Ratio, Proportion, and Similarity

Objectives

1. Express a ratio in simplest form.
2. Solve for an unknown term in a given proportion.
3. Express a given proportion in an equivalent form.
4. State and apply the properties of similar polygons.

5-1 Ratio and Proportion

The **ratio** of one number to another is the quotient when the first number is divided by the second. This quotient is usually expressed in *simplest form.*

The ratio of 8 to 12 is $\frac{8}{12}$, or $\frac{2}{3}$.

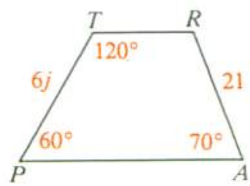
If $y \neq 0$, the ratio of x to y is $\frac{x}{y}$.

Since we cannot divide by zero, a ratio $\frac{r}{s}$ is defined only if $s \neq 0$. When an expression such as $\frac{r}{s}$ appears in this book, you may assume that $s \neq 0$.

Ratios can be used to compare two numbers. To find the ratio of the lengths of two segments, the segments must be measured in terms of the same unit.

Example 1 **a.** Find the ratio of TP to RA.

b. Find the ratio of the measure of the largest angle of the trapezoid to that of the smallest angle.



Solution **a.** $\frac{TP}{RA} = \frac{6j}{21} = \frac{2j}{7}$.

The ratio of TP to RA is $2j$ to 7.

b. $\angle R$ has measure $180 - 70$, or 110. Thus $\angle T$ is the largest angle and $\angle P$ is the smallest angle.

$$\frac{m\angle T}{m\angle P} = \frac{120}{60} = \frac{2}{1}$$

The ratio of the measure of the largest angle of the trapezoid to that of the smallest angle is 2 to 1.

Example 2 A poster is 1 m long and 65 cm wide. Find the ratio of the width to the length.

Solution *Method 1*

Use centimeters.

1 m = 100 cm

$$\frac{\text{width}}{\text{length}} = \frac{65}{100} = \frac{13}{20}$$

Method 2

Use meters.

65 cm = 0.65 m

$$\frac{\text{width}}{\text{length}} = \frac{0.65}{1} = \frac{65}{100} = \frac{13}{20}$$

Example 2 shows that the ratio of two quantities is not affected by the unit chosen.

Sometimes the ratio of a to b is written in the form $a{:}b$. This form can also be used to compare three or more numbers. The statement that three numbers are in the ratio $c{:}d{:}e$ (read "c to d to e") means:

(1) The ratio of the first two numbers is $c{:}d$.
(2) The ratio of the last two numbers is $d{:}e$.
(3) The ratio of the first and last numbers is $c{:}e$.

Example 3 The measures of the three angles of a triangle are in the ratio 2:2:5. Find the measure of each angle.

Solution Let $2x$, $2x$, and $5x$ represent the measures.

$$\begin{aligned} 2x + 2x + 5x &= 180 \\ 9x &= 180 \\ x &= 20 \end{aligned}$$

Then $2x = 40$ and $5x = 100$.

The measures of the angles are 40, 40, and 100.

A **proportion** is an equation stating that two ratios are equal. For example,

$$\frac{a}{b} = \frac{c}{d} \quad \text{and} \quad a{:}b = c{:}d$$

are equivalent forms of the same proportion. Either form can be read "a is to b as c is to d." The number a is called the first *term* of the proportion. The numbers b, c, and d are the second, third, and fourth terms, respectively.

When three or more ratios are equal, you can write an *extended proportion:*

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$$

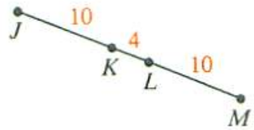
Classroom Exercises

Express the ratio in simplest form.

1. $\frac{15}{20}$ **2.** $\frac{4j}{7j}$ **3.** $\frac{4n}{n^2}$ **4.** $\frac{n^2}{4n}$

5. Compare your answers to Exercises 3 and 4. Is the ratio $a:b$ of two numbers always, sometimes, or never the same as the ratio $b:a$?

Express the ratio in simplest form.



6. $JK:KL$ **7.** $KL:JK$ **8.** $KL:JM$

9. $KM:LK$ **10.** $JL:LM$ **11.** $JK:KL:LM$

12. What is the ratio of 750 milliliters to 1.5 liters?

13. Can you find the ratio of 2 liters to 4 kilometers? Explain.

14. The ratio of the lengths of two segments is 4:3 when they are measured in centimeters. What is their ratio when they are measured in inches?

15. Three numbers aren't known, but the ratio of the numbers is 1:2:5. Is it possible that the numbers are 1, 2, and 5? 10, 20, and 50? 3, 6, and 20? x, $2x$, and $5x$?

16. What is the second term of the proportion $\frac{a}{b} = \frac{x}{y}$?

Written Exercises

***ABCD* is a parallelogram. Find the value of each ratio.**

A

1. $AB:BC$ **2.** $AB:CD$

3. $m\angle C:m\angle D$ **4.** $m\angle B:m\angle C$

5. AD: perimeter of $ABCD$

In Exercises 6–14, $x = 12$, $y = 8$, and $z = 24$. Write each ratio in simplest form.

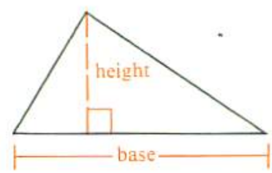
6. x to y **7.** z to x **8.** $x + y$ to z

9. $\frac{y}{x+z}$ **10.** $\frac{z+x}{z-x}$ **11.** $\frac{x-y}{x+y}$

12. $x:y:z$ **13.** $z:x:y$ **14.** $x:(x+y):(y+z)$

Exercises 15–20 refer to a triangle. Express the ratio of the height to the base in simplest form.

	15.	16.	17.	18.	19.	20.
height	1 m	0.4 km	40 cm	2 cm	3 km	80 mm
base	0.8 m	0.3 km	2 m	5 mm	150 m	0.5 m



Write the algebraic ratio in simplest form.

21. $\frac{3a}{4ab}$
22. $\frac{2cd}{5c^2}$
23. $\frac{3(x+4)}{a(x+4)}$
24. $\frac{10x}{5x}$
25. $\frac{3(x-y)}{(x-y)(x+y)}$
26. $\frac{a+5}{4a+20}$

In Exercises 27–32, find the measure of each angle.

B 27. The ratio of the measures of two complementary angles is 4:5. (*Hint:* Let $4x$ and $5x$ represent the measures.)

28. The ratio of the measures of two supplementary angles is 11:4.

29. The measures of the angles of a triangle are in the ratio 3:4:5.

30. The measures of the acute angles of a right triangle are in the ratio 5:7.

31. The measures of the angles of an isosceles triangle are in the ratio 3:3:4.

32. The measures of the angles of a hexagon are in the ratio 4:6:6:7:8:9.

33. The perimeter of a triangle is 96 cm and the lengths of its sides are in the ratio 9:11:12. Find the length of each side.

34. The measures of the consecutive angles of a quadrilateral are in the ratio 6:7:11:12. Find the measure of each angle, draw a quadrilateral that satisfies the requirements, and explain why two sides must be parallel.

35. What is the ratio of the measure of an interior angle to the measure of an exterior angle in a regular decagon? A regular n-gon?

36. A team's best hitter has a lifetime batting average of .320. He has been at bat 325 times.
   a. How many hits has he made?
   b. The player goes into a slump and doesn't get any hits at all in his next ten times at bat. Now what is his batting average to the nearest thousandth?

C 37. A basketball player has made 24 points out of 30 free throws. She hopes to make all her next free throws until her free-throw percentage is 85 or better. How many consecutive free throws will she have to make?

38. Points B and C lie on $\overline{AD}$. $\frac{AB}{BD} = \frac{3}{4}$, $\frac{AC}{CD} = \frac{5}{6}$, and $BD = 66$. Find AC.

39. Find the ratio of x to y: $\frac{4}{y} + \frac{3}{x} = 44$

$$\frac{12}{y} - \frac{2}{x} = 44$$

5-2 Properties of Proportions

The first and last terms of a proportion are called the *extremes.* The middle terms are the *means.* In the proportions below, the extremes are shown in red. The means are shown in black.

$$a:b = c:d \qquad 6:9 = 2:3 \qquad \frac{6}{9} = \frac{2}{3}$$

Notice that $6 \cdot 3 = 9 \cdot 2$. This illustrates a property of all proportions, called the *means-extremes* property of proportions:

The product of the extremes equals the product of the means.

$\frac{a}{b} = \frac{c}{d}$ is *equivalent* to $ad = bc$.

The two equations are equivalent because we can change either of them into the other by multiplying (or dividing) each side by bd. Try this yourself.

It is often necessary to replace one proportion by an equivalent proportion. When you do so in a proof, you may use the reason "A property of proportions." The following properties will be justified in the exercises.

Properties of Proportions

1. $\frac{a}{b} = \frac{c}{d}$ is equivalent to:

   a. $ad = bc$ b. $\frac{a}{c} = \frac{b}{d}$ c. $\frac{b}{a} = \frac{d}{c}$ d. $\frac{a+b}{b} = \frac{c+d}{d}$

2. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots$, then $\frac{a+c+e+\cdots}{b+d+f+\cdots} = \frac{a}{b} = \cdots$.

Example Use the proportion $\frac{x}{y} = \frac{3}{4}$ to complete each statement.

a. $3y = \underline{\ ?\ }$ b. $\frac{x+y}{y} = \frac{?}{?}$

c. $\frac{x}{3} = \frac{?}{?}$ d. $\frac{4}{3} = \frac{?}{?}$

Solution a. $3y = 4x$ b. $\frac{x+y}{y} = \frac{7}{4}$

c. $\frac{x}{3} = \frac{y}{4}$ d. $\frac{4}{3} = \frac{y}{x}$

Classroom Exercises

1. If $\frac{e}{f} = \frac{g}{h}$, which equation is correct?
   a. $ef = gh$ b. $eh = fg$ c. $eg = fh$
2. Which proportions are equivalent to $\frac{x}{12} = \frac{3}{4}$?
   a. $\frac{x}{3} = \frac{12}{4}$ b. $\frac{x}{4} = \frac{12}{3}$ c. $\frac{12}{x} = \frac{4}{3}$ d. $\frac{x + 12}{12} = \frac{7}{4}$

Complete the statement.

