F(1) - F(0)] \\ &= -\left[\left(-\frac{2}{3}\right) - 0\right] = \frac{2}{3} \end{aligned}$$

$$\text{Total shaded area} = 3 + \frac{5}{3} + \frac{2}{3} = \frac{16}{3}$$

18. Divide the shaded area as follows.



Note that an antiderivative of $3 - 3x^2$ is $F(x) = 3x - x^3$.

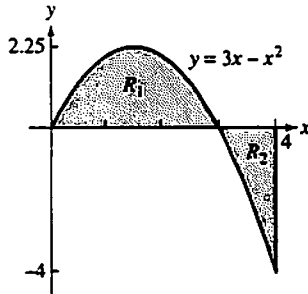
$$\text{Area of } R_1 = \int_0^1 (3 - 3x^2) dx = F(1) - F(0) = 2$$

$$\text{Area of } R_2 = (9)(1) = 9$$

$$\begin{aligned} \text{Area of } R_3 &= (9)(1) + \int_1^2 (3 - 3x^2) dx \\ &= 9 + [F(2) - F(1)] \\ &= 9 + (-2 - 2) = 5 \end{aligned}$$

$$\text{Total shaded area} = 2 + 9 + 5 = 16$$

19. Divide the shaded area as follows.



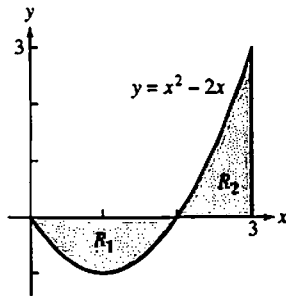
Note that an antiderivative of $3x - x^2$ is $F(x) = \frac{3}{2}x^2 - \frac{1}{3}x^3$.

$$\text{Area of } R_1 = \int_0^3 (3x - x^2) dx = F(3) - F(0) = \frac{9}{2} - 0 = \frac{9}{2}$$

$$\begin{aligned} \text{Area of } R_2 &= -\int_3^4 (3x - x^2) dx \\ &= -[F(4) - F(3)] \\ &= -\left(\frac{8}{3} - \frac{9}{2}\right) = \frac{11}{6} \end{aligned}$$

$$\text{Total shaded area} = \frac{9}{2} + \frac{11}{6} = \frac{19}{3}$$

20. Divide the shaded area as follows.

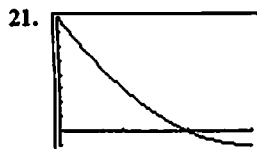


Note that an antiderivative of $x^2 - 2x$ is $F(x) = \frac{1}{3}x^3 - x^2$.

$$\begin{aligned} \text{Area of } R_1 &= -\int_0^2 (x^2 - 2x) dx \\ &= -[F(2) - F(0)] \\ &= -\left(-\frac{4}{3} - 0\right) = \frac{4}{3} \end{aligned}$$

$$\begin{aligned} \text{Area of } R_2 &= \int_2^3 (x^2 - 2x) dx \\ &= F(3) - F(2) \\ &= 0 - \left(-\frac{4}{3}\right) = \frac{4}{3} \end{aligned}$$

$$\text{Total shaded area} = \frac{4}{3} + \frac{4}{3} = \frac{8}{3}$$



$[0, 3]$ by $[-1, 8]$

An antiderivative of $x^2 - 6x + 8$ is $F(x) = \frac{1}{3}x^3 - 3x^2 + 8x$.

$$(a) \int_0^3 (x^2 - 6x + 8) dx = F(3) - F(0) = 6 - 0 = 6$$

$$\begin{aligned} (b) \text{ Area} &= \int_0^2 (x^2 - 6x + 8) dx - \int_2^3 (x^2 - 6x + 8) dx \\ &= [F(2) - F(0)] - [F(3) - F(2)] \\ &= \left(\frac{20}{3} - 0\right) - \left(6 - \frac{20}{3}\right) = \frac{22}{3} \end{aligned}$$



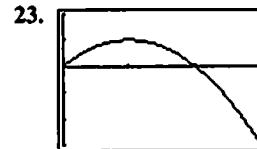
$[0, 2]$ by $[-5, 3]$

An antiderivative of $-x^2 + 5x - 4$ is

$$F(x) = -\frac{1}{3}x^3 + \frac{5}{2}x^2 - 4x.$$

$$(a) \int_0^2 (-x^2 + 5x - 4) dx = F(2) - F(0) = -\frac{2}{3} - 0 = -\frac{2}{3}$$

$$\begin{aligned} (b) \text{ Area} &= -\int_0^1 (-x^2 + 5x - 4) dx + \int_1^2 (-x^2 + 5x - 4) dx \\ &= -[F(1) - F(0)] + [F(2) - F(1)] \\ &= -\left(-\frac{11}{6} - 0\right) + \left[-\frac{2}{3} - \left(-\frac{11}{6}\right)\right] = 3 \end{aligned}$$

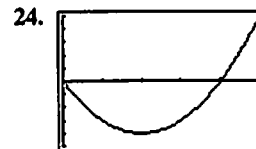


$[0, 3]$ by $[-3, 2]$

An antiderivative of $2x - x^2$ is $F(x) = x^2 - \frac{1}{3}x^3$.

$$(a) \int_0^3 (2x - x^2) dx = F(3) - F(0) = 0 - 0 = 0$$

$$\begin{aligned} (b) \text{ Area} &= \int_0^2 (2x - x^2) dx - \int_2^3 (2x - x^2) dx \\ &= [F(2) - F(0)] - [F(3) - F(2)] \\ &= \left(\frac{4}{3} - 0\right) - \left(0 - \frac{4}{3}\right) = \frac{8}{3} \end{aligned}$$



$[0, 5]$ by $[-5, 5]$

An antiderivative of $x^2 - 4x$ is $F(x) = \frac{1}{3}x^3 - 2x^2$.

$$(a) \int_0^5 (x^2 - 4x) dx = F(5) - F(0) = -\frac{25}{3} - 0 = -\frac{25}{3}$$

$$\begin{aligned} \text{(b) Area} &= -\int_0^4 (x^2 - 4x) dx + \int_4^5 (x^2 - 4x) dx \\ &= -[F(4) - F(0)] + [F(5) - F(4)] \\ &= -\left(-\frac{32}{3} - 0\right) + \left[-\frac{25}{3} - \left(-\frac{32}{3}\right)\right] = 13 \end{aligned}$$

25. An antiderivative of $x^2 - 1$ is $F(x) = \frac{1}{3}x^3 - x$.

$$\begin{aligned} av &= \frac{1}{\sqrt{3}} \int_0^{\sqrt{3}} (x^2 - 1) dx \\ &= \frac{1}{\sqrt{3}} [F(\sqrt{3}) - F(0)] \\ &= \frac{1}{\sqrt{3}} (0 - 0) = 0 \end{aligned}$$

Find $x = c$ in $[0, \sqrt{3}]$ such that $c^2 - 1 = 0$

$$c^2 = 1$$

$$c = \pm 1$$

Since 1 is in $[0, \sqrt{3}]$, $x = 1$.

26. An antiderivative of $-\frac{x^2}{2}$ is $F(x) = -\frac{x^3}{6}$.

$$av = \frac{1}{3} \int_0^3 \left(-\frac{x^2}{2}\right) dx = \frac{1}{3} [F(3) - F(0)] = \frac{1}{3} \left(-\frac{9}{2}\right) = -\frac{3}{2}$$

Find $x = c$ in $[0, 3]$ such that $-\frac{c^2}{2} = -\frac{3}{2}$.

$$c^2 = 3$$

$$c = \pm\sqrt{3}$$

Since $\sqrt{3}$ is in $[0, 3]$, $x = \sqrt{3}$.

27. An antiderivative of $-3x^2 - 1$ is $F(x) = -x^3 - x$.

$$av = \frac{1}{1} \int_0^1 (-3x^2 - 1) dx = F(1) - F(0) = -2$$

Find $x = c$ in $[0, 1]$ such that $-3c^2 - 1 = -2$

$$-3c^2 = -1$$

$$c^2 = \frac{1}{3}$$

$$c = \pm\frac{1}{\sqrt{3}}$$

Since $\frac{1}{\sqrt{3}}$ is in $[0, 1]$, $x = \frac{1}{\sqrt{3}}$.

28. An antiderivative of $(x - 1)^2$ is $F(x) = \frac{1}{3}(x - 1)^3$.

$$av = \frac{1}{3} \int_0^3 (x - 1)^2 dx = \frac{1}{3} [F(3) - F(0)] = \frac{1}{3} \left(\frac{8}{3} + \frac{1}{3}\right) = 1$$

Find $x = c$ in $[0, 3]$ such that $(c - 1)^2 = 1$.

$$c - 1 = \pm 1$$

$$c = 2 \text{ or } c = 0.$$

Since both are in $[0, 3]$, $x = 0$ or $x = 2$.

29. The region between the graph and the x -axis is a triangle of

height 3 and base 6, so the area of the region

$$\text{is } \frac{1}{2}(3)(6) = 9.$$

$$av(f) = \frac{1}{6} \int_{-4}^2 f(x) dx = \frac{9}{6} = \frac{3}{2}$$

30. The region between the graph and the x -axis is a rectangle with a half circle of radius 1 cut out. The area of the region is $2(1) - \frac{1}{2}\pi(1)^2 = \frac{4 - \pi}{2}$.

$$av(f) = \frac{1}{2} \int_{-1}^1 f(t) dt = \frac{1}{2} \left(\frac{4 - \pi}{2}\right) = \frac{4 - \pi}{4}$$

31. There are equal areas above and below the x -axis.

$$av(f) = \frac{1}{2\pi} \int_0^{2\pi} f(t) dt = \frac{1}{2\pi} \cdot 0 = 0$$

32. Since $\tan \theta$ is an odd function, there are equal areas above and below the x -axis.

$$av(f) = \frac{1}{\pi/2 - (-\pi/4)} \int_{-\pi/4}^{\pi/4} f(\theta) d\theta = \frac{2}{\pi} \cdot 0 = 0$$

33. $\min f = \frac{1}{2}$ and $\max f = 1$

$$\frac{1}{2} \leq \int_0^1 \frac{1}{1+x^4} dx \leq 1$$

34. $f(0.5) = \frac{16}{17}$

$$\left(\frac{1}{2}\right)\left(\frac{16}{17}\right) \leq \int_0^{0.5} \frac{1}{1+x^4} dx \leq \left(\frac{1}{2}\right)(1)$$

$$\frac{8}{17} \leq \int_0^{0.5} \frac{1}{1+x^4} dx \leq \frac{1}{2}$$

$$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \leq \int_{0.5}^1 \frac{1}{1+x^4} dx \leq \left(\frac{1}{2}\right)\left(\frac{16}{17}\right)$$

$$\frac{1}{4} \leq \int_{0.5}^1 \frac{1}{1+x^4} dx \leq \frac{8}{17}$$

$$\frac{8}{17} + \frac{1}{4} \leq \int_0^1 \frac{1}{1+x^4} dx \leq \frac{1}{2} + \frac{8}{17}$$

$$\frac{49}{68} \leq \int_0^1 \frac{1}{1+x^4} dx \leq \frac{33}{34}$$

35. $\max \sin(x^2) = \sin(1)$ on $[0, 1]$

$$\int_0^1 \sin(x^2) dx \leq \sin(1) < 1$$

36. $\max \sqrt{x+8} = 3$ and $\min \sqrt{x+8} = 2\sqrt{2}$ on $[0, 1]$

$$2\sqrt{2} \leq \int_0^1 \sqrt{x+8} dx \leq 3$$

37. $(b - a) \min f(x) \geq 0$ on $[a, b]$

$$0 \leq (b - a) \min f(x) \leq \int_a^b f(x) dx$$

38. $(b - a) \max f(x) \leq 0$ on $[a, b]$

$$\int_a^b f(x) dx \leq (b - a) \max f(x) \leq 0$$

39. Yes, $\int_a^b av(f) dx = \int_a^b f(x) dx$.

This is because $av(f)$ is a constant, so

$$\begin{aligned} \int_a^b av(f) dx &= [av(f) \cdot x]_a^b \\ &= av(f) \cdot b - av(f) \cdot a \\ &= (b - a)av(f) \\ &= (b - a) \left[\frac{1}{b - a} \int_a^b f(x) dx \right] \\ &= \int_a^b f(x) dx \end{aligned}$$

40. (a) 300 mi

(b) $\frac{150 \text{ mi}}{30 \text{ mph}} + \frac{150 \text{ mi}}{50 \text{ mph}} = 8 \text{ h}$

(c) $\frac{300 \text{ mi}}{8 \text{ h}} = 37.5 \text{ mph}$

(d) The average speed is the total distance divided by the

total time. Algebraically, $\frac{d_1 + d_2}{t_1 + t_2}$. The driver computed

$\frac{1}{2} \left(\frac{d_1}{t_1} + \frac{d_2}{t_2} \right)$. The two expressions are not equal.

41. Time for first release = $\frac{1000 \text{ m}^3}{10 \text{ m}^3/\text{min}} = 100 \text{ min}$

Time for second release = $\frac{1000 \text{ m}^3}{20 \text{ m}^3/\text{min}} = 50 \text{ min}$

Average rate = $\frac{\text{total released}}{\text{total time}} = \frac{2000 \text{ m}^3}{150 \text{ min}} = 13\frac{1}{3} \text{ m}^3/\text{min}$

42. $\int_0^1 \sin x dx \leq \int_0^1 x dx = \left[\frac{1}{2}x^2 \right]_0^1 = \frac{1}{2}$

43. $\int_0^1 \sec x dx \geq \int_0^1 \left(1 + \frac{x^2}{2} \right) dx = \left[x + \frac{x^3}{6} \right]_0^1 = \frac{7}{6}$

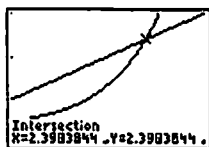
44. (a) Area = $\frac{1}{2}bh$

(b) $\frac{h}{2b}x^2 + C$

(c) $\int_0^b y(x) dx = \left[\frac{h}{2b}x^2 \right]_0^b = \frac{hb^2}{2b} = \frac{1}{2}bh$

45. $av(x^k) = \frac{1}{k} \int_0^k x^k dx = \frac{1}{k} \left[\frac{1}{k+1} x^{k+1} \right]_0^k = \frac{k^{k+1}}{k(k+1)}$

Graph $y_1 = \frac{x^{k+1}}{x(x+1)}$ and $y_2 = x$ on a graphing calculator and find the point of intersection for $x > 1$.



[1.3] by [0.3]

Thus, $k \approx 2.39838$

46. An antiderivative of $F'(x)$ is $F(x)$ and an antiderivative of

$G'(x)$ is $G(x)$.

$$\int_a^b F'(x) dx = F(b) - F(a)$$

$$\int_a^b G'(x) dx = G(b) - G(a)$$

Since $F'(x) = G'(x)$, $\int_a^b F'(x) dx = \int_a^b G'(x) dx$, so

$$F(b) - F(a) = G(b) - G(a).$$

Section 5.4 Fundamental Theorem of Calculus (pp. 277–288)

Exploration 1 Graphing NINT f

2. The function $y = \tan x$ has vertical asymptotes at all odd multiples of $\frac{\pi}{2}$. There are six of these between -10 and 10 .

3. In attempting to find $F(-10) = \int_3^{-10} \tan(t) dt + 5$, the calculator must find a limit of Riemann sums for the integral, using values of $\tan t$ for t between -10 and 3 . The large positive and negative values of $\tan t$ found near the asymptotes cause the sums to fluctuate erratically so that no limit is approached. (We will see in Section 8.3 that the “areas” near the asymptotes are infinite, although NINT is not designed to determine this.)

4. $y = \tan x$



[1.6, 4.7] by [-2, 2]

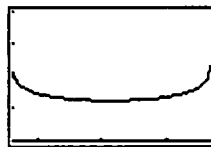
5. The domain of this continuous function is the open interval

$$\left(\frac{\pi}{2}, \frac{3\pi}{2} \right).$$

6. The domain of F is the same as the domain of the continuous function in step 4, namely $\left(\frac{\pi}{2}, \frac{3\pi}{2} \right)$.

7. We need to choose a closed window narrower than

$$\left(\frac{\pi}{2}, \frac{3\pi}{2} \right) \text{ to avoid the asymptotes.}$$

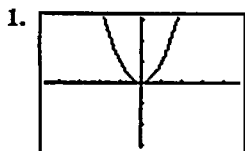


[1.6, 4.7] by [0, 16]

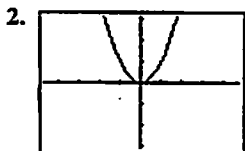
8. The graph of F looks like the graph in step 7. It would be decreasing on $\left(\frac{\pi}{2}, \pi \right)$ and increasing on $\left[\pi, \frac{3\pi}{2} \right)$, with vertical asymptotes at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

Exploration 2 The Effect of Changing a in

$$\int_a^x f(t) dt$$



$[-4.7, 4.7]$ by $[-3.1, 3.1]$



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

3. Since $\text{NINT}(x^2, x, 0, 0) = 0$, the x -intercept is 0.
 4. Since $\text{NINT}(x^2, x, 5, 5) = 0$, the x -intercept is 5.
 5. Changing a has no effect on the graph of $y = \frac{d}{dx} \int_a^x f(t) dt$.
 It will always be the same as the graph of $y = f(x)$.
 6. Changing a shifts the graph of $y = \int_a^x f(t) dt$ vertically in such a way that a is always the x -intercept. If we change from a_1 to a_2 , the distance of the vertical shift is $\int_{a_2}^{a_1} f(t) dt$.

Quick Review 5.4

1. $\frac{dy}{dx} = \cos(x^2) \cdot 2x = 2x \cos(x^2)$
 2. $\frac{dy}{dx} = 2(\sin x)(\cos x) = 2 \sin x \cos x$
 3. $\frac{dy}{dx} = 2(\sec x)(\sec x \tan x) - 2(\tan x)(\sec^2 x)$
 $= 2 \sec^2 x \tan x - 2 \tan x \sec^2 x = 0$
 4. $\frac{dy}{dx} = \frac{3}{3x} - \frac{7}{7x} = 0$
 5. $\frac{dy}{dx} = 2^x \ln 2$
 6. $\frac{dy}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$
 7. $\frac{dy}{dx} = \frac{(-\sin x)(x) - (\cos x)(1)}{x^2} = -\frac{x \sin x + \cos x}{x^2}$
 8. $\frac{dy}{dt} = \cos t, \frac{dx}{dt} = -\sin t$
 $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t}{-\sin t} = -\cot t$
 9. Implicitly differentiate:
 $x \frac{dy}{dx} + (1)y + 1 = 2y \frac{dy}{dx}$
 $\frac{dy}{dx}(x - 2y) = -(y + 1)$
 $\frac{dy}{dx} = \frac{-(y + 1)}{x - 2y} = \frac{y + 1}{2y - x}$
10. $\frac{dy}{dx} = \frac{1}{dx/dy} = \frac{1}{3x}$
- Section 5.4 Exercises**
1. $\int_{1/2}^3 \left(2 - \frac{1}{x}\right) dx = \left[2x - \ln|x|\right]_{1/2}^3$
 $= (6 - \ln 3) - \left(1 - \ln \frac{1}{2}\right)$
 $= 5 - \ln 3 + \ln \frac{1}{2}$
 $= 5 - \ln 3 - \ln 2$
 $= 5 - \ln 6 \approx 3.208$
2. $\int_2^{-1} 3^x dx = \left[\frac{1}{\ln 3} 3^x\right]_2^{-1}$
 $= \left(\frac{1}{\ln 3}\right)\left(\frac{1}{3} - 9\right)$
 $= -\frac{26}{3 \ln 3} \approx -7.889$
3. $\int_0^1 (x^2 + \sqrt{x}) dx = \left[\frac{1}{3}x^3 + \frac{2}{3}x^{3/2}\right]_0^1 = \left(\frac{1}{3} + \frac{2}{3}\right) - (0 + 0) = 1$
4. $\int_0^5 x^{3/2} dx = \left[\frac{2}{5}x^{5/2}\right]_0^5 = \frac{2}{5}(25\sqrt{5} - 0) = 10\sqrt{5} \approx 22.361$
5. $\int_1^{32} x^{-6/5} dx = \left[-5x^{-1/5}\right]_1^{32} = -5\left(\frac{1}{2} - 1\right) = \frac{5}{2}$
6. $\int_{-2}^{-1} \frac{2}{x^2} dx = 2 \int_{-2}^{-1} x^{-2} dx = 2\left[-x^{-1}\right]_{-2}^{-1} = 2\left[1 - \frac{1}{2}\right] = 1$
7. $\int_0^\pi \sin x dx = \left[-\cos x\right]_0^\pi = 1 - (-1) = 2$
8. $\int_0^\pi (1 + \cos x) dx = \left[x + \sin x\right]_0^\pi$
 $= (\pi + 0) - (0 + 0)$
 $= \pi \approx 3.142$
9. $\int_0^{\pi/3} 2 \sec^2 \theta d\theta = 2\left[\tan \theta\right]_0^{\pi/3}$
 $= 2(\sqrt{3} - 0)$
 $= 2\sqrt{3} \approx 3.464$
10. $\int_{\pi/6}^{5\pi/6} \csc^2 \theta d\theta = \left[-\cot \theta\right]_{\pi/6}^{5\pi/6}$
 $= \sqrt{3} - (-\sqrt{3})$
 $= 2\sqrt{3} \approx 3.464$
11. $\int_{\pi/4}^{3\pi/4} \csc x \cot x dx = \left[-\csc x\right]_{\pi/4}^{3\pi/4} = (-\sqrt{2}) - (-\sqrt{2}) = 0$
12. $\int_0^{\pi/3} 4 \sec x \tan x dx = 4\left[\sec x\right]_0^{\pi/3} = 4(2 - 1) = 4$

$$13. \int_{-1}^1 (r+1)^2 dr = \left[\frac{1}{3}(r+1)^3 \right]_{-1}^1 = \frac{8}{3} - 0 = \frac{8}{3}$$

$$14. \int_0^4 \frac{1-\sqrt{u}}{\sqrt{u}} du = \int_0^4 (u^{-1/2} - 1) du$$

$$= \left[2u^{1/2} - u \right]_0^4$$

$$= (4 - 4) - (0 - 0) = 0$$

15. Graph $y = 2 - x$.



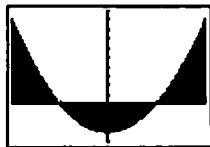
$[0, 3]$ by $[-2, 3]$

$$\text{Over } [0, 2]: \int_0^2 (2-x) dx = \left[2x - \frac{1}{2}x^2 \right]_0^2 = 2$$

$$\text{Over } [2, 3]: \int_2^3 (2-x) dx = \left[2x - \frac{1}{2}x^2 \right]_2^3 = \frac{3}{2} - 2 = -\frac{1}{2}$$

$$\text{Total area} = |2| + \left| -\frac{1}{2} \right| = \frac{5}{2}$$

16. Graph $y = 3x^2 - 3$.



$[-2, 2]$ by $[-4, 10]$

Over $[-2, -1]$:

$$\int_{-2}^{-1} (3x^2 - 3) dx = \left[x^3 - 3x \right]_{-2}^{-1} = 2 - (-2) = 4$$