3. If $\frac{a}{b} = \frac{6}{5}$, then $5a = \underline{\ ?\ }$.
4. If $\frac{c}{d} = \frac{9}{4}$, then $\frac{d}{c} = \frac{?}{?}$.
5. If $\frac{e}{f} = \frac{7}{11}$, then $\frac{e}{7} = \frac{?}{?}$.
6. If $\frac{w}{x} = \frac{y}{z}$, then $\frac{w}{y} = \frac{?}{?}$.
7. If $\frac{a}{b} = \frac{2}{3}$, then $\frac{a + b}{b} = \frac{?}{?}$.
8. If $\frac{a}{b} = \frac{j}{k} = \frac{4}{7}$, then $\frac{a + j + 4}{b + k + 7} = \frac{?}{?}$.
9. a. Apply the means-extremes property of proportions to the proportion $\frac{e}{f} = \frac{g}{5}$ and you get $5e = \underline{\ ?\ }$.
   b. Apply the property to the proportion $\frac{5}{f} = \frac{g}{e}$ and you get $\underline{\ ?\ } = \underline{\ ?\ }$.
   c. Are the proportions $\frac{e}{f} = \frac{g}{5}$ and $\frac{5}{f} = \frac{g}{e}$ equivalent?
10. Explain an easy way to show that the proportions $\frac{x}{7} = \frac{2}{3}$ and $\frac{x}{2} = \frac{3}{7}$ are not equivalent.

What can you conclude from the given information?

11. $\frac{a}{b} = \frac{c}{n}$ and $\frac{b}{a} = \frac{x}{c}$
12. $\frac{3}{4} = \frac{y}{k}$ and $\frac{3}{v} = \frac{4}{k}$
13. Apply the means-extremes property to $\frac{a}{b} = \frac{c}{d}$ and also to $\frac{a}{c} = \frac{b}{d}$. (Note that you have justified Property 1(b) on page 209 by showing that each proportion is equivalent to the same equation.)
14. Explain why $\frac{a}{b} = \frac{c}{d}$ and $\frac{b}{a} = \frac{d}{c}$ are equivalent. (This justifies Property 1(c) on page 209.)

Written Exercises

Complete each statement.

A

1. If $\frac{x}{5} = \frac{3}{4}$, then $4x = \underline{\ ?\ }$.
2. If $\frac{7}{x} = \frac{3}{8}$, then $3x = \underline{\ ?\ }$.
3. If $n:3 = 7:8$, then $8n = \underline{\ ?\ }$.
4. If $4:g = 5:6$, then $5g = \underline{\ ?\ }$.

5. If $\frac{a}{4} = \frac{b}{7}$, then $\frac{a}{b} = \frac{?}{?}$.

6. If $\frac{x}{y} = \frac{3}{8}$, then $\frac{y}{x} = \frac{?}{?}$.

7. If $\frac{x}{2} = \frac{y}{3}$, then $\frac{x+2}{2} = \underline{\ ?\ }$.

8. If $\frac{a}{b} = \frac{5-x}{x}$, then $\frac{a+b}{b} = \underline{\ ?\ }$.

Find the value of x.

9. $\frac{x}{3} = \frac{4}{5}$

10. $\frac{x}{7} = \frac{3}{8}$

11. $\frac{2x}{5} = \frac{3}{4}$

12. $\frac{8}{x} = \frac{2}{5}$

13. $\frac{x+5}{4} = \frac{1}{2}$

14. $\frac{x+3}{2} = \frac{4}{3}$

15. $\frac{x+2}{x+3} = \frac{4}{5}$

16. $\frac{2x+1}{4x-1} = \frac{2}{3}$

17. $\frac{x+3}{2} = \frac{2x-1}{3}$

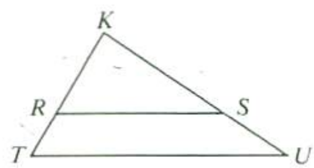
18. $\frac{10}{7x+5} = \frac{7}{6x-2}$

19. $\frac{x+5}{x-5} = \frac{7}{4}$

20. $\frac{4x-5}{4} = \frac{20x+1}{7}$

For the figure shown, it is given that $\frac{KR}{RT} = \frac{KS}{SU}$. Copy and complete the table.

	KR	RT	KT	KS	SU	KU
21.	12	9	?	16	?	?
22.	8	?	10	12	?	?
23.	16	?	?	?	10	30
24.	?	2	?	9	?	12
B 25.	?	?	12	10	5	?
26.	12	4	?	?	?	20
27.	?	9	36	?	?	48
28.	?	?	30	28	?	42



(*Hint for Ex. 25*: Let $KR = x$; then $RT = 12 - x$.)

29. Show that the proportions $\frac{a+b}{b} = \frac{c+d}{d}$ and $\frac{a}{b} = \frac{c}{d}$ are equivalent. (Note that this exercise justifies property 1(d) on page 209.)

30. Given the proportions $\frac{x+y}{y} = \frac{r}{s}$ and $\frac{x-y}{x+y} = \frac{s}{y}$, what can you conclude?

31. Show that the proportions $\frac{a-b}{a+b} = \frac{c-d}{c+d}$ and $\frac{a}{b} = \frac{c}{d}$ are equivalent.

32. Show that the proportions $\frac{a+c}{b+d} = \frac{a-c}{b-d}$ and $\frac{a}{b} = \frac{c}{d}$ are equivalent.

Find the value of x.

33. $\frac{x}{x-3} = \frac{x+4}{x}$

34. $\frac{x+2}{x+6} = \frac{x-1}{x+2}$

35. $\frac{x+1}{x-2} = \frac{x+5}{x-6}$

Find the value of x.

C 36. $\dfrac{x-2}{x-5} = \dfrac{2x+1}{x-1}$

37. $\dfrac{x(x+5)}{4x+4} = \dfrac{9}{5}$

38. $\dfrac{x-1}{x+2} = \dfrac{10}{3x-2}$

Find the values of x and y.

39. $\dfrac{x}{y+1} = \dfrac{3}{2}$

$\dfrac{x+y}{x-y} = \dfrac{7}{2}$

40. $\dfrac{x-3}{4} = \dfrac{y+2}{2}$

$\dfrac{x+y-1}{6} = \dfrac{x-y+1}{5}$

41. Prove: If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then $\frac{a+c+e}{b+d+f} = \frac{a}{b}$.

(*Hint:* Let $\frac{a}{b} = r$. Then $a = br$, $c = dr$, and $e = \underline{\ ?\ }$.)

42. Explain how to extend the proof of Exercise 41 to justify Property 2 on page 209.

43. If $\dfrac{4a-9b}{4a} = \dfrac{a-2b}{b}$, find the numerical value of the ratio $a:b$.

5-3 Similar Polygons

When you draw a diagram of a soccer field, you don't need an enormous piece of paper. You use a convenient sheet and draw *to scale*. That is, you show the right shape, but in a convenient size. Two figures, such as those below, that have the same shape are called *similar*.

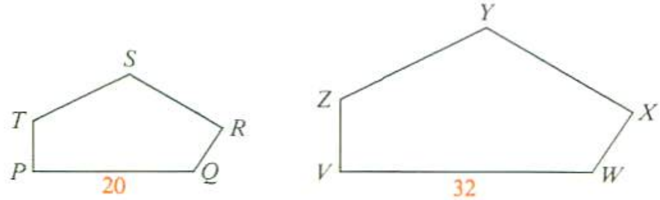
Two polygons are **similar** if their vertices can be paired so that:

(1) Corresponding angles are congruent.

(2) Corresponding sides are in proportion. (Their lengths have the same ratio.)

When you refer to similar polygons, their corresponding vertices must be in the same order. Given that polygon $PQRST$ is similar to polygon $VWXYZ$, you write:

$$\text{polygon } PQRST \sim \text{polygon } VWXYZ$$



From the definition of similar polygons, we have:

(1) $\angle P \cong \angle V$ $\quad \angle Q \cong \angle W$ $\quad \angle R \cong \angle X$ $\quad \angle S \cong \angle Y$ $\quad \angle T \cong \angle Z$

(2) $\dfrac{PQ}{VW} = \dfrac{QR}{WX} = \dfrac{RS}{XY} = \dfrac{ST}{YZ} = \dfrac{TP}{ZV}$

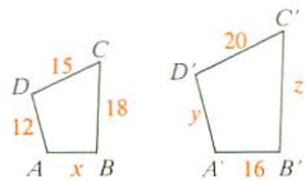
The ratio of the lengths of two corresponding sides is called the **scale factor** of the similarity. Since $\dfrac{PQ}{VW} = \dfrac{20}{32} = \dfrac{5}{8}$, the scale factor of pentagon $PQRST$ to pentagon $VWXYZ$ is $\dfrac{5}{8}$, or 5:8.

The example that follows shows one convenient way to label corresponding vertices: A and A' (read A prime), B and B', and so on.