Over $[-1, 1]$:

$$\int_{-1}^1 (3x^2 - 3) dx = \left[x^3 - 3x \right]_{-1}^1 = -2 - 2 = -4$$

$$\text{Over } [1, 2]: \int_1^2 (3x^2 - 3) dx = \left[x^3 - 3x \right]_1^2 = 2 - (-2) = 4$$

$$\text{Total area} = |4| + |-4| + |4| = 12$$

17. Graph $y = x^3 - 3x^2 + 2x$.



$[0, 2]$ by $[-1, 1]$

Over $[0, 1]$:

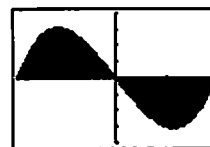
$$\int_0^1 (x^3 - 3x^2 + 2x) dx = \left[\frac{1}{4}x^4 - x^3 + x^2 \right]_0^1 = \frac{1}{4} - 0 = \frac{1}{4}$$

Over $[1, 2]$:

$$\int_1^2 (x^3 - 3x^2 + 2x) dx = \left[\frac{1}{4}x^4 - x^3 + x^2 \right]_1^2 = 0 - \frac{1}{4} = -\frac{1}{4}$$

$$\text{Total area} = \left| \frac{1}{4} \right| + \left| -\frac{1}{4} \right| = \frac{1}{2}$$

18. Graph $y = x^3 - 4x$.



$[-2, 2]$ by $[-4, 4]$

Over $[-2, 0]$:

$$\int_{-2}^0 (x^3 - 4x) dx = \left[\frac{1}{4}x^4 - 2x^2 \right]_{-2}^0 = 0 - (-4) = 4$$

Over $[0, 2]$:

$$\int_0^2 (x^3 - 4x) dx = \left[\frac{1}{4}x^4 - 2x^2 \right]_0^2 = -4 - 0 = -4$$

$$\text{Total area} = |4| + |4| = 8$$

19. (a) No, $f(x) = \frac{x^2 - 1}{x + 1}$ is discontinuous at $x = -1$.

$$(b) \frac{x^2 - 1}{x + 1} = x - 1, x \neq -1$$

The area between the graph of f and the x -axis over

$[-2, 1]$ where f is negative is $\frac{1}{2}(3)(3) = \frac{9}{2}$. The area

between the graph of f and the x -axis over $(1, 3]$ where

f is positive is $\frac{1}{2}(2)(2) = 2$.

$$\int_{-2}^3 \frac{x^2 - 1}{x + 1} dx = -\frac{9}{2} + 2 = -\frac{5}{2}$$

20. (a) No, $f(x) = \frac{9 - x^2}{3x - 9}$ is discontinuous at $x = 3$.

$$(b) \frac{9 - x^2}{3x - 9} = -\frac{3 + x}{3}, x \neq 3$$

Note that f is negative for x in $[0, 5]$. $f(0) = -1$ and

$f(5) = -\frac{8}{3}$. The area between the graph of f and the

x -axis over $[0, 5]$ is $\frac{1}{2}(5 - 0)\left(1 + \frac{8}{3}\right) = \frac{55}{6}$

$$\int_0^5 \frac{9 - x^2}{3x - 9} dx = -\frac{55}{6}$$

21. (a) No, $f(x) = \tan x$ is discontinuous at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

(b) The integral does not have a value. If $0 < b < \frac{\pi}{2}$, then

$$\int_0^b \tan x \, dx = \left[-\ln |\cos x| \right]_0^b = -\ln |\cos b|$$

since the Fundamental Theorem applies for $[0, b]$. As $b \rightarrow \frac{\pi}{2}^-$, $\cos b \rightarrow 0^+$ so $-\ln |\cos b| \rightarrow \infty$ or $\int_0^b \tan x \, dx \rightarrow \infty$.

Hence the integral does not exist over a subinterval of $[0, 2\pi]$, so it doesn't exist over $[0, 2\pi]$.

22. (a) No, $f(x) = \frac{x+1}{x^2-1}$ is discontinuous at $x = 1$.

(b) The integral does not have a value. If $0 < b < 1$, then

$$\int_0^b \frac{x+1}{x^2-1} \, dx = \int_0^b \frac{1}{x-1} \, dx = \left[\ln |x-1| \right]_0^b = \ln |b-1|,$$

since $\frac{x+1}{x^2-1} = \frac{1}{x-1}$ and the Fundamental Theorem applies for $[0, b]$. As $b \rightarrow 1^-$, $\ln |b-1| \rightarrow -\infty$ or $\int_0^b \frac{x+1}{x^2-1} \, dx \rightarrow -\infty$. Hence the integral does not exist

over a subinterval of $[0, 2]$, so it does not exist over $[0, 2]$.

23. (a) No, $f(x) = \frac{\sin x}{x}$ is discontinuous at $x = 0$.

(b) $\text{NINT}\left(\frac{\sin x}{x}, x, -1, 2\right) \approx 2.55$. The integral exists since the area is finite because $\frac{\sin x}{x}$ is bounded.

24. (a) No, $f(x) = \frac{1-\cos x}{x^2}$ is discontinuous at $x = 0$.

(b) $\text{NINT}\left(\frac{1-\cos x}{x^2}, x, -2, 3\right) \approx 2.08$. The integral exists since the area is finite because $\frac{1-\cos x}{x^2}$ is bounded.

25. First, find the area under the graph of $y = x^2$.

$$\int_0^1 x^2 \, dx = \left[\frac{1}{3}x^3 \right]_0^1 = \frac{1}{3}$$

Next find the area under the graph of $y = 2 - x$.

$$\int_1^2 (2-x) \, dx = \left[2x - \frac{1}{2}x^2 \right]_1^2 = 2 - \frac{3}{2} = \frac{1}{2}$$

$$\text{Area of the shaded region} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

26. First find the area under the graph of $y = \sqrt{x}$.

$$\int_0^1 x^{1/2} \, dx = \left[\frac{2}{3}x^{3/2} \right]_0^1 = \frac{2}{3}$$

Next find the area under the graph of $y = x^2$.

$$\int_1^2 x^2 \, dx = \left[\frac{1}{3}x^3 \right]_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$

$$\text{Area of the shaded region} = \frac{2}{3} + \frac{7}{3} = 3$$

27. First find the area under the graph of $y = 1 + \cos x$.

$$\int_0^\pi (1 + \cos x) \, dx = \left[x + \sin x \right]_0^\pi = \pi$$

The area of the rectangle is 2π .

$$\text{Area of the shaded region} = 2\pi - \pi = \pi.$$

28. First, find the area of the region between $y = \sin x$ and the

x -axis for $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$.

$$\int_{\pi/6}^{5\pi/6} \sin x \, dx = \left[-\cos x \right]_{\pi/6}^{5\pi/6} = \frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2} \right) = \sqrt{3}$$

The area of the rectangle is $\left(\sin \frac{\pi}{6}\right)\left(\frac{2\pi}{3}\right) = \frac{\pi}{3}$

$$\text{Area of the shaded region} = \sqrt{3} - \frac{\pi}{3}$$

29. $\text{NINT}\left(\frac{1}{3+2\sin x}, x, 0, 10\right) \approx 3.802$

30. $\text{NINT}\left(\frac{2x^4-1}{x^4-1}, x, -0.8, 0.8\right) \approx 1.427$

31. $\frac{1}{2}\text{NINT}(\sqrt{\cos x}, x, -1, 1) \approx 0.914$

32. $\sqrt{8-2x^2} \geq 0$ between $x = -2$ and $x = 2$

$$\text{NINT}(\sqrt{8-2x^2}, x, -2, 2) \approx 8.886$$

33. Plot $y_1 = \text{NINT}(e^{-t^2}, t, 0, x)$, $y_2 = 0.6$ in a $[0, 1]$ by $[0, 1]$ window, then use the intersect function to find $x \approx 0.699$.

34. When $y = 0$, $x = 1$.

$$y^3 = 1 - x^3$$

$$y = \sqrt[3]{1-x^3}$$

$$\text{NINT}(\sqrt[3]{1-x^3}, x, 0, 1) \approx 0.883$$

$$35. \int_a^x f(t) dt + K = \int_b^x f(t) dt$$

$$K = -\int_a^x f(t) dt + \int_b^x f(t) dt$$

$$= \int_1^a f(t) dt + \int_b^x f(t) dt$$

$$= \int_b^a f(t) dt$$

$$K = \int_2^{-1} (t^2 - 3t + 1) dt$$

$$= \left[\frac{1}{3}t^3 - \frac{3}{2}t^2 + t \right]_2^{-1}$$

$$= \left[-\frac{1}{3} - \frac{3}{2} + (-1) \right] - \left[\frac{8}{3} - 6 + 2 \right] = -\frac{3}{2}$$