Example Quad. $ABCD \sim$ quad. $A'B'C'D'$. Find:

a. the scale factor

b. the values of x, y, and z



Solution **a.** scale factor $= \dfrac{15}{20} = \dfrac{3}{4}$

b. $\dfrac{x}{16} = \dfrac{3}{4}$ $\qquad \dfrac{12}{y} = \dfrac{3}{4}$ $\qquad \dfrac{18}{z} = \dfrac{3}{4}$

$4x = 48$ $\qquad 3y = 48$ $\qquad 3z = 72$

$x = 12$ $\qquad y = 16$ $\qquad z = 24$

Classroom Exercises

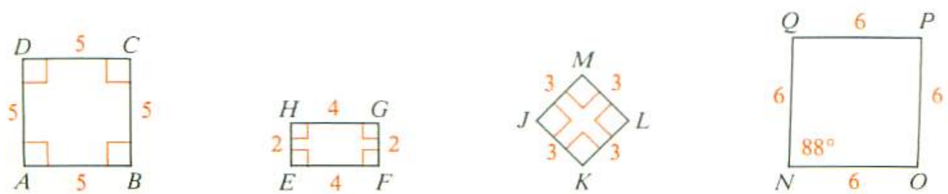
Are the quadrilaterals similar? If they aren't, tell why not.

1. $ABCD$ and $EFGH$

2. $ABCD$ and $JKLM$

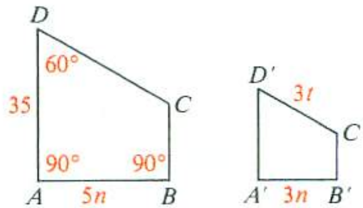
3. $ABCD$ and $NOPQ$

4. $JKLM$ and $NOPQ$



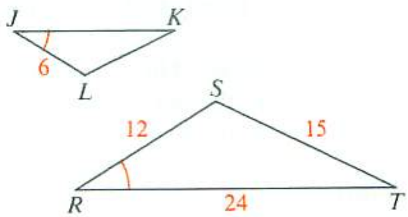
5. If the corresponding angles of two polygons are congruent, must the polygons be similar?
6. If the corresponding sides of two polygons are in proportion, must the polygons be similar?
7. Two polygons are similar. Do they have to be congruent?
8. Two polygons are congruent. Do they have to be similar?
9. Are all regular pentagons similar?
10. Are all isosceles right triangles similar?

11. Quad. $ABCD \sim$ quad. $A'B'C'D'$. Complete.
   a. $m\angle C' = \underline{\ ?\ }$ b. $A'D' = \underline{\ ?\ }$
   c. $DC = \underline{\ ?\ }$ d. Quad. $CBAD \sim \underline{\ ?\ }$
   e. Explain why
      quad. $ABCD \sim$ quad. $B'C'D'A'$
   is not a correct statement.



12. The lengths of the sides of a quadrilateral are 4, 6, 6, and 8. The lengths of the sides of a similar quadrilateral are 6, 9, 9, and 12.
   a. What is the scale factor?
   b. What are the perimeters of the two quadrilaterals?
   c. What is the ratio of the perimeters?

13. The triangles are similar. Complete.
   a. $\triangle RST \sim \underline{\ ?\ }$
   b. The scale factor is $\underline{\ ?\ }$.
   c. $JK = \underline{\ ?\ }$ and $KL = \underline{\ ?\ }$
   d. The perimeter of $\triangle JKL$ is $\underline{\ ?\ }$.
      The perimeter of $\triangle RST$ is $\underline{\ ?\ }$.
   e. The ratio of the perimeters is $\underline{\ ?\ }$.



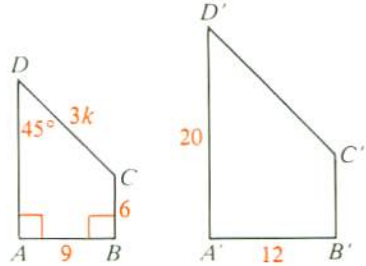
Written Exercises

Tell whether the two polygons are *always, sometimes,* or *never* similar.

A

1. Two equilateral triangles
2. Two right triangles
3. Two isosceles triangles
4. Two scalene triangles
5. Two squares
6. Two rectangles
7. Two rhombuses
8. Two isosceles trapezoids
9. A right triangle and an acute triangle
10. An isosceles triangle and a scalene triangle
11. A right triangle and a scalene triangle
12. An equilateral triangle and an equiangular triangle

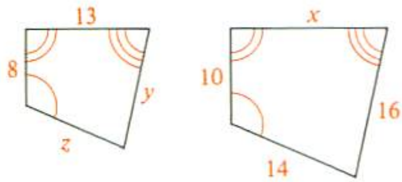
In Exercises 13–20, quad. $ABCD \sim$ quad. $A'B'C'D'$.



13. What is the scale factor of quad. $ABCD$ to quad. $A'B'C'D'$?
14. What special kind of figure must quad. $A'B'C'D'$ be? Explain.
15. Find $m\angle D'$.
16. Find $m\angle C'$.
17. Find $B'C'$.
18. Find AD.
19. Find $C'D'$.
20. Find the ratio of the perimeters.

Two similar polygons are shown. Find the values of x, y, and z. (In Exercise 23 find the values of x and y.)

B 21.



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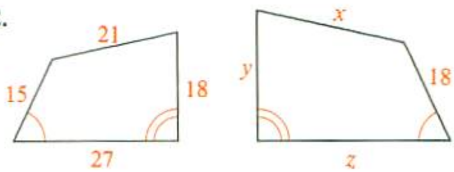

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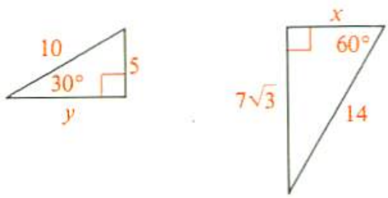

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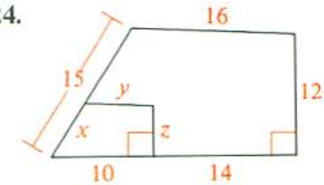
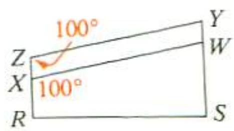
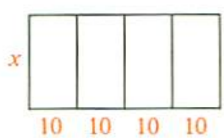


25. Draw two equilateral hexagons that are clearly not similar.
26. Draw two equiangular hexagons that are clearly not similar.
27. If $\triangle ABC \sim \triangle DEF$, express AB in terms of other lengths. (There are two possible answers.)
28. Explain how you can tell at once that quadrilateral $RSWX$ is not similar to quadrilateral $RSYZ$.

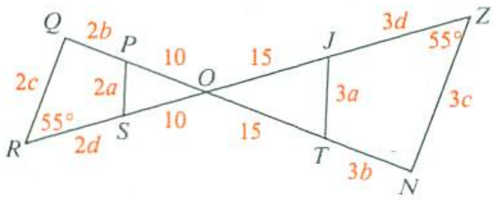


Plot the given points on graph paper. Draw quadrilateral $ABCD$ and $\overline{A'B'}$. Locate points C' and D' so that $A'B'C'D'$ is similar to $ABCD$.

29. $A(0, 0)$, $B(4, 0)$, $C(2, 4)$, $D(0, 2)$, $A'(-10, -2)$, $B'(-2, -2)$
30. $A(0, 0)$, $B(4, 0)$, $C(2, 4)$, $D(0, 2)$, $A'(7, 2)$, $B'(7, 0)$
31. The card shown was cut into four congruent pieces with each piece similar to the original. Find the value of x.



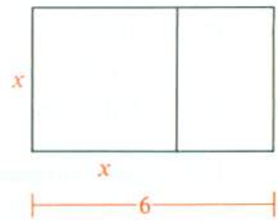
C **32.** What can you deduce from the diagram shown below? Explain.



33. The large rectangle shown is a *golden rectangle*. This means that when a square is cut off, the rectangle that remains is similar to the original rectangle.

a. How wide is the original rectangle?

b. The ratio of length to width in a golden rectangle is called the *golden ratio*. Write the golden ratio in simplified radical form. Then use a calculator to find an approximation to the nearest hundredth.



Self-Test 1

Express the ratio in simplest form.

1. 9 : 15 **2.** 60 cm to 2 m **3.** $\frac{4ab}{6b^2}$

Solve for x.

4. $\frac{x}{8} = \frac{9}{12}$ **5.** $\frac{x-2}{2} = \frac{x+6}{4}$ **6.** $\frac{x}{5-x} = \frac{12}{8}$

Tell whether the equation is equivalent to the proportion $\frac{a}{b} = \frac{5}{7}$.

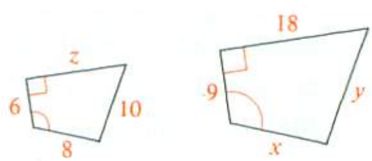
7. $\frac{a}{7} = \frac{b}{5}$ **8.** $7a = 5b$ **9.** $\frac{a+b}{b} = \frac{12}{7}$

10. If $\triangle ABC \sim \triangle RST$, $m\angle A = 45$, and $m\angle C = 60$, then $m\angle R =$ ___?___, $m\angle S =$ ___?___, and $m\angle T =$ ___?___.

The quadrilaterals shown are similar.

11. The scale factor of the smaller quadrilateral to the larger quadrilateral is $\frac{?}{?}$.

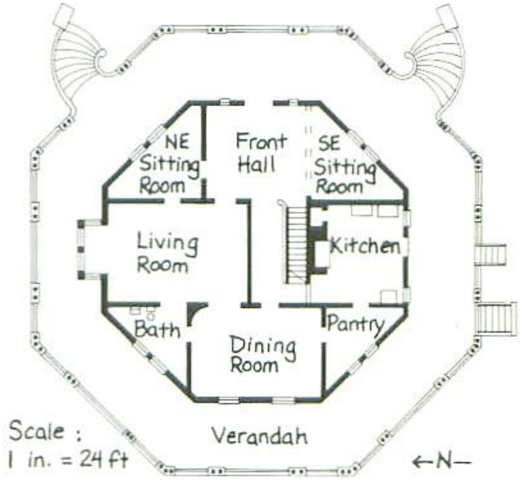
12. $x =$ ___?___ **13.** $y =$ ___?___ **14.** $z =$ ___?___



15. The measures of the angles of a hexagon are in the ratio 5 : 5 : 5 : 6 : 7 : 8. Find the measures.

Application

SCALE DRAWINGS



This "octagon house" was built in Irvington, New York, in 1860. The plan shows the rooms on the first floor. The scale on this *scale drawing* tells you that a length of 1 in. on the plan represents a true length of 24 ft.

$$\frac{\text{Plan length in inches}}{\text{True length in feet}} = \frac{1}{24}$$

The following examples show how you can use this formula to find actual dimensions of the house from the plan or to convert dimensions of full-sized objects to plan size.