36. To find an antiderivative of $\sin^2 x$, recall from trigonometry

$$\text{that } \cos 2x = 1 - 2 \sin^2 x, \text{ so } \sin^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x.$$

$$K = \int_2^0 \sin^2 t dt$$

$$= \int_2^0 \left[\frac{1}{2} - \frac{1}{2} \cos(2x) \right] dx$$

$$= \left[\frac{1}{2}x - \frac{1}{4} \sin(2x) \right]_2^0$$

$$= \left[\frac{1}{2}x - \frac{1}{2} \sin x \cos x \right]_2^0$$

$$= 0 - \left(1 - \frac{\sin 2 \cos 2}{2} \right) = \frac{\sin 2 \cos 2 - 2}{2} \approx -1.189$$

$$37. \frac{dy}{dx} = \sqrt{1+x^2}$$

$$38. \frac{dy}{dx} = \frac{d}{dx} \left(\int_1^x \frac{1}{t} dt \right) = \frac{d}{dx} \left(-\int_1^x \frac{1}{t} dt \right) = -\frac{1}{x}$$

$$39. \frac{dy}{dx} = \sin \left[(\sqrt{x})^2 \right] \frac{d}{dx} (\sqrt{x}) = (\sin x) \cdot \frac{1}{2} x^{-1/2} = \frac{\sin x}{2\sqrt{x}}$$

$$40. \frac{dy}{dx} = \cos(2x) \frac{d}{dx}(2x) = 2 \cos(2x)$$

$$41. \frac{dy}{dx} = \frac{d}{dx} \left(\int_0^{x^3} \cos(2t) dt - \int_0^{x^2} \cos(2t) dt \right)$$

$$= \cos(2x^3) \cdot 3x^2 - \cos(2x^2) \cdot 2x$$

$$= 3x^2 \cos(2x^3) - 2x \cos(2x^2)$$

$$42. \frac{dy}{dx} = \frac{d}{dx} \left(\int_{\sin x}^{\cos x} t^2 dt \right)$$

$$= \frac{d}{dx} \left(\int_0^{\cos x} t^2 dt - \int_0^{\sin x} t^2 dt \right)$$

$$= \cos^2 x \cdot (-\sin x) - \sin^2 x \cdot \cos x$$

$$= -\sin x \cos^2 x - \sin^2 x \cos x$$

43. Choose (d).

$$\frac{dy}{dx} = \frac{d}{dx} \left(\int_{\pi}^x e^{-t^2} dt - 3 \right) = e^{-x^2}$$

$$y(\pi) = \int_{\pi}^{\pi} e^{-t^2} dt - 3 = 0 - 3 = -3$$

44. Choose (c).

$$\frac{dy}{dx} = \frac{d}{dx} \left(\int_{-1}^x \sec t dt + 4 \right) = \sec x$$

$$y(-1) = \int_{-1}^{-1} \sec t dt + 4 = 0 + 4 = 4$$

45. Choose (b).

$$\frac{dy}{dx} = \frac{d}{dx} \left(\int_0^x \sec t dt + 4 \right) = \sec x$$

$$y(0) = \int_0^0 \sec t dt + 4 = 0 + 4 = 4$$

46. Choose (a).

$$\frac{dy}{dx} = \frac{d}{dx} \left(\int_1^x e^{-t^2} dt - 3 \right) = e^{-x^2}$$

$$y(1) = \int_1^1 e^{-t^2} dt - 3 = 0 - 3 = -3$$

$$47. x = a \text{ since } \int_a^a f(t) dt = 0$$

$$48. f(x) = \frac{d}{dx} \left(\int_1^x f(t) dt \right) = \frac{d}{dx} (x^2 - 2x + 1) = 2x - 2$$

$$49. f'(x) = \frac{d}{dx} \left(2 + \int_0^x \frac{10}{1+t} dt \right) = \frac{10}{1+x}$$

$$f'(0) = 10$$

$$f(0) = 2 + \int_0^0 \frac{10}{1+t} dt = 2$$

$$L(x) = 2 + 10x$$

$$50. f(x) = \frac{d}{dx} \left(\int_0^x f(t) dt \right)$$

$$= \frac{d}{dx} (x \cos \pi x)$$

$$= x(-\pi \sin \pi x) + 1 \cdot \cos \pi x$$

$$= -\pi x \sin \pi x + \cos \pi x$$

$$f(4) = -4\pi \sin 4\pi + \cos 4\pi = 1$$

51. One arch of $\sin kx$ is from $x = 0$ to $x = \frac{\pi}{k}$.

$$\text{Area} = \int_0^{\pi/k} \sin kx dx = \left[-\frac{1}{k} \cos kx \right]_0^{\pi/k} = \frac{1}{k} - \left(-\frac{1}{k} \right) = \frac{2}{k}$$

$$52. (a) \int_{-3}^2 (6 - x - x^2) dx = \left[6x - \frac{1}{2}x^2 - \frac{1}{3}x^3 \right]_{-3}^2$$

$$= \frac{22}{3} - \left(-\frac{27}{2} \right)$$

$$= \frac{125}{6}$$

(b) The vertex is at $x = \frac{-(-1)}{2(-1)} = -\frac{1}{2}$. (Recall that the vertex of a parabola $y = ax^2 + bx + c$ is at $x = -\frac{b}{2a}$.)
 $y\left(-\frac{1}{2}\right) = \frac{25}{4}$, so the height is $\frac{25}{4}$.

(c) The base is $2 - (-3) = 5$.

$$\frac{2}{3}(\text{base})(\text{height}) = \frac{2}{3}(5)\left(\frac{25}{4}\right) = \frac{125}{6}$$

53. (a) $H(0) = \int_0^0 f(t) dt = 0$

(b) $H'(x) = \frac{d}{dx}\left(\int_0^x f(t) dt\right) = f(x)$

$$H'(x) > 0 \text{ when } f(x) > 0.$$

H is increasing on $[0, 6]$.

(c) H is concave up on the open interval where

$$H''(x) = f'(x) > 0.$$

$$f'(x) > 0 \text{ when } 9 < x \leq 12.$$

H is concave up on $(9, 12)$.

(d) $H(12) = \int_0^{12} f(t) dt > 0$ because there is more area above the x -axis than below the x -axis.

$H(12)$ is positive.

(e) $H'(x) = f(x) = 0$ at $x = 6$ and $x = 12$. Since $H'(x) = f(x) > 0$ on $[0, 6)$, the values of H are increasing to the left of $x = 6$, and since $H'(x) = f(x) < 0$ on $(6, 12]$, the values of H are decreasing to the right of $x = 6$. H achieves its maximum value at $x = 6$.

(f) $H(x) > 0$ on $(0, 12]$. Since $H(0) = 0$, H achieves its minimum value at $x = 0$.

54. (a) $s'(t) = f(t)$. The velocity at $t = 5$ is $f(5) = 2$ units/sec.

(b) $s''(t) = f'(t) < 0$ at $t = 5$ since the graph is decreasing, so acceleration at $t = 5$ is negative.

(c) $s(3) = \int_0^3 f(x) dx = \frac{1}{2}(3)(3) = 4.5$ units

(d) s has its largest value at $t = 6$ sec since $s'(6) = f(6) = 0$ and $s''(6) = f'(6) < 0$.

(e) The acceleration is zero when $s''(t) = f'(t) = 0$. This occurs when $t = 4$ sec and $t = 7$ sec.

(f) Since $s(0) = 0$ and $s'(t) = f(t) > 0$ on $(0, 6)$, the particle moves away from the origin in the positive direction on $(0, 6)$. The particle then moves in the negative direction, towards the origin, on $(6, 9)$ since $s'(t) = f(t) < 0$ on $(6, 9)$ and the area below the x -axis is smaller than the area above the x -axis.

(g) The particle is on the positive side since

$$s(9) = \int_0^9 f(x) dx > 0 \text{ (the area below the } x\text{-axis is smaller than the area above the } x\text{-axis).}$$

55. (a) $s'(3) = f(3) = 0$ units/sec

(b) $s''(3) = f'(3) > 0$ so acceleration is positive.

(c) $s(3) = \int_0^3 f(x) dx = \frac{1}{2}(-6)(3) = -9$ units

(d) $s(6) = \int_0^6 f(x) dx = \frac{1}{2}(-6)(3) + \frac{1}{2}(6)(3) = 0$, so the particle passes through the origin at $t = 6$ sec.

(e) $s''(t) = f'(t) = 0$ at $t = 7$ sec

(f) The particle is moving away from the origin in the negative direction on $(0, 3)$ since $s(0) = 0$ and $s'(t) < 0$ on $(0, 3)$. The particle is moving toward the origin on $(3, 6)$ since $s'(t) > 0$ on $(3, 6)$ and $s(6) = 0$. The particle moves away from the origin in the positive direction for $t > 6$ since $s'(t) > 0$.

(g) The particle is on the positive side since

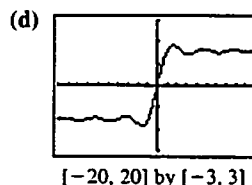
$$s(9) = \int_0^9 f(x) dx > 0 \text{ (the area below the } x\text{-axis is smaller than the area above the } x\text{-axis).}$$

56. (a) $f(t)$ is an even function so $\int_{-x}^0 \frac{\sin(t)}{t} dt = \int_0^x \frac{\sin(t)}{t} dt$.

$$\begin{aligned} \text{Si}(-x) &= \int_0^{-x} \frac{\sin(t)}{t} dt \\ &= -\int_0^x \frac{\sin(t)}{t} dt \\ &= -\int_0^x \frac{\sin(t)}{t} dt = -\text{Si}(x) \end{aligned}$$

(b) $\text{Si}(0) = \int_0^0 \frac{\sin t}{t} dt = 0$

(c) $\text{Si}'(x) = f(x) = 0$ when $t = \pi k$, k a nonzero integer.



57. (a) $c(100) - c(1) = \int_1^{100} \left(\frac{dc}{dx}\right) dx$
 $= \int_1^{100} \frac{1}{2\sqrt{x}} dx = \left[\sqrt{x}\right]_1^{100}$
 $= 10 - 1 = 9$ or \$9

(b) $c(400) - c(100) = \int_{100}^{400} \left(\frac{dc}{dx}\right) dx$
 $= \int_{100}^{400} \frac{1}{2\sqrt{x}} dx = \left[\sqrt{x}\right]_{100}^{400}$
 $= 20 - 10 = 10$ or \$10

$$\begin{aligned}
 58. \int_0^3 \left(2 - \frac{2}{(x+1)^2}\right) dx &= \left[2x + 2(x+1)^{-1}\right]_0^3 \\
 &= \left[6 + \frac{1}{2}\right] - 2 = \frac{9}{2} \\
 &= 4.5 \text{ thousand}
 \end{aligned}$$

The company should expect \$4500.

$$\begin{aligned}
 59. \text{(a)} \frac{1}{30-0} \int_0^{30} \left(450 - \frac{x^2}{2}\right) dx &= \frac{1}{30} \left[450x - \frac{x^3}{6}\right]_0^{30} \\
 &= 300 \text{ drums}
 \end{aligned}$$

$$\text{(b)} (300 \text{ drums})(\$0.02 \text{ per drum}) = \$6$$

60. (a) True, because $h'(x) = f(x)$ and therefore $h''(x) = f'(x)$.

(b) True, because h and h' are both differentiable by part (a).

(c) True, because $h'(1) = f(1) = 0$.

(d) True, because $h'(1) = f(1) = 0$ and $h''(1) = f'(1) < 0$.

(e) False, because $h''(1) = f'(1) < 0$.

(f) False, because $h''(1) = f'(1) \neq 0$.

(g) True, because $h'(x) = f(x)$, and f is a decreasing function that includes the point $(1, 0)$.

61. Since $f(t)$ is odd, $\int_{-x}^0 f(t) dt = -\int_0^x f(t) dt$ because the area between the curve and the x -axis from 0 to x is the opposite of the area between the curve and the x -axis from $-x$ to 0, but it is on the opposite side of the x -axis.

$$\begin{aligned}
 \int_0^{-x} f(t) dt &= -\int_{-x}^0 f(t) dt = -\left[-\int_0^x f(t) dt\right] = \int_0^x f(t) dt \\
 \text{Thus } \int_0^x f(t) dt &\text{ is even.}
 \end{aligned}$$

62. Since $f(t)$ is even, $\int_{-x}^0 f(t) dt = \int_0^x f(t) dt$ because the area between the curve and the x -axis from 0 to x is the same as the area between the curve and the x -axis from $-x$ to 0.

$$\begin{aligned}
 \int_0^{-x} f(t) dt &= -\int_{-x}^0 f(t) dt = -\int_0^x f(t) dt \\
 \text{Thus } \int_0^x f(t) dt &\text{ is odd.}
 \end{aligned}$$

63. If f is an even continuous function, then $\int_0^x f(t) dt$ is odd, but $\frac{d}{dx} \int_0^x f(t) dt = f(x)$. Therefore, f is the derivative of the odd continuous function $\int_0^x f(t) dt$.