The verandah measures $\frac{3}{8}$ in. wide on the plan. Find its true width, T.

$\frac{\frac{3}{8}}{T} = \frac{1}{24}$, so $1 \cdot T = \frac{3}{8} \cdot 24$

$T = 9$ The real verandah is 9 ft wide.

A sofa is 6 ft long. Find its plan length, P.

$\frac{P}{6} = \frac{1}{24}$, so $24 \cdot P = 6 \cdot 1$

$P = \frac{1}{4}$ The plan length is $\frac{1}{4}$ in.

Exercises

1. Find the true length and width of the dining room.
2. A rug measures 9 ft by $7\frac{1}{2}$ ft. What would its dimensions be on the floor plan? Would it fit in the northeast sitting room?
3. If a new floor plan is drawn with a scale of 1 in. = 10 ft, how many times longer is each line segment on the new plan than the corresponding segment on the plan shown?
4. Suppose that on the architect's drawings each side of the verandah (the outer octagon) measured 12 in. What was the scale of these drawings?

Independent Reading

When you begin a new chapter or lesson, it is a good idea to have some goals in mind. The objectives listed at the beginning of each main chapter division help you see where you are headed. For example, the title below, "Working with Similar Triangles," and the objectives tell you that pages 218–238 cover several methods of proving that triangles are similar and discuss some important applications of theorems related to similar triangles. You may want to skim all three of the following sections to see what important ideas are covered in each section.

As you read, pay particular attention to new words and phrases, in heavy type, and to important results such as theorems, postulates, and summaries, which are set off by special type and colored rules or boxes. Be sure to study all the worked-out examples. You may want to try solving some of them before you look at the printed solutions. If there is anything you don't understand after you have read and reread it, make a note and ask your teacher about it later.

When you have finished one of the main chapter divisions, look back at the words in heavy type and make sure you know their meanings. When you believe that you understand the material, try the Self-Test. The answers are printed at the back of the book, so you can check your own work. The Chapter Reviews, Chapter Tests, Cumulative Reviews, and, of course, the exercises will also help you check your mastery of a chapter or a group of chapters.

Working with Similar Triangles

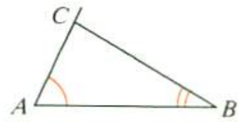
Objectives

1. Use the AA Similarity Postulate, the SAS Similarity Theorem, and the SSS Similarity Theorem to prove triangles similar.
2. Deduce information about segments or angles by first proving that two triangles are similar.
3. Apply the Triangle Proportionality Theorem and its corollary.
4. State and apply the Triangle Angle-Bisector Theorem.

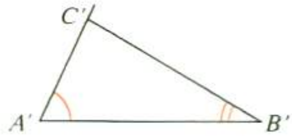
5-4 A Postulate for Similar Triangles

You can always prove that two triangles are similar by showing that they satisfy the definition of similar polygons. However, there are simpler methods. For example, the following experiment suggests that two triangles are similar whenever two pairs of angles are congruent.

1. Draw any two segments, $\overline{AB}$ and $\overline{A'B'}$.



2. Draw any angle at A and a congruent angle at A'. Draw any angle at B and a congruent angle at B'. Label points C and C' as shown. $\angle ACB \cong \angle A'C'B'$. (Why?)



3. Measure each pair of corresponding sides and compute an approximate decimal value for the ratio of their lengths:

$$\frac{AB}{A'B'} \qquad \frac{BC}{B'C'} \qquad \frac{AC}{A'C'}$$

4. Are the ratios computed in Step 3 approximately equal?

If you worked carefully, your answer in Step 4 was *yes*. Thus, corresponding angles of the two triangles are congruent and corresponding sides are in proportion. By the definition of similar polygons, $\triangle ABC \sim \triangle A'B'C'$.

Whenever you draw two triangles with two angles of one triangle congruent to two angles of the other, you will find that the third angles are also congruent and that corresponding sides are in proportion.

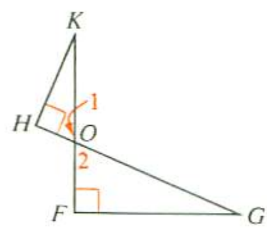
Postulate 15 AA Similarity Postulate

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Example

Given: $\angle H$ and $\angle F$ are rt. $\angle$s.

Prove: $HK \cdot GO = FG \cdot KO$



Plan for Proof: We can prove that $HK \cdot GO = FG \cdot KO$ if we can show that $\frac{HK}{FG} = \frac{KO}{GO}$. To do this, we will show that $\triangle HKO \sim \triangle FGO$.

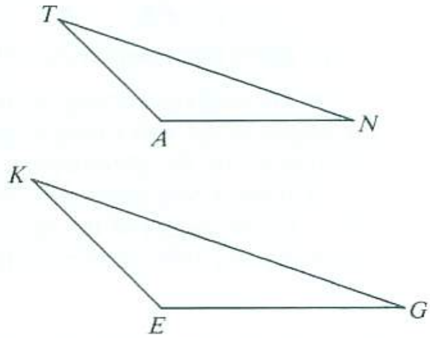
Proof:

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Vertical $\angle$s are $\cong$.
2. $\angle H$ and $\angle F$ are rt. $\angle$s.	2. Given
3. $m\angle H = 90 = m\angle F$	3. Def. of a rt. $\angle$
4. $\triangle HKO \sim \triangle FGO$	4. AA Similarity Postulate
5. $\frac{HK}{FG} = \frac{KO}{GO}$	5. Corr. sides of $\sim$ $\triangle$s are in proportion.
6. $HK \cdot GO = FG \cdot KO$	6. A property of proportions

The example shows one way to prove that the product of the lengths of two segments is equal to the product of the lengths of two other segments. You can prove two triangles similar, write a proportion, and then apply the means-extremes property of proportions.

Classroom Exercises

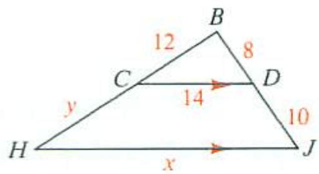
In Exercises 1-8, $\triangle TAN \sim \triangle KEG$. Tell whether each statement must be true.



Exs. 1-8

1. $\triangle NTA \sim \triangle GEK$
2. If $m\angle A = 120$, then $m\angle E = 120$.
3. If $m\angle T = 35$, then $m\angle G = 35$.
4. $AT : EK = EG : AN$
5. If $\frac{TA}{KE} = \frac{2}{3}$, then $\frac{TN}{KG} = \frac{2}{3}$.
6. If $\frac{TA}{KE} = \frac{2}{3}$, then $\frac{m\angle T}{m\angle K} = \frac{2}{3}$.
7. If the scale factor of $\triangle TAN$ to $\triangle KEG$ is 4 to 5, then the scale factor of $\triangle KEG$ to $\triangle TAN$ is 5 to 4.
8. If $\overline{KG}$ is twice as long as $\overline{KE}$, then $\overline{TN}$ is twice as long as $\overline{TA}$.

9. Name all pairs of congruent angles in the figure.

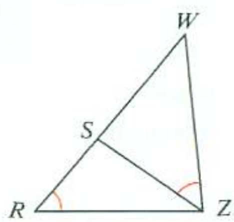


Exs. 9-12

Complete.

10. $\triangle BCD \sim$ __?__
11. $\frac{14}{x} = \frac{8}{?}$ and $x =$ __?__
12. $\frac{12}{12 + y} = \frac{?}{?}$ and $y =$ __?__

13. The diagram shows three triangles. Name two pairs of congruent angles in two of the triangles.
14. Complete: $\triangle RWZ \sim$ __?__
15. Name a third pair of congruent angles.
16. Complete: $\frac{RW}{?} = \frac{WZ}{?} = \frac{RZ}{?}$



Exs. 13-16

Suppose your goal is to show that $EF \cdot KL = EG \cdot KJ$. Complete each proportion so that it will lead to this equation.

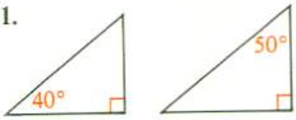
17. $\frac{EF}{KJ} = \frac{?}{?}$

18. $\frac{KL}{EG} = \frac{?}{?}$

Written Exercises

Tell whether the triangles are similar or not similar. If you can't reach a conclusion, write *no conclusion is possible.*

A **1.**



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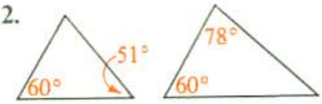

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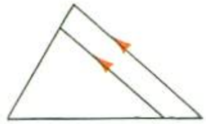

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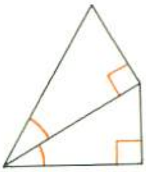

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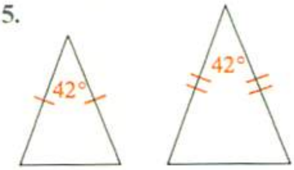

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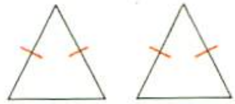

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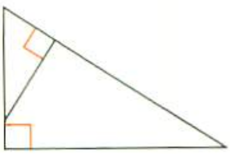

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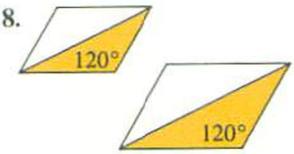
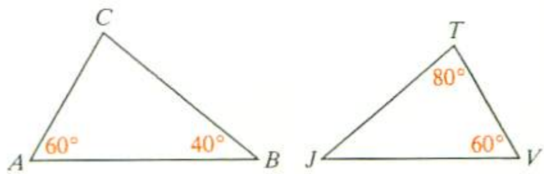


Parallelograms given

9.

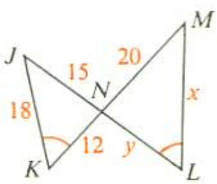
Trapezoid given

Complete.



10. a. $\triangle ABC \sim$ __?__

b. $\frac{AB}{?} = \frac{BC}{?} = \frac{AC}{?}$



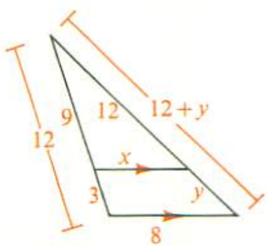
11. a. $\triangle JKN \sim$ __?__

b. $\frac{15}{?} = \frac{18}{?}$ and $\frac{15}{?} = \frac{12}{?}$

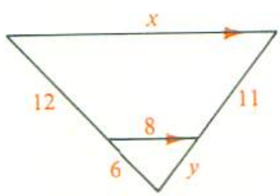
c. $x =$ __?__ and $y =$ __?__

Find the values of x and y.

12.



13.



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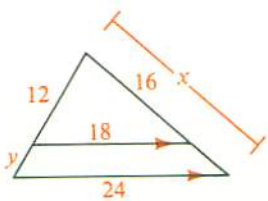
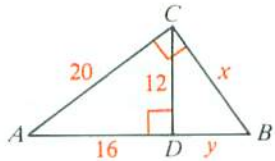
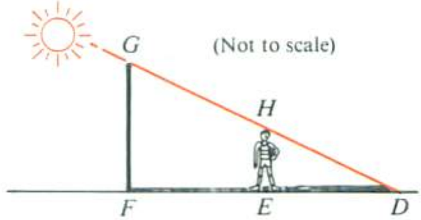


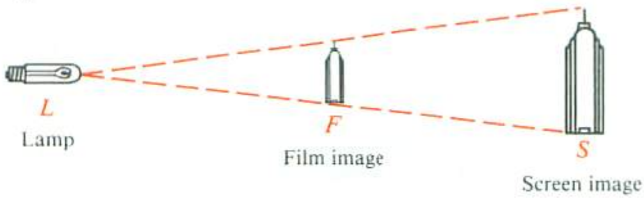
B **15.** **a.** Name two triangles that are similar to $\triangle ABC$.
b. Find the values of x and y.



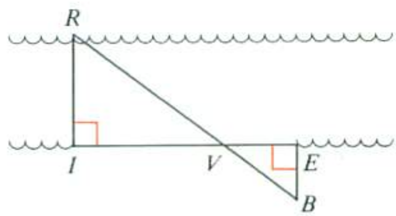
16. To estimate the height of a pole, a basketball player exactly 2 m tall stood so that the ends of the shadows coincided. He found that $\overline{DE}$ and $\overline{DF}$ measured 1.6 m and 4.4 m, respectively. About how tall was the pole?



17. The diagram, *not* drawn to scale, shows a film being projected on a screen. $LF = 6$ cm and $LS = 24$ m. The screen image is 2.2 m tall. How tall is the film image?

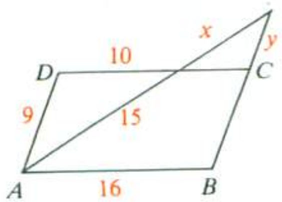


18. If $IV = 36$ m, $VE = 20$ m, and $EB = 15$ m, find the width, RI, of the river.

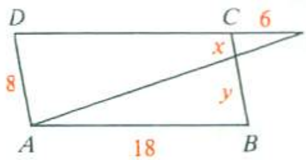


In Exercises 19 and 20, $ABCD$ is a parallelogram. Find the values of x and y.

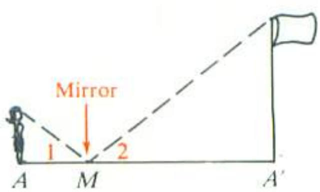
19.



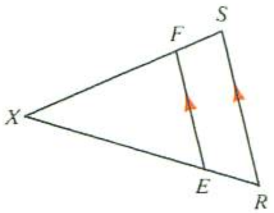
20.



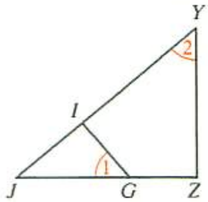
21. You can estimate the height of a flagpole by placing a mirror on level ground so that you see the top of the flagpole in it. The girl shown is 172 cm tall. Her eyes are about 12 cm from the top of her head. By measurement, AM is about 120 cm and $A'M$ is about 4.5 m. From physics it is known that $\angle 1 \cong \angle 2$. Explain why the triangles are similar and find the approximate height of the pole.



22. Given: $\overline{EF} \parallel \overline{RS}$
Prove: **a.** $\triangle FXE \sim \triangle SXR$
b. $\frac{FX}{SX} = \frac{EF}{RS}$

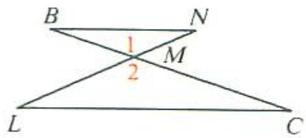


23. Given: $\angle 1 \cong \angle 2$
Prove: **a.** $\triangle JIG \sim \triangle JZY$
b. $\frac{JG}{JY} = \frac{GI}{YZ}$

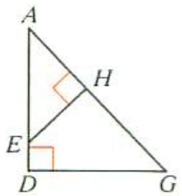


24. Given: $\angle B \cong \angle C$
Prove: $NM \cdot CM = LM \cdot BM$

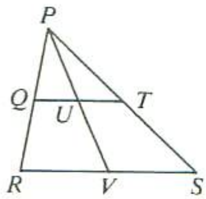
25. Given: $\overline{BN} \parallel \overline{LC}$
Prove: $BN \cdot LM = CL \cdot NM$



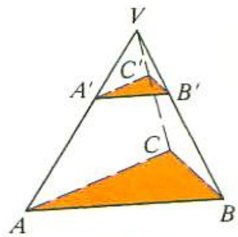
26. Given: $\angle D$ and $\angle AHE$ are right angles.
a. Prove two triangles similar.
b. Prove $AE \cdot DG = AG \cdot HE$



27. Given: $\overline{QT} \parallel \overline{RS}$
Prove: $\frac{QU}{RV} = \frac{UT}{VS}$



In the diagram for Exercises 28 and 29, the plane of $\triangle A'B'C'$ is parallel to the plane of $\triangle ABC$.



Exs. 28, 29

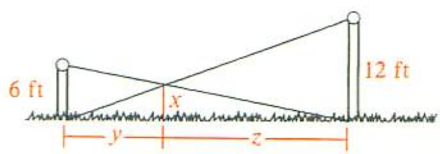
28. $VA' = 15$ and $A'A = 20$
a. If $VC' = 18$, then $VC = \underline{\ ?\ }$.
b. If $VB = 49$, then $BB' = \underline{\ ?\ }$.
c. If $A'B' = 24$, then $AB = \underline{\ ?\ }$.

29. If $VA' = 10$, $VA = 25$, $AB = 20$, $BC = 14$, and $AC = 16$, find the perimeter of $\triangle A'B'C'$.

30. Prove that the lengths of corresponding altitudes of similar triangles have the same ratio as the lengths of corresponding sides.

C 31. Prove that in any triangle the product of the lengths of one side and the altitude to that side is equal to the product of the lengths of another side and the altitude to that side.

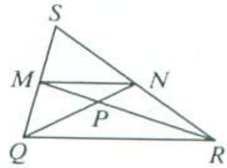
32. Two vertical poles have heights 6 ft and 12 ft. A rope is stretched from the top of each pole to the bottom of the other. How far above the ground do the ropes cross? (*Hint:* The lengths y and z do not affect the answer.)



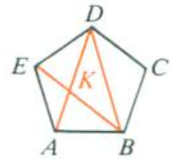
In Exercises 33–36 write a paragraph proof for anything you are asked to prove.

33. Given: $\overline{QN}$ and $\overline{RM}$ are medians of $\triangle QRS$.

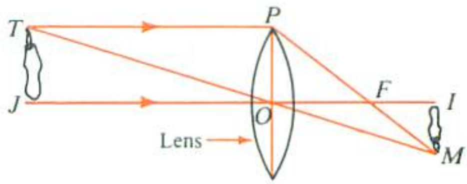
Prove: $\frac{QP}{PN} = \frac{2}{1}$ and $\frac{RP}{PM} = \frac{2}{1}$



34. Given: Regular pentagon $ABCDE$
   a. Make a large copy of the diagram.
   b. Write the angle measures on your diagram.
   c. Prove that $\frac{DA}{DK} = \frac{DK}{AK}$.



★ 35. Related to any doubly convex lens there is a focal distance OF. Physicists have determined experimentally that a vertical lens, a vertical object $\overline{JT}$ (with $\overline{JO}$ horizontal), a vertical image $\overline{IM}$, and a focus F are related as shown in the diagram. Once the relationship is known, geometry can be used to establish a lens law:

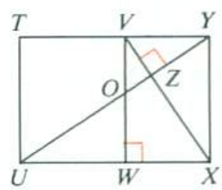


$$\frac{1}{\text{object distance}} + \frac{1}{\text{image distance}} = \frac{1}{\text{focal distance}}$$

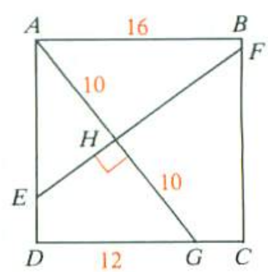
   a. Prove that $\frac{1}{OJ} + \frac{1}{OI} = \frac{1}{OF}$.
   b. Show algebraically that $OF = \frac{OJ \cdot OI}{OJ + OI}$.