Similarly, if f is an odd continuous function, then f is the derivative of the even continuous function $\int_0^x f(t) dt$.

64. Solving $\text{NINT}\left(\frac{\sin t}{t}, t, 0, x\right) = 1$ graphically, the solution is $x \approx 1.0648397$. We now argue that there are no other

solutions, using the functions $\text{Si}(x)$ and $f(t)$ as defined in

Exercise 56. Since $\frac{d}{dx} \text{Si}(x) = f(x) = \frac{\sin x}{x}$, $\text{Si}(x)$ is

increasing on each interval $[2k\pi, (2k+1)\pi]$ and

decreasing on each interval $[(2k+1)\pi, (2k+2)\pi]$, where

k is a nonnegative integer. Thus, for $x > 0$, $\text{Si}(x)$ has its

local minima at $x = 2k\pi$, where k is a positive integer.

Furthermore, each arch of $y = f(x)$ is smaller in height than

the previous one, so $\int_{2k\pi}^{(2k+1)\pi} |f(x)| dx > \int_{(2k+1)\pi}^{(2k+2)\pi} |f(x)| dx$.

This means that

$$\text{Si}((2k+2)\pi) - \text{Si}(2k\pi) = \int_{2k\pi}^{(2k+2)\pi} f(x) dx > 0, \text{ so each}$$

successive minimum value is greater than the previous one.

Since $f(2\pi) \approx \text{NINT}\left(\frac{\sin x}{x}, x, 0, 2\pi\right) \approx 1.42$ and $\text{Si}(x)$ is

continuous for $x > 0$, this means $\text{Si}(x) > 1.42$ (and hence

$\text{Si}(x) \neq 1$) for $x \geq 2\pi$. Now, $\text{Si}(x) = 1$ has exactly one

solution in the interval $[0, \pi]$ because $\text{Si}(x)$ is increasing on

this interval and $x \approx 1.065$ is a solution. Furthermore,

$\text{Si}(x) = 1$ has no solution on the interval $[\pi, 2\pi]$ because

$\text{Si}(x)$ is decreasing on this interval and $\text{Si}(2\pi) \approx 1.42 > 1$.

Thus, $\text{Si}(x) = 1$ has exactly one solution in the interval

$[0, \infty)$. Also, there is no solution in the interval $(-\infty, 0]$

because $\text{Si}(x)$ is odd by Exercise 56 (or 62), which means

that $\text{Si}(x) \leq 0$ for $x \leq 0$ (since $\text{Si}(x) \geq 0$ for $x \geq 0$).

■ Section 5.5 Trapezoidal Rule (pp. 289–297)

Exploration 1 Area Under a Parabolic Arc

1. Let $y = f(x) = Ax^2 + Bx + C$

Then $y_0 = f(-h) = Ah^2 - Bh + C$,

$y_1 = f(0) = A(0)^2 + B(0) + C = C$, and

$y_2 = f(h) = Ah^2 + Bh + C$.

2. $y_0 + 4y_1 + y_2 = Ah^2 - Bh + C + 4C + Ah^2 + Bh + C$
 $= 2Ah^2 + 6C$.

3. $A_p = \int_{-h}^h (Ax^2 + Bx + C) dx$

$$= \left[\frac{Ax^3}{3} + \frac{Bx^2}{2} + Cx \right]_{-h}^h$$

$$= \frac{Ah^3}{3} + \frac{Bh^2}{2} + Ch - \left(-\frac{Ah^3}{3} + \frac{Bh^2}{2} - Ch \right)$$

$$= 2A \frac{h^3}{3} + 2Ch$$

$$= \frac{h}{3}(2Ah^2 + 6C)$$

4. Substitute the expression in step 2 for the parenthetically enclosed expression in step 3:

$$A_p = \frac{h}{3}(2Ah^2 + 6C)$$

$$= \frac{h}{3}(y_0 + 4y_1 + y_2).$$

Quick Review 5.5

- $y' = -\sin x$
 $y'' = -\cos x$
 $y'' < 0$ on $[-1, 1]$, so the curve is concave down on $[-1, 1]$.
- $y' = 4x^3 - 12$
 $y'' = 12x^2$
 $y'' > 0$ on $[8, 17]$, so the curve is concave up on $[8, 17]$.
- $y' = 12x^2 - 6x$
 $y'' = 24x - 6$
 $y'' < 0$ on $[-8, 0]$, so the curve is concave down on $[-8, 0]$.
- $y' = \frac{1}{2}\cos \frac{x}{2}$
 $y'' = -\frac{1}{4}\sin \frac{x}{2}$
 $y'' \leq 0$ on $[48\pi, 50\pi]$, so the curve is concave down on $[48\pi, 50\pi]$.
- $y' = 2e^{2x}$
 $y'' = 4e^{2x}$
 $y'' > 0$ on $[-5, 5]$, so the curve is concave up on $[-5, 5]$.
- $y' = \frac{1}{x}$
 $y'' = -\frac{1}{x^2}$
 $y'' < 0$ on $[100, 200]$, so the curve is concave down on $[100, 200]$.
- $y' = -\frac{1}{x^2}$
 $y'' = \frac{2}{x^3}$
 $y'' > 0$ on $[3, 6]$, so the curve is concave up on $[3, 6]$.
- $y' = -\csc x \cot x$
 $y'' = (-\csc x)(-\csc^2 x) + (\csc x \cot x)(\cot x)$
 $= \csc^3 x + \csc x \cot^2 x$
 $y'' > 0$ on $[0, \pi]$, so the curve is concave up on $[0, \pi]$.
- $y' = -100x^9$
 $y'' = -900x^8$
 $y'' < 0$ on $[10, 10^{10}]$, so the curve is concave down on $[10, 10^{10}]$.
- $y' = \cos x + \sin x$
 $y'' = -\sin x + \cos x$
 $y'' < 0$ on $[1, 2]$, so the curve is concave down.

Section 5.5 Exercises

1. (a) $f(x) = x, h = \frac{2-0}{4} = \frac{1}{2}$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2

$$T = \frac{1}{4}\left(0 + 2\left(\frac{1}{2}\right) + 2(1) + 2\left(\frac{3}{2}\right) + 2\right) = 2$$

(b) $f'(x) = 1, f''(x) = 0$

The approximation is exact.

(c) $\int_0^2 x \, dx = \left[\frac{1}{2}x^2\right]_0^2 = 2$

2. (a) $f(x) = x^2, h = \frac{2-0}{4} = \frac{1}{2}$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	0	$\frac{1}{4}$	1	$\frac{9}{4}$	4

$$T = \frac{1}{4}\left(0 + 2\left(\frac{1}{4}\right) + 2(1) + 2\left(\frac{9}{4}\right) + 4\right) = 2.75$$

(b) $f'(x) = 2x, f''(x) = 2 > 0$ on $[0, 2]$

The approximation is an overestimate.

(c) $\int_0^2 x^2 \, dx = \left[\frac{1}{3}x^3\right]_0^2 = \frac{8}{3}$

3. (a) $f(x) = x^3, h = \frac{2-0}{4} = \frac{1}{2}$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	0	$\frac{1}{8}$	1	$\frac{27}{8}$	8

$$T = \frac{1}{4}\left(0 + 2\left(\frac{1}{8}\right) + 2(1) + 2\left(\frac{27}{8}\right) + 8\right) = 4.25$$

(b) $f'(x) = 3x^2, f''(x) = 6x > 0$ on $[0, 2]$

The approximation is an overestimate.

(c) $\int_0^2 x^3 \, dx = \left[\frac{1}{4}x^4\right]_0^2 = 4$

4. (a) $f(x) = \frac{1}{x}, h = \frac{2-1}{4} = \frac{1}{4}$

x	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	2
$f(x)$	1	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{4}{7}$	$\frac{1}{2}$

$$T = \frac{1}{8}\left(1 + 2\left(\frac{4}{5}\right) + 2\left(\frac{2}{3}\right) + 2\left(\frac{4}{7}\right) + \frac{1}{2}\right) \approx 0.697$$

(b) $f'(x) = -\frac{1}{x^2}, f''(x) = \frac{2}{x^3} > 0$ on $[1, 2]$

The approximation is an overestimate.

(c) $\int_1^2 \frac{1}{x} \, dx = \left[\ln|x|\right]_1^2 = \ln 2 \approx 0.693$

5. (a) $f(x) = \sqrt{x}$, $h = \frac{4-0}{4} = 1$

x	0	1	2	3	4
$f(x)$	0	1	$\sqrt{2}$	$\sqrt{3}$	2

$$T = \frac{1}{2}(0 + 2(1) + 2(\sqrt{2}) + 2(\sqrt{3}) + 2) \approx 5.146$$

(b) $f'(x) = \frac{1}{2}x^{-1/2}$, $f''(x) = -\frac{1}{4}x^{-3/2} < 0$ on $[0, 4]$

The approximation is an underestimate.

(c) $\int_0^4 \sqrt{x} dx = \left[\frac{2}{3}x^{3/2} \right]_0^4 = \frac{16}{3} \approx 5.333$

6. (a) $f(x) = \sin x$, $h = \frac{\pi-0}{4} = \frac{\pi}{4}$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$f(x)$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0

$$T = \frac{\pi}{8}\left(0 + 2\left(\frac{\sqrt{2}}{2}\right) + 2(1) + 2\left(\frac{\sqrt{2}}{2}\right) + 0\right) \approx 1.896$$

(b) $f'(x) = \cos x$, $f''(x) = -\sin x < 0$ on $[0, \pi]$

The approximation is an underestimate.

(c) $\int_0^\pi \sin x dx = [-\cos x]_0^\pi = 2$

7. $\frac{5}{2}(6.0 + 2(8.2) + 2(9.1) + \dots + 2(12.7) + 13.0)(30)$
 $= 15,990 \text{ ft}^3$

8. (a) $\frac{200}{2}(0 + 2(520) + 2(800) + 2(1000) + \dots + 2(860) + 0)(20) = 26,360,000 \text{ ft}^3$

(b) You plan to start with 26,360 fish. You intend to have

$$(0.75)(26,360) = 19,770 \text{ fish to be caught. Since}$$

$$\frac{19,770}{20} = 988.5, \text{ the town can sell at most 988 licenses.}$$

9. Sum the trapezoids and multiply by $\frac{1}{3600}$ to change seconds to hours

$$\begin{aligned} & \frac{1}{2}(2.2(0 + 30) + (3.2 - 2.2)(30 + 40) \\ & + (4.5 - 3.2)(40 + 50) + (5.9 - 4.5)(50 + 60) \\ & + (7.8 - 5.9)(60 + 70) + (10.2 - 7.8)(70 + 80) \\ & + (12.7 - 10.2)(80 + 90) + (16.0 - 12.7)(90 + 100) \\ & + (20.6 - 16.0)(100 + 110) \\ & + (26.2 - 20.6)(110 + 120) \\ & + (37.1 - 26.2)(120 + 130))\left(\frac{1}{3600}\right) \approx 0.9785 \text{ miles} \end{aligned}$$

10. (a) $f(x) = x^3 - 2x$, $h = \frac{3 - (-1)}{4} = 1$

x	-1	0	1	2	3
$f(x)$	1	0	-1	4	21

$$S = \frac{1}{3}(1 + 4(0) + 2(-1) + 4(4) + 21) = 12$$

(b) $\int_{-1}^3 (x^3 - 2x) dx = \left[\frac{1}{4}x^4 - x^2 \right]_{-1}^3$
 $= \left(\frac{81}{4} - 9\right) - \left(\frac{1}{4} - 1\right)$
 $= 12$

$$|E_s| = 0$$

(c) For $f(x) = x^3 - 2x$, $M_{f(4)} = 0$ since $f^{(4)} = 0$.