★ 36. Given: rectangle $XYTU$;
$\overline{XV} \perp \overline{UY}$; $\overline{VW} \perp \overline{UX}$

Prove: quad. $XYVW \sim$ quad. $YTUX$



★ 37. $ABCD$ is a square.
   a. Find the distance from H to each side of the square.
   b. Find BF, FC, CG, DE, EA, EH, and HF.



COMPUTER KEY-IN

1. a. Refer to Exercise 35, page 224. Write a computer program that will compute the focal distance when the object distance and image distance are given. Use the formula derived in Exercise 35(b).

   b. Trials with a particular lens resulted in the measurements (given in centimeters) shown in the table below. RUN your program for the values and complete the table.

OJ	60	55	50	45	40	35	30	25	20
OI	20	20.5	21	22.5	24	26	30.5	42	61
OF	?	?	?	?	?	?	?	?	?

   c. Modify your program so that it will compute the *average* focal distance using the nine experimental values of OF found in part (b).

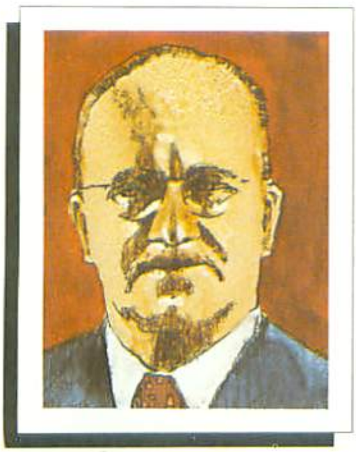
2. a. Let the average focal distance, correct to the nearest tenth, found in Exercise 1(c) be *the* focal distance of the lens. Substitute this value for OF in the equation $\frac{1}{OJ} + \frac{1}{OI} = \frac{1}{OF}$. Then solve for OI in terms of OJ.

   b. Write and RUN a computer program to complete the table below that gives values for OJ, the object distance.

OJ	100	90	80	70	65	15	10	5
OI	?	?	?	?	?	?	?	?

BIOGRAPHICAL NOTE

R. Buckminster Fuller



The early curiosity shown by R. Buckminster Fuller (1895–1983) about the world around him led to a life of invention and philosophy. As a mathematician he made many contributions to the fields of engineering, architecture, and cartography. His ultimate goal was always "to do more with less." Thus his discoveries often had economic and ecological implications.

Fuller's inventions include the geodesic dome (see pages 450 and 451), the 3-wheeled Dymaxion car, and the Dymaxion Air-ocean World Map on which he was able to project the spherical earth as a flat surface without any visible distortions. He also designed other structures that were based upon triangles and circles instead of the usual rectangular surfaces.

5-5 Theorems for Similar Triangles

You can prove two triangles similar by using the definition of similar polygons or by using the AA Postulate. Of course, in practice you would always use the AA Postulate instead of the definition. Two additional methods are established in the theorems below. Proofs are left as Exercises 19 and 20.

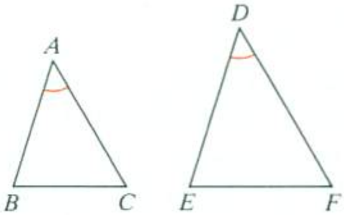
Theorem 5-1 SAS Similarity Theorem

If an angle of one triangle is congruent to an angle of another triangle and the sides including those angles are in proportion, then the triangles are similar.

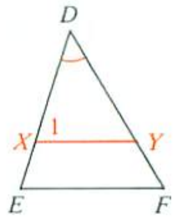
Given: $\angle A \cong \angle D$;

$$\frac{AB}{DE} = \frac{AC}{DF}$$

Prove: $\triangle ABC \sim \triangle DEF$



Plan for Proof: Assume $AB < DE$. Take X on $\overline{DE}$ so that $DX = AB$. Through X draw a line parallel to $\overleftrightarrow{EF}$. Then $\triangle DXY \sim \triangle DEF$ and $\frac{DX}{DE} = \frac{DY}{DF}$. Use this proportion, the given proportion, and the fact that $DX = AB$ to show that $DY = AC$. Then $\triangle ABC \cong \triangle DXY$ and $\angle B \cong \angle 1$. But $\angle 1 \cong \angle E$, so $\angle B \cong \angle E$. Also, $\angle A \cong \angle D$. Use the AA Similarity Postulate to show that $\triangle ABC \sim \triangle DEF$.

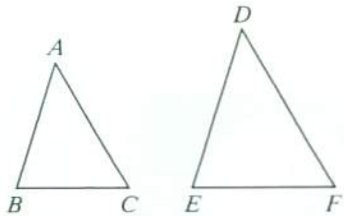


Theorem 5-2 SSS Similarity Theorem

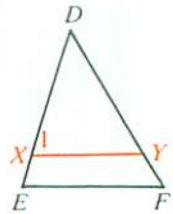
If the sides of two triangles are in proportion, then the two triangles are similar.

Given: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

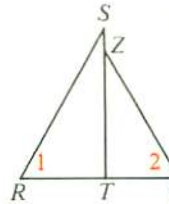
Prove: $\triangle ABC \sim \triangle DEF$



Plan for Proof: Assume $AB < DE$. Take X on $\overline{DE}$ so that $DX = AB$ and draw a line parallel to $\overleftrightarrow{EF}$. $\triangle DXY \sim \triangle DEF$ and $\frac{DX}{DE} = \frac{XY}{EF} = \frac{DY}{DF}$. Use this extended proportion, the given extended proportion, and the fact that $DX = AB$ to show that $BC = XY$ and $AC = DY$. Then $\triangle ABC \cong \triangle DXY$, and $\angle B \cong \angle 1$. Since $\angle 1 \cong \angle E$, $\angle B \cong \angle E$. Finally, apply the SAS Similarity Theorem.



Example Can the given information be used to prove $\triangle RST \sim \triangle WZT$? If so, how?



a. $RS = 18,\ \ ST = 15,\ \ RT = 10,\ \ WT = 6,\ \ ZT = 9,$
$WZ = 10.8$

b. $\overline{ST} \perp \overline{RW}$, $SZ = 8$, $ZT = 24$, $RT = 20$, $WT = 15$

c. $\angle 1 \cong \angle 2$, $\dfrac{ST}{ZT} = \dfrac{RS}{WZ}$

Solution **a.** $\dfrac{RS}{WZ} = \dfrac{18}{10.8} = \dfrac{5}{3},\ \dfrac{ST}{ZT} = \dfrac{15}{9} = \dfrac{5}{3},\ \dfrac{RT}{WT} = \dfrac{10}{6} = \dfrac{5}{3}$

Thus $\dfrac{RS}{WZ} = \dfrac{ST}{ZT} = \dfrac{RT}{WT}$.

$\triangle RST \sim \triangle WZT$ by the SSS Similarity Theorem.

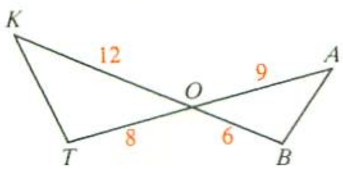
b. $\dfrac{ST}{ZT} = \dfrac{32}{24} = \dfrac{4}{3}$ and $\dfrac{RT}{WT} = \dfrac{20}{15} = \dfrac{4}{3}$; $\angle STR \cong \angle ZTW$

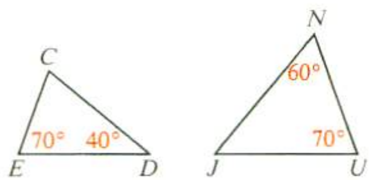
$\triangle RST \sim \triangle WZT$ by the SAS Similarity Theorem.

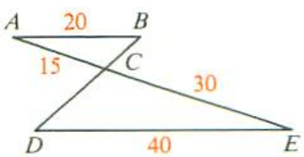
c. The triangles cannot be proved similar. (Notice that $\angle 1$ and $\angle 2$ are not the angles included by the sides that are known to be in proportion.)

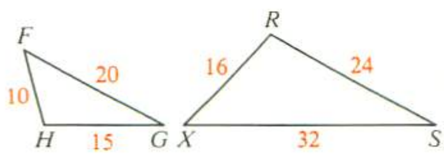
Classroom Exercises

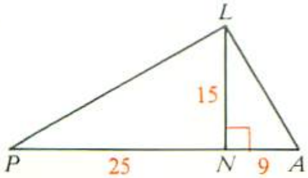
Can two triangles shown be proved similar? If so, state the similarity and tell which similarity postulate or theorem you would use.

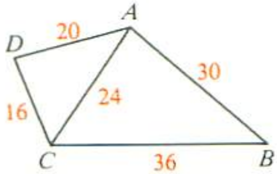
1. 

2. 

3. 

4. 

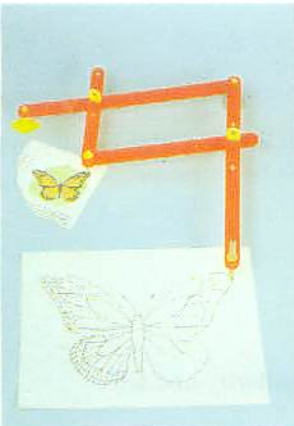
5. 

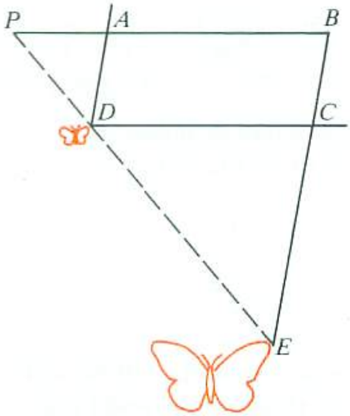
6. 

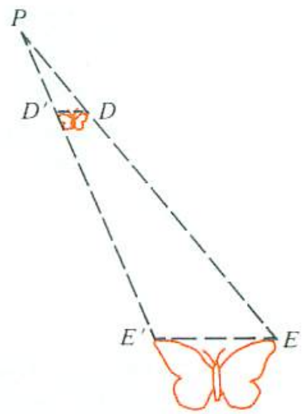
7. Suppose you want to prove that $\triangle RST \sim \triangle XYZ$ by the SSS Similarity Theorem. State the extended proportion you would need to prove first.

8. Suppose you want to prove that $\triangle RST \sim \triangle XYZ$ by the SAS Similarity Theorem. If you know that $\angle R \cong \angle X$, what else would you need to prove?

9. A *pantograph* is a tool for enlarging or reducing maps and drawings. Four bars are pinned together at A, B, C, and D so that $ABCD$ is a parallelogram and points P, D, and E lie on a line. Point P is fixed to the drawing board. To enlarge a figure, the artist inserts a stylus at D and guides the pen or pencil at E so that the stylus traces the original. As E moves, the angles of the parallelogram change, but P, D, and E remain collinear. Suppose PA is 3 units and AB is 7 units.





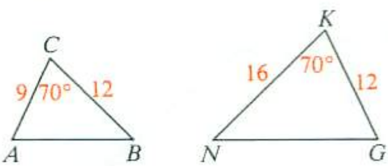


a. Explain why $\triangle PBE \sim \triangle PAD$.
b. What is the ratio of PB to PA?
c. What is the ratio of PE to PD?
d. What is the ratio of the butterfly's wingspan, $E'E$, in the enlargement to its wingspan, $D'D$, in the original?

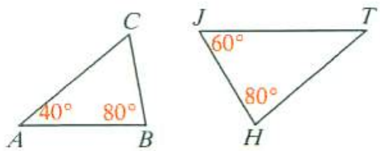
Written Exercises

Name two similar triangles. Also name the postulate or theorem that justifies your answer.

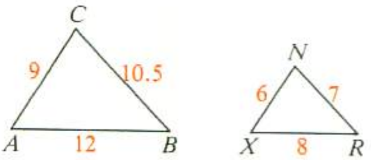
A 1.



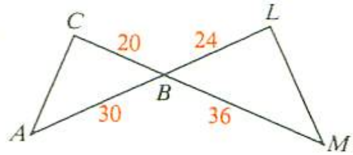
2.



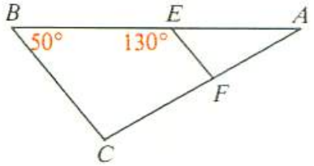
3.



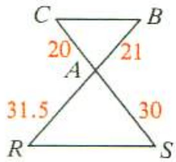
4.



5.



6.



One triangle has vertices *A*, *B*, and *C*. Another triangle has vertices *P*, *K*, and *N*. Are two triangles similar? If so, state the similarity and the scale factor.

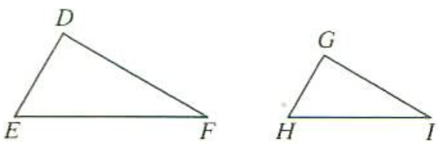
	AB	*BC*	*AC*	*PK*	*KN*	*PN*
7.	6	8	10	9	12	15
8.	6	8	10	15	9	12
9.	6	8	10	25	20	16
10.	12	16	18	20	22.5	15

11. Given: $\frac{DE}{GH} = \frac{DF}{GI} = \frac{EF}{HI}$

Prove: $\angle E \cong \angle H$

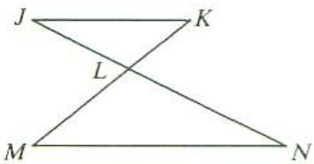
12. Given: $\frac{DE}{GH} = \frac{EF}{HI}$; $\angle E \cong \angle H$

Prove: $\frac{EF}{HI} = \frac{DF}{GI}$



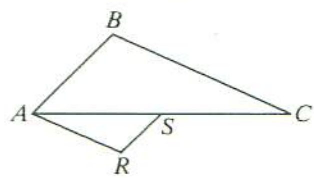
B **13.** Given: $\frac{JL}{NL} = \frac{KL}{ML}$

Prove: $\angle J \cong \angle N$



14. Given: $\frac{AB}{SR} = \frac{BC}{RA} = \frac{CA}{AS}$

Prove: $\overline{BC} \parallel \overline{AR}$



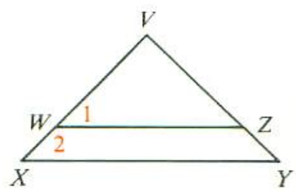
15. Given: $\frac{VW}{VX} = \frac{VZ}{VY}$

Prove: $\overline{WZ} \parallel \overline{XY}$

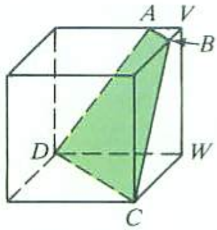
16. Given: $\frac{VW}{VY} = \frac{VZ}{VX}$

Which one(s) of the following *must* be true?

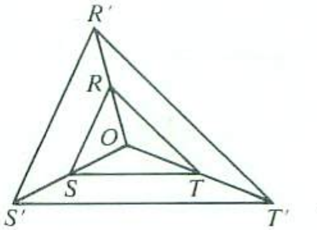
(1) $\triangle VWZ \sim \triangle VXY$ (2) $\overline{WZ} \parallel \overline{XY}$ (3) $\angle 1 \cong \angle Y$



17. The faces of a cube are congruent squares. The cube shown is cut by plane $ABCD$. $VA = VB$ and $VW = 4 \cdot VA$. Find, in terms of AB, the length of the median of trap. $ABCD$.



18. Given: $OR' = 2 \cdot OR$;
$OS' = 2 \cdot OS$;
$OT' = 2 \cdot OT$
Prove: $\triangle RST \sim \triangle R'S'T'$



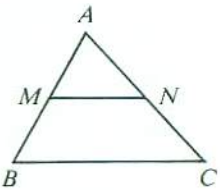
19. Prove the SAS Similarity Theorem.

20. Prove the SSS Similarity Theorem.

21. Prove: If the vertex angle of one isosceles triangle is congruent to the vertex angle of another isosceles triangle, then the triangles are similar.

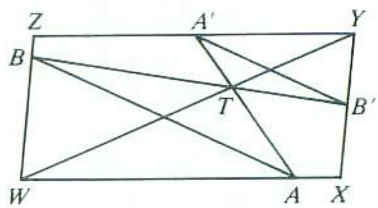
22. Prove Theorem 4-17 on page 174: The segment that joins the midpoints of two sides of a triangle is parallel to the third side and is half as long as the third side.

Given: M is the midpoint of $\overline{AB}$;
N is the midpoint of $\overline{AC}$.
Prove: $\overline{MN} \parallel \overline{BC}$; $MN = \frac{1}{2}BC$



C 23. Prove that the lengths of corresponding medians of similar triangles have the same ratio as the lengths of corresponding sides.

24. Given: $\square WXYZ$
Prove: $\triangle ATB \sim \triangle A'TB'$
(*Hint:* Show that $\frac{AT}{A'T}$ and $\frac{BT}{B'T}$ both equal $\frac{TW}{TY}$.)



★ 25. In $\triangle CAT$, it is known that $AC = 20$, $AT = 25$, and $\overline{AM}$ is a median. P is any point on $\overline{AT}$. Q lies on $\overline{AC}$ with $AQ = \frac{3}{5}AP$. $\overline{QP}$ and $\overline{AM}$ intersect at K.

a. Draw the diagram for the case when P is point T. Then find the ratio $QK : PK$.

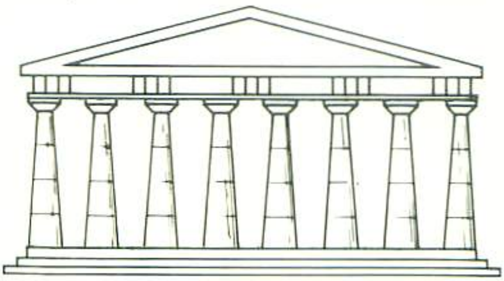
b. Show that the ratio $QK : PK$ is the same for any position of P on $\overline{AT}$.

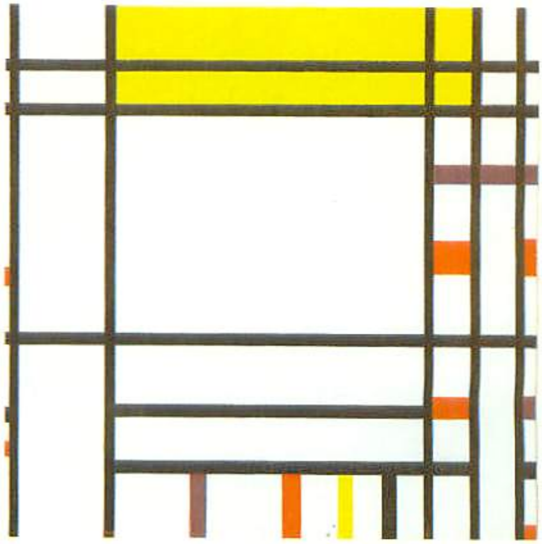
Challenge

Explain how to pass a plane through a cube in such a way that the intersection is (**a**) an equilateral triangle; (**b**) a trapezoid; (**c**) a pentagon; (**d**) a hexagon.

CALCULATOR KEY-IN

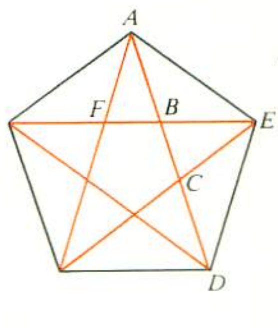
Before the arch on top of the Parthenon in Athens was destroyed, the front of the building fit almost exactly into a *golden rectangle*. A **golden rectangle** is such that its length l and width w satisfy the equation $\frac{l}{w} = \frac{l + w}{l}$. The ratio $\frac{l}{w}$ is called the **golden ratio.**





Over the centuries, artists and architects have found the golden rectangle to be especially pleasing to the eye. How many golden rectangles can you find in the painting by Piet Mondrian (1872–1944) that is shown?