(d) Simpson's Rule for cubic polynomials will always give exact values since $f^{(4)} = 0$ for all cubic polynomials.

11. The average of the 13 discrete temperatures gives equal weight to the low values at the end.

12. (a) $\frac{1}{2}(126 + 2 \cdot 65 + 2 \cdot 66 + \dots + 2 \cdot 58 + 110) = 841$
 $av(f) \approx \frac{1}{12} \cdot 841 \approx 70.08$

(b) We are approximating the area under the temperature graph. By doubling the endpoints, the error in the first and last trapezoids increases.

13. $S_{50} \approx 3.13791$, $S_{100} \approx 3.14029$

14. $S_{50} \approx 1.08943$, $S_{100} \approx 1.08943$

15. $S_{50} = 1.37066$, $S_{100} = 1.37066$ using $a = 0.0001$ as lower limit

$S_{50} = 1.37076$, $S_{100} = 1.37076$ using $a = 0.000000001$ as lower limit

16. $S_{50} \approx 0.82812$, $S_{100} \approx 0.82812$

17. (a) $T_{10} \approx 1.983523538$

$$T_{100} \approx 1.999835504$$

$$T_{1000} \approx 1.999998355$$

(b)

n	$ E_T = 2 - T_n$
10	$0.016476462 = 1.6476462 \times 10^{-2}$
100	1.64496×10^{-4}
1000	1.645×10^{-6}

(c) $|E_{T_{10n}}| \approx 10^{-2}|E_{T_n}|$

(d) $b - a = \pi$, $h^2 = \frac{\pi^2}{n^2}$, $M = 1$

$$|E_{T_n}| \leq \frac{\pi}{12} \left(\frac{\pi^2}{n^2} \right) = \frac{\pi^3}{12n^2}$$

$$|E_{T_{10n}}| \leq \frac{\pi^3}{12(10n)^2} = 10^{-2}|E_{T_n}|$$

18. (a) $S_{10} \approx 2.000109517$
 $S_{100} \approx 2.000000011$
 $S_{1000} \approx 2.000000000$

(b)

n	$ E_S = 2 - S_n$
10	1.09517×10^{-4}
100	1.1×10^{-8}
1000	0

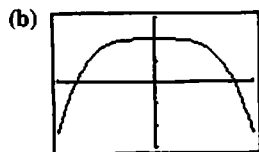
(c) $|E_{S_{10n}}| \approx 10^{-4} |E_{S_n}|$

(d) $b - a = \pi, h^4 = \frac{\pi^4}{n^4}, M = 1$

$$|E_{S_n}| \leq \frac{\pi}{180} \left(\frac{\pi^4}{n^4} \right) = \frac{\pi^5}{180n^4}$$

$$|E_{S_{10n}}| \leq \frac{\pi^5}{180(10n)^4} = 10^{-4} |E_{S_n}|$$

19. (a) $f'(x) = 2x \cos(x^2)$
 $f''(x) = 2x \cdot -2x \sin(x^2) + 2 \cos(x^2)$
 $= -4x^2 \sin(x^2) + 2 \cos(x^2)$



$[-1, 1]$ by $[-3, 3]$

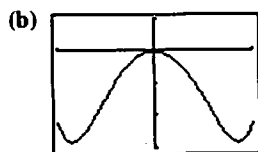
(c) The graph shows that $-3 \leq f''(x) \leq 2$ so $|f''(x)| \leq 3$ for $-1 \leq x \leq 1$.

(d) $|E_T| \leq \frac{1 - (-1)}{12} (h^2)(3) = \frac{h^2}{2}$

(e) For $0 < h \leq 0.1$, $|E_T| \leq \frac{h^2}{2} \leq \frac{0.1^2}{2} = 0.005 < 0.01$

(f) $n \geq \frac{1 - (-1)}{h} \geq \frac{2}{0.1} = 20$

20. (a) $f'''(x) = -4x^2 \cdot 2x \cos(x^2) - 8x \sin(x^2) - 4x \sin(x^2)$
 $= -8x^3 \cos(x^2) - 12x \sin(x^2)$
 $f^{(4)}(x) = -8x^3 \cdot -2x \sin(x^2) - 24x^2 \cos(x^2)$
 $- 12x \cdot 2x \cos(x^2) - 12 \sin(x^2)$
 $= (16x^4 - 12) \sin(x^2) - 48x^2 \cos(x^2)$



$[-1, 1]$ by $[-30, 10]$

(c) The graph shows that $-30 \leq f^{(4)}(x) \leq 10$ so $|f^{(4)}(x)| \leq 30$ for $-1 \leq x \leq 1$.

(d) $|E_S| \leq \frac{1 - (-1)}{180} (h^4)(30) = \frac{h^4}{3}$

(e) For $0 < h \leq 0.4$, $|E_S| \leq \frac{h^4}{3} \leq \frac{0.4^4}{3} \approx 0.00853 < 0.01$

(f) $n \geq \frac{1 - (-1)}{h} \geq \frac{2}{0.4} = 5$

21. $h = \frac{24 \text{ in.}}{6} = 4 \text{ in.}$

Estimate the area to be

$$\frac{4}{3} [0 + 4(18.75) + 2(24) + 4(26) + 2(24) + 4(18.75) + 0] \approx 466.67 \text{ in}^2$$

22. Note that the tank cross-section is represented by the shaded area, not the entire wing cross-section. Using Simpson's Rule, estimate the cross-section area to be

$$\frac{1}{3} [y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + y_6]$$

$$= \frac{1}{3} [1.5 + 4(1.6) + 2(1.8) + 4(1.9) + 2(2.0)$$

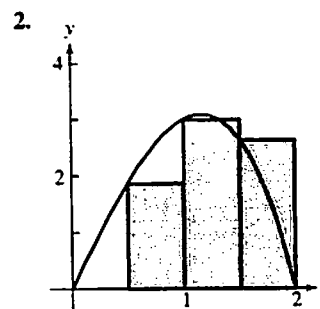
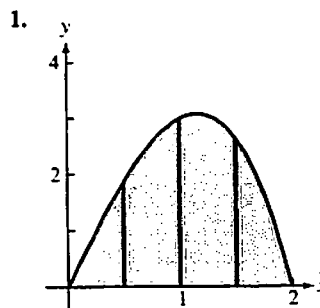
$$+ 4(2.1) + 2.1] = 11.2 \text{ ft}^2$$

$$\text{Length} \approx (5000 \text{ lb}) \left(\frac{1}{42 \text{ lb/ft}^3} \right) \left(\frac{1}{11.2 \text{ ft}^2} \right) \approx 10.63 \text{ ft}$$

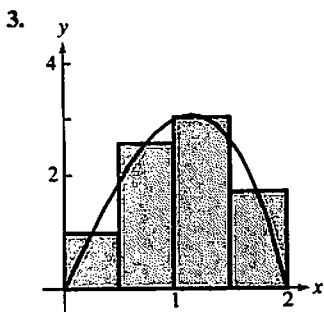
23. $T_n = \frac{h}{2} [y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n]$
 $= \frac{h[y_0 + y_1 + \dots + y_{n-1}] + h[y_1 + y_2 + \dots + y_n]}{2}$
 $= \frac{\text{LRAM}_n + \text{RRAM}_n}{2}$

24. $S_{2n} = \frac{h}{3} (y_0 + 4y_1 + 2y_2 + 4y_3 + \dots + 2y_{2n-2}$
 $+ 4y_{2n-1} + y_{2n})$
 $= \frac{1}{3} [h(y_0 + 2y_1 + 2y_2 + \dots + 2y_{2n-1} + y_{2n})$
 $+ (2h)(y_1 + y_3 + y_5 + \dots + y_{2n-1})]$
 $= \frac{2T_{2n} + \text{MRAM}_n}{3}$, where $h = \frac{b-a}{2n}$.

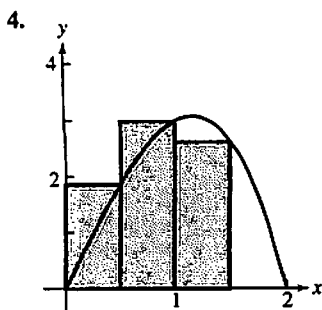
Chapter 5 Review (pp. 298–301)



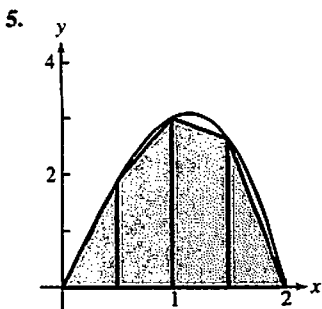
$$\text{LRAM}_4: \frac{1}{2} \left(0 + \frac{15}{8} + 3 + \frac{21}{8} \right) = \frac{15}{4} = 3.75$$



$$\text{MRAM}_4: \frac{1}{2} \left(\frac{63}{64} + \frac{165}{64} + \frac{195}{64} + \frac{105}{64} \right) = 4.125$$



$$\text{RRAM}_4: \frac{1}{2} \left(\frac{15}{8} + 3 + \frac{21}{8} + 0 \right) = \frac{15}{4} = 3.75$$



$$T_4 = \frac{1}{2}(\text{LRAM}_4 + \text{RRAM}_4) = \frac{1}{2} \left(\frac{15}{4} + \frac{15}{4} \right) = 3.75$$

6. $\int_0^2 (4x - x^3) dx = \left[2x^2 - \frac{1}{4}x^4 \right]_0^2 = 8 - 4 = 4$

7.

n	LRAM_n	MRAM_n	RRAM_n
10	1.78204	1.60321	1.46204
20	1.69262	1.60785	1.53262
30	1.66419	1.60873	1.55752
50	1.64195	1.60918	1.57795
100	1.62557	1.60937	1.59357
1000	1.61104	1.60944	1.60784

8. $\int_1^5 \frac{1}{x} dx = \left[\ln |x| \right]_1^5 = \ln 5 - \ln 1 = \ln 5 \approx 1.60944$

9. (a) $\int_5^2 f(x) dx = -\int_2^5 f(x) dx = -3$

The statement is true.

(b) $\int_{-2}^5 [f(x) + g(x)] dx$
 $= \int_{-2}^5 f(x) dx + \int_{-2}^5 g(x) dx$
 $= \int_{-2}^2 f(x) dx + \int_2^5 f(x) dx + \int_{-2}^5 g(x) dx$
 $= 4 + 3 + 2 = 9$

The statement is true.

(c) If $f(x) \leq g(x)$ on $[-2, 5]$, then $\int_{-2}^5 f(x) dx \leq \int_{-2}^5 g(x) dx$,

but this is not true since

$$\int_{-2}^5 f(x) dx = \int_{-2}^2 f(x) dx + \int_2^5 f(x) dx = 4 + 3 = 7 \text{ and}$$

$$\int_{-2}^5 g(x) dx = 2. \text{ The statement is false.}$$

10. (a) Volume of one cylinder: $\pi r^2 h = \pi \sin^2(m_i) \Delta x$

Total volume: $V = \lim_{n \rightarrow \infty} \sum_{i=1}^n \pi \sin^2(m_i) \Delta x$

(b) Use $\pi \sin^2 x$ on $[0, \pi]$.

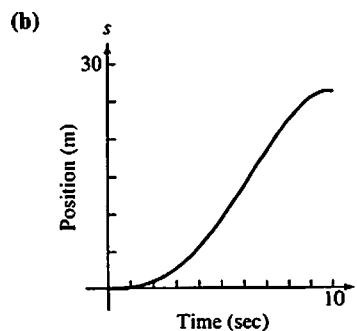
$$\text{NINT}(\pi \sin^2 x, x, 0, \pi) \approx 4.9348$$

11. (a) Approximations may vary. Using Simpson's Rule, the

area under the curve is approximately

$$\frac{1}{3} [0 + 4(0.5) + 2(1) + 4(2) + 2(3.5) + 4(4.5) + 2(4.75) + 4(4.5) + 2(3.5) + 4(2) + 0] = 26.5$$

The body traveled about 26.5 m.



The curve is always increasing because the velocity is always positive, and the graph is steepest when the velocity is highest, at $t = 6$.

12. (a) $\int_0^{10} x^3 dx$

(b) $\int_0^{10} x \sin x dx$

(c) $\int_0^{10} x(3x-2)^2 dx$

(d) $\int_0^{10} \frac{1}{1+x^2} dx$

(e) $\int_0^{10} \pi(9 - \sin^2 \frac{\pi x}{10}) dx$

13. The graph is above the x -axis for $0 \leq x < 4$ and below the x -axis for $4 < x \leq 6$

$$\begin{aligned} \text{Total area} &= \int_0^4 (4-x) dx - \int_4^6 (4-x) dx \\ &= \left[4x - \frac{1}{2}x^2\right]_0^4 - \left[4x - \frac{1}{2}x^2\right]_4^6 \\ &= [8-0] - [6-8] = 10 \end{aligned}$$

14. The graph is above the x -axis for $0 \leq x < \frac{\pi}{2}$ and below the x -axis for $\frac{\pi}{2} < x \leq \pi$

$$\begin{aligned} \text{Total area} &= \int_0^{\pi/2} \cos x dx - \int_{\pi/2}^{\pi} \cos x dx \\ &= \left[\sin x\right]_0^{\pi/2} - \left[\sin x\right]_{\pi/2}^{\pi} \\ &= (1-0) - (0-1) = 2 \end{aligned}$$

15. $\int_{-2}^2 5 dx = \left[5x\right]_{-2}^2 = 10 - (-10) = 20$

16. $\int_2^5 4x dx = \left[2x^2\right]_2^5 = 50 - 8 = 42$

17. $\int_0^{\pi/4} \cos x dx = \left[\sin x\right]_0^{\pi/4} = \frac{\sqrt{2}}{2} - 0 = \frac{\sqrt{2}}{2}$

18. $\int_{-1}^1 (3x^2 - 4x + 7) dx = \left[x^3 - 2x^2 + 7x\right]_{-1}^1 = 6 - (-10) = 16$

19. $\int_0^1 (8s^3 - 12s^2 + 5) ds = \left[2s^4 - 4s^3 + 5s\right]_0^1 = 3 - 0 = 3$

20. $\int_1^2 \frac{4}{x^2} dx = \left[-\frac{4}{x}\right]_1^2 = -2 - (-4) = 2$

21. $\int_1^{27} y^{-4/3} dy = \left[-3y^{-1/3}\right]_1^{27} = -1 - (-3) = 2$

22. $\int_1^4 \frac{dt}{t\sqrt{t}} = \int_1^4 t^{-3/2} dt = \left[-2t^{-1/2}\right]_1^4 = -1 - (-2) = 1$

23. $\int_0^{\pi/3} \sec^2 \theta d\theta = \left[\tan \theta\right]_0^{\pi/3} = \sqrt{3} - 0 = \sqrt{3}$

24. $\int_1^e \frac{1}{x} dx = \left[\ln |x|\right]_1^e = 1 - 0 = 1$

25. $\int_0^1 \frac{36}{(2x+1)^3} dx = \int_0^1 36(2x+1)^{-3} dx = \left[-9(2x+1)^{-2}\right]_0^1 = -1 - (-9) = 8$

26. $\int_1^2 \left(x + \frac{1}{x^2}\right) dx = \int_1^2 (x + x^{-2}) dx = \left[\frac{1}{2}x^2 - x^{-1}\right]_1^2 = \frac{3}{2} - \left(-\frac{1}{2}\right) = 2$

27. $\int_{-\pi/3}^0 \sec x \tan x dx = \left[\sec x\right]_{-\pi/3}^0 = 1 - 2 = -1$

28. $\int_{-1}^1 2x \sin(1-x^2) dx = \left[\cos(1-x^2)\right]_{-1}^1 = 1 - 1 = 0$

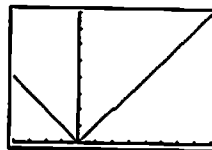
29. $\int_0^2 \frac{2}{y+1} dy = \left[2 \ln |y+1|\right]_0^2 = 2 \ln 3 - 0 = 2 \ln 3$

30. Graph $y = \sqrt{4-x^2}$ on $[0, 2]$.

[-1.35, 3.35] by [-0.5, 2.6]

The region under the curve is a quarter of a circle of radius 2.

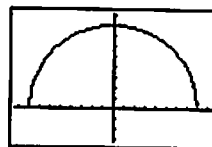
$$\int_0^2 \sqrt{4-x^2} dx = \frac{1}{4}\pi(2)^2 = \pi$$

31. Graph $y = |x|$ on $[-4, 8]$.

[-4, 8] by [0, 8]

The region under the curve consists of two triangles.

$$\int_{-4}^8 |x| dx = \frac{1}{2}(4)(4) + \frac{1}{2}(8)(8) = 40$$

32. Graph $y = \sqrt{64-x^2}$ on $[-8, 8]$.

[-9.4, 9.4] by [-3.2, 9.2]

The region under the curve $y = \sqrt{64-x^2}$ is half a circle of radius 8.

$$\int_{-8}^8 2\sqrt{64-x^2} dx = 2\int_{-8}^8 \sqrt{64-x^2} dx = 2\left[\frac{1}{2}\pi(8)^2\right] = 64\pi$$

33. (a) Note that each interval is 1 day = 24 hours
 Upper estimate:
 $24(0.020 + 0.021 + 0.023 + 0.025 + 0.028$
 $+ 0.031 + 0.035) = 4.392 \text{ L}$
 Lower estimate:
 $24(0.019 + 0.020 + 0.021 + 0.023 + 0.025$
 $+ 0.028 + 0.031) = 4.008 \text{ L}$

(b) $\frac{24}{2}[0.019 + 2(0.020) + 2(0.021) + \dots + 2(0.031)$
 $+ 0.035] = 4.2 \text{ L}$

34. (a) Upper estimate:
 $3(5.30 + 5.25 + 5.04 + \dots + 1.11) = 103.05 \text{ ft}$
 Lower estimate:
 $3(5.25 + 5.04 + 4.71 + \dots + 0) = 87.15 \text{ ft}$

(b) $\frac{3}{2}[5.30 + 2(5.25) + 2(5.04) + \dots + 2(1.11) + 0]$
 $= 95.1 \text{ ft}$

35. One possible answer:
 The Δx is important because it corresponds to the actual physical quantity Δx in a Riemann sum. Without the Δx , our integral approximations would be way off.

36. $\int_{-4}^4 f(x) dx = \int_{-4}^0 f(x) dx + \int_0^4 f(x) dx$
 $= \int_{-4}^0 (x - 2) dx + \int_0^4 x^2 dx$
 $= \left[\frac{1}{2}x^2 - 2x \right]_{-4}^0 + \left[\frac{1}{3}x^3 \right]_0^4$
 $= [0 - 16] + \left[\frac{64}{3} - 0 \right] = \frac{16}{3}$

37. Let $f(x) = \sqrt{1 + \sin^2 x}$
 $\max f = \sqrt{2}$ since $\max \sin^2 x = 1$
 $\min f = 1$ since $\min \sin^2 x = 0$
 $(\min f)(1 - 0) \leq \int_0^1 \sqrt{1 + \sin^2 x} dx \leq (\max f)(1 - 0)$
 $0 < 1 \leq \int_0^1 \sqrt{1 + \sin^2 x} dx \leq \sqrt{2}$

38. (a) $av(y) = \frac{1}{4-0} \int_0^4 \sqrt{x} dx = \frac{1}{4} \left[\frac{2}{3} x^{3/2} \right]_0^4 = \frac{1}{4} \left(\frac{16}{3} - 0 \right) = \frac{4}{3}$

(b) $av(y) = \frac{1}{a-0} \int_0^a \sqrt{x} dx = \frac{1}{a} \left[\frac{2}{3} ax^{3/2} \right]_0^a = \frac{2}{3} a^{3/2}$

39. $\frac{dy}{dx} = \sqrt{2 + \cos^3 x}$

40. $\frac{dy}{dx} = \sqrt{2 + \cos^3(7x^2)} \cdot \frac{d}{dx}(7x^2) = 14x\sqrt{2 + \cos^3(7x^2)}$

41. $\frac{dy}{dx} = \frac{d}{dx} \left(- \int_1^x \frac{6}{3+t^4} dt \right) = - \frac{6}{3+x^4}$

42. $\frac{dy}{dx} = \frac{d}{dx} \left(\int_0^{2x} \frac{1}{t^2+1} dt - \int_0^x \frac{1}{t^2+1} dt \right)$
 $= \frac{1}{(2x)^2+1} \cdot 2 - \frac{1}{x^2+1}$
 $= \frac{2}{4x^2+1} - \frac{1}{x^2+1}$

43. $c(x) = \int_{25}^x \frac{2}{\sqrt{t}} dt + 50$
 $= \left[4t^{1/2} \right]_{25}^x + 50$
 $= 4\sqrt{x} - 20 + 50$
 $= 4\sqrt{x} + 30$

$c(2500) = 4\sqrt{2500} + 30 = 230$

The total cost for printing 2500 newsletters is \$230.

44. $av(I) = \frac{1}{14} \int_0^{14} (600 + 600t) dt$
 $= \frac{1}{14} [600t + 300t^2]_0^{14} = 4800$

Rich's average daily inventory is 4800 cases.

$c(t) = 0.04I(t) = 24 + 24t$

$av(c) = \frac{1}{14} \int_0^{14} (24 + 24t) dt = \frac{1}{14} [24t + 12t^2]_0^{14} = 192$

Rich's average daily holding cost is \$192.

We could also say $(0.04)4800 = 192$.

45. $\int_0^x (t^3 - 2t + 3) dt = \left[\frac{1}{4}t^4 - t^2 + 3t \right]_0^x$
 $= \frac{1}{4}x^4 - x^2 + 3x$

$\frac{1}{4}x^4 - x^2 + 3x = 4$

$\frac{1}{4}x^4 - x^2 + 3x - 4 = 0$

$x^4 - 4x^2 + 12x - 16 = 0$

Using a graphing calculator, $x \approx -3.09131$

or $x \approx 1.63052$.

46. (a) True, because $g'(x) = f(x)$.
 (b) True, because g is differentiable.
 (c) True, because $g'(1) = f(1) = 0$.
 (d) False, because $g''(1) = f'(1) > 0$.
 (e) True, because $g'(1) = f(1) = 0$ and $g''(1) = f'(1) > 0$.
 (f) False, because $g''(1) = f'(1) \neq 0$.
 (g) True, because $g'(x) = f(x)$, and f is an increasing function which includes the point $(1, 0)$.

47. $\int_0^1 \sqrt{1+x^4} dx = F(1) - F(0)$

48. $y(x) = \int_5^x \frac{\sin t}{t} dt + 3$

$$49. y' = 2x + \frac{1}{x}$$

$$y'' = 2 - \frac{1}{x^2}$$

Thus, it satisfies condition i.

$$y(1) = 1 + \int_1^1 \frac{1}{t} dt + 1 = 1 + 0 + 1 = 2$$

$$y'(1) = 2 + \frac{1}{1} = 2 + 1 = 3$$

Thus, it satisfies condition ii.

50. Graph (b).

$$y = \int_1^x 2t dt + 4 = \left[t^2 \right]_1^x + 4 = (x^2 - 1) + 4 = x^2 + 3$$

51. (a) Each interval is $5 \text{ min} = \frac{1}{12} \text{ h}$.

$$\frac{1}{24}[2.5 + 2(2.4) + 2(2.3) + \cdots + 2(2.4) + 2.3]$$

$$= \frac{29}{12} \approx 2.42 \text{ gal}$$

$$(b) (60 \text{ mi/h})\left(\frac{12}{29} \text{ h/gal}\right) \approx 24.83 \text{ mi/gal}$$

52. (a) Using the freefall equation $s = \frac{1}{2}gt^2$ from Section 3.4, the distance A falls in 4 seconds is $\frac{1}{2}(32)(4^2) = 256 \text{ ft}$.

When her parachute opens, her altitude is

$$6400 - 256 = 6144 \text{ ft.}$$

(b) The distance B falls in 13 seconds is

$$\frac{1}{2}(32)(13^2) = 2704 \text{ ft. When her parachute opens, her altitude is } 7000 - 2704 = 4296 \text{ ft.}$$

(c) Let t represent the number of seconds after A jumps.

For $t \geq 4$ sec, A's position is given by

$$S_A(t) = 6144 - 16(t - 4) = 6208 - 16t, \text{ so A lands at}$$

$t = \frac{6208}{16} = 388 \text{ sec}$. For $t \geq 45 + 13 = 58 \text{ sec}$, B's position is given by

$$S_B(t) = 4296 - 16(t - 58) = 5224 - 16t,$$

so B lands at $t = \frac{5224}{16} = 326.5 \text{ sec}$. B lands first.

53. (a) Area of the trapezoid $= \frac{1}{2}(2h)(y_1 + y_3) = h(y_1 + y_3)$

$$\text{Area of the rectangle} = (2h)y_2 = 2hy_2$$

$$h(y_1 + y_3) + 2(2hy_2) = h(y_1 + 4y_2 + y_3)$$

(b) Let $h = \frac{b-a}{2n}$.

$$S_{2n} = \frac{h}{3}[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 2y_{2n-2} + 4y_{2n-1} + y_{2n}]$$

$$= \frac{1}{3}[h(y_0 + 4y_1 + y_2) + h(y_2 + 4y_3 + y_4) + \cdots + h(y_{2n-2} + 4y_{2n-1} + y_{2n})]$$

Since each expression of the form

$h(y_{2i-2} + 4y_{2i-1} + y_{2i})$ is equal to twice the area of the i th of n rectangles plus the area of the i th of n trapezoids, $S_{2n} = \frac{2 \cdot \text{MRAM}_n + T_n}{3}$.

$$54. (a) g(1) = \int_1^1 f(t) dt = 0$$

$$(b) g(3) = \int_1^3 f(t) dt = -\frac{1}{2}(2)(1) = -1$$

$$(c) g(-1) = \int_1^{-1} f(t) dt = -\int_{-1}^1 f(t) dt = -\frac{1}{4}\pi(2)^2 = -\pi$$

(d) $g'(x) = f(x)$; Since $f(x) > 0$ for $-3 < x < 1$ and $f(x) < 0$ for $1 < x < 3$, $g(x)$ has a relative maximum at $x = 1$.

$$(e) g'(-1) = f(-1) = 2$$

The equation of the tangent line is

$$y - (-\pi) = 2(x + 1) \text{ or } y = 2x + 2 - \pi$$

(f) $g''(x) = f'(x)$, $f'(x) = 0$ at $x = -1$ and $f'(x)$ is not defined at $x = 2$. The inflection points are at $x = -1$ and $x = 2$. Note that $g''(x) = f'(x)$ is undefined at $x = 1$ as well, but since $g''(x) = f'(x)$ is negative on both sides of $x = 1$, $x = 1$ is not an inflection point.

(g) Note that the absolute maximum is $g(1) = 0$ and the absolute minimum is

$$g(-3) = \int_1^{-3} f(t) dt = -\int_{-3}^1 f(t) dt = -\frac{1}{2}\pi(2)^2 = -2\pi.$$

The range of g is $[-2\pi, 0]$.

$$55. (a) \text{NINT}(e^{-x^2/2}, x, -10, 10) \approx 2.506628275$$

$$\text{NINT}(e^{-x^2/2}, x, -20, 20) \approx 2.506628275$$

(b) The area is $\sqrt{2\pi}$.

56. First estimate the surface area of the swamp.

$$\frac{20}{2}[146 + 2(122) + 2(76) + 2(54) + 2(40) + 2(30) + 13] = 8030 \text{ ft}^2$$

$$(5 \text{ ft})(8030 \text{ ft}^2) \cdot \frac{1 \text{ yd}^3}{27 \text{ ft}^3} \approx 1500 \text{ yd}^3$$

57. (a) $V^2 = (V_{\max})^2 \sin^2(120\pi t)$

Using NINT:

$$\begin{aligned} \text{av}(V^2) &= \frac{1}{1} \int_0^1 (V_{\max})^2 \sin^2(120\pi t) dt \\ &= (V_{\max})^2 \int_0^1 \sin^2(120\pi t) dt = (V_{\max})^2 \frac{1}{2} = \frac{(V_{\max})^2}{2} \\ V_{\text{rms}} &= \sqrt{\frac{(V_{\max})^2}{2}} = \frac{V_{\max}}{\sqrt{2}} \end{aligned}$$

(b) $V_{\max} = 240\sqrt{2} \approx 339.41$ volts

Chapter 6

Differential Equations and Mathematical Modeling

Section 6.1 Antiderivatives and Slope Fields (pp. 303–315)

Exploration 1 Constructing a Slope Field

- As i and j vary from 1 to 10, 100 ordered pairs are produced. Each ordered pair represents a distinct point in the viewing window.
- The distance between the points with j fixed and $i = r$ and $i = r + 1$ is the distance between their x -coordinates.

$$\begin{aligned} & \left[X_{\min} + \left(2(r+1) - 1\right) \frac{h}{2} \right] - \left[X_{\min} + (2r - 1) \frac{h}{2} \right] \\ &= (X_{\min} - X_{\min}) + (2r + 2 - 1 - 2r + 1) \frac{h}{2} = h \end{aligned}$$

- The distance between the points with i fixed and $j = r$ and $j = r + 1$ is the distance between their y -coordinates.

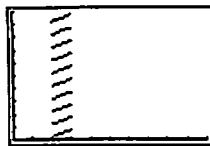
$$\begin{aligned} & \left[Y_{\min} + (2(r+1) - 1) \frac{k}{2} \right] - \left[Y_{\min} + (2r - 1) \frac{k}{2} \right] \\ &= (Y_{\min} - Y_{\min}) + (2r + 2 - 1 - 2r + 1) \frac{k}{2} = k \end{aligned}$$

- Here $h = k = 1$. Each line segment in the third column has slope $\frac{4}{7}$, because the x -coordinate of the midpoint of each line segment is 2.5. The y -coordinates are $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots, \frac{19}{2}$.

The 10 graphs are graphs of the functions

$$y = \left(\frac{4}{7}\right)(x - 2.5) + \frac{n}{2}, \quad 2 \leq x \leq 3, \quad \text{for } n = 1, 3, 5, \dots, 19.$$

The length of the line segment can be increased or decreased by adjusting the restriction $2 \leq x \leq 3$.



[0, 10] by [0, 10]

- Again $h = k = 1$. The y -coordinate of the midpoint of each line segment is $\frac{7}{2}$. The x -coordinates of the midpoint of

each line segment are $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots, \frac{19}{2}$. From left to right the slopes of the line segments are

$$\frac{2}{\frac{1}{2} + 1}, \frac{2}{\frac{3}{2} + 1}, \frac{2}{\frac{5}{2} + 1}, \dots, \frac{2}{\frac{19}{2} + 1}$$

The 10 graphs are graphs of the functions.

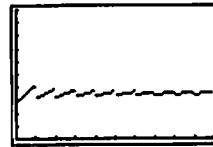
$$y_1 = \left(\frac{2}{\frac{1}{2} + 1}\right)\left(x - \frac{1}{2}\right) + \frac{7}{2}, \quad 0 \leq x \leq 1,$$

$$y_2 = \left(\frac{2}{\frac{3}{2} + 1}\right)\left(x - \frac{3}{2}\right) + \frac{7}{2}, \quad 1 \leq x \leq 2,$$

$$y_3 = \left(\frac{2}{\frac{5}{2} + 1}\right)\left(x - \frac{5}{2}\right) + \frac{7}{2}, \quad 2 \leq x \leq 3,$$

⋮

$$y_{10} = \left(\frac{2}{\frac{19}{2} + 1}\right)\left(x - \frac{19}{2}\right) + \frac{7}{2}, \quad 9 \leq x \leq 10.$$



[0, 10] by [0, 10]

- For each line segment in part (5), make a column of parallel line segments as in part (4).

7. WZ

Quick Review 6.1

- $100(1.06) = \$106.00$
- $100\left(1 + \frac{0.06}{4}\right)^4 \approx \106.14
- $100\left(1 + \frac{0.06}{12}\right)^{12} \approx \106.17
- $100\left(1 + \frac{0.06}{365}\right)^{365} \approx \106.18
- $\frac{dy}{dx} = \frac{d}{dx} \sin 3x = (\cos 3x)(3) = 3 \cos 3x$
- $\frac{dy}{dx} = \frac{d}{dx} \tan \frac{5}{2}x = \left(\sec^2 \frac{5}{2}x\right)\left(\frac{5}{2}\right) = \frac{5}{2} \sec^2 \frac{5}{2}x$
- $\frac{dy}{dx} = \frac{d}{dx} Ce^{2x} = (Ce^{2x})(2) = 2Ce^{2x}$
- $\frac{dy}{dx} = \frac{d}{dx} \ln(x+2) = \frac{1}{x+2}$