Exercises



1. A regular pentagon is shown. It happens to be true that $\frac{AD}{AC}$, $\frac{AC}{AB}$, and $\frac{AB}{BC}$ all equal the golden ratio. Measure the appropriate lengths to the nearest millimeter and compute the ratios with a calculator.

2. From the equation $\frac{l}{w} = \frac{l + w}{l}$ it can be shown that the numerical value of $\frac{l}{w}$ is $\frac{1 + \sqrt{5}}{2}$. Express the value of $\frac{l}{w}$, the golden ratio, as a decimal.

3. Sometimes the golden ratio is expressed as $\frac{w}{l}$ rather than $\frac{l}{w}$. From Exercise 2 you see that $\frac{w}{l} = \frac{2}{1 + \sqrt{5}}$. Express $\frac{w}{l}$ as a decimal.

COMPUTER KEY-IN

The sequence 1, 1, 2, 3, 5, 8, 13, 21, . . . is called a *Fibonacci sequence* after its discoverer, Leonardo Fibonacci, a 13th-century mathematician. The first two terms are 1 and 1. You then add two consecutive terms to get the next term.

1st term	+	2nd term	=	3rd term	2nd term	+	3rd term	=	4th term
1	+	1	=	2	1	+	2	=	3
3rd	+	4th	=	5th	4th	+	5th	=	6th
2	+	3	=	5	3	+	5	=	8

The following computer program computes the first twenty-five terms of the Fibonacci sequence shown above and finds the ratio of any term to its preceding term. For example, we want to look at the ratios

$$\frac{1}{1} = 1, \quad \frac{2}{1} = 2, \quad \frac{3}{2} = 1.5, \quad \frac{5}{3} \approx 1.66667, \text{ and so on.}$$

```
10 PRINT "TERM NO.","TERM","RATIO TO PRECEDING TERM"
20 LET A = 1
30 LET B = 1
40 PRINT 1,A,"-"
50 FOR N = 2 TO 25
60 PRINT N,B,B/A
70 LET C = B + A
80 LET A = B
90 LET B = C
100 NEXT N
110 END
```

Exercises

1. RUN the given computer program. As the terms become larger, what happens to the values of the ratios?

2. Suppose another sequence is formed by choosing starting numbers different from 1 and 1. For example, suppose the sequence is 3, 11, 14, 25, 39, . . . , where the pattern for creating the terms of the sequence is still the same. Change lines 20 and 30 to:

```
20 LET A = 3
30 LET B = 11
```

RUN the modified program. What happens to the values of the ratios as the terms become larger and larger?

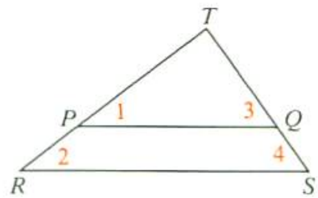
3. Modify the program again so that another pair of starting numbers is used and the first fifty terms are computed. RUN the program. What can you conclude from the results?

5-6 Proportional Lengths

Points L and M lie on $\overline{AB}$ and $\overline{CD}$, respectively. If $\frac{AL}{LB} = \frac{CM}{MD}$, we say that $\overline{AB}$ and $\overline{CD}$ are **divided proportionally.**

Theorem 5-3 Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally.



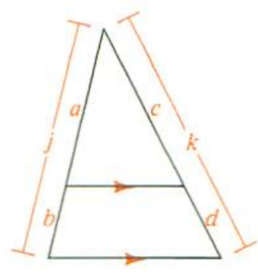
Given: $\triangle RST$; $\overleftrightarrow{PQ} \parallel \overline{RS}$

Prove: $\frac{RP}{PT} = \frac{SQ}{QT}$

Proof:

Statements	Reasons
1. $\overleftrightarrow{PQ} \parallel \overline{RS}$	1. ?
2. $\angle 1 \cong \angle 2$; $\angle 3 \cong \angle 4$	2. ?
3. $\triangle RST \sim \triangle PQT$	3. ?
4. $\frac{RT}{PT} = \frac{ST}{QT}$	4. Corr. sides of $\sim$ ⚠ are in proportion.
5. $RT = RP + PT$; $ST = SQ + QT$	5. ?
6. $\frac{RP + PT}{PT} = \frac{SQ + QT}{QT}$	6. ?
7. $\frac{RP}{PT} = \frac{SQ}{QT}$	7. A property of proportions (Property 1(d), p. 209)

We will use the Triangle Proportionality Theorem to justify any proportion equivalent to $\frac{RP}{PT} = \frac{SQ}{QT}$. For the diagram at the right, some of the proportions that may be justified by the Triangle Proportionality Theorem include:

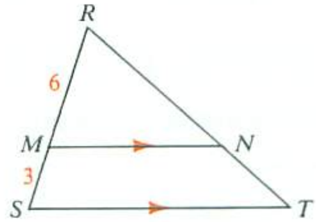


$$\frac{a}{j} = \frac{c}{k} \qquad \frac{a}{c} = \frac{j}{k} \qquad \frac{b}{j} = \frac{d}{k}$$

$$\frac{a}{b} = \frac{c}{d} \qquad \frac{a}{c} = \frac{b}{d} \qquad \frac{b}{d} = \frac{j}{k}$$

Example Find the numerical value.

a. $\frac{TN}{NR}$ b. $\frac{TR}{NR}$ c. $\frac{RN}{RT}$



Solution a. $\frac{TN}{NR} = \frac{SM}{MR} = \frac{3}{6} = \frac{1}{2}$

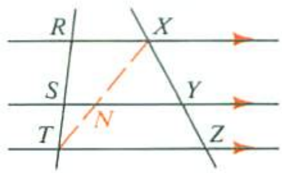
b. $\frac{TR}{NR} = \frac{SR}{MR} = \frac{9}{6} = \frac{3}{2}$

c. $\frac{RN}{RT} = \frac{RM}{RS} = \frac{6}{9} = \frac{2}{3}$

Compare the following corollary with Theorem 4-8 on page 164.

Corollary

If three parallel lines intersect two transversals, then they divide the transversals proportionally.



Given: $\overleftrightarrow{RX} \parallel \overleftrightarrow{SY} \parallel \overleftrightarrow{TZ}$

Prove: $\frac{RS}{ST} = \frac{XY}{YZ}$

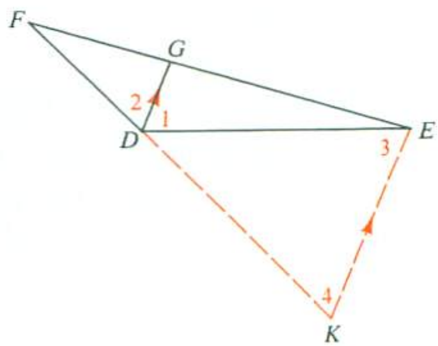
■ ***Plan for Proof:*** Draw $\overline{TX}$, intersecting $\overleftrightarrow{SY}$ at N. Note that $\overleftrightarrow{SY}$ is parallel to one side of $\triangle RTX$, and also to one side of $\triangle TXZ$. You can apply the Triangle Proportionality Theorem to show that $\frac{RS}{ST} = \frac{XN}{NT}$ and $\frac{XY}{YZ} = \frac{XN}{NT}$. Then $\frac{RS}{ST} = \frac{XY}{YZ}$.

Theorem 5-4 Triangle Angle-Bisector Theorem

If a ray bisects an angle of a triangle, then it divides the opposite side into segments proportional to the other two sides.

Given: $\triangle DEF$; $\overrightarrow{DG}$ bisects $\angle FDE$.

Prove: $\frac{GF}{GE} = \frac{DF}{DE}$

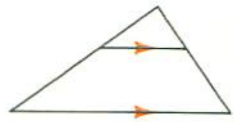


■ ***Plan for Proof:*** By drawing a line through E parallel to $\overrightarrow{DG}$ you can form a triangle to which you can apply the Triangle Proportionality Theorem. Using alternate interior angles, corresponding angles, and what is given, show that $\angle 3 \cong \angle 4$. Then $DK = DE$. But $\frac{GF}{GE} = \frac{DF}{DK}$ by the Triangle Proportionality Theorem, and substitution of DE for DK completes the proof.

Classroom Exercises

1. The two segments are divided proportionally. State several correct proportions.

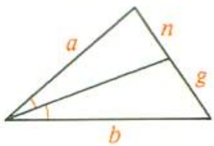
2. State several proportions informally as shown in the proportion at the left below.



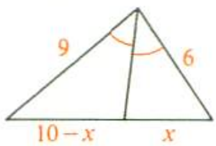
$$\frac{\text{upper left}}{\text{whole left}} = \frac{\text{upper right}}{\text{whole right}} \qquad \frac{\text{lower left}}{\text{upper left}} = \frac{?}{?}$$

State a proportion for each diagram.

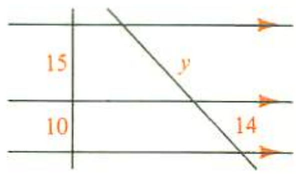
3.



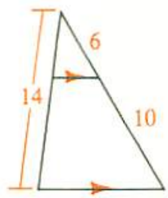
4.



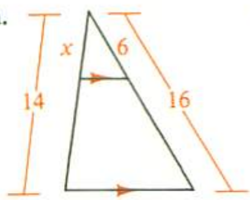
5.



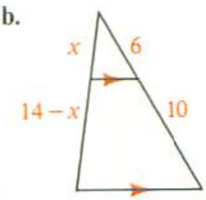
6. Suppose you want to find the length of the segment on the upper left. Three methods are suggested below. Complete each solution.



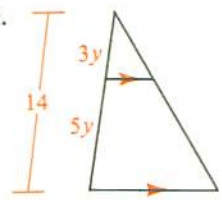
a.



b.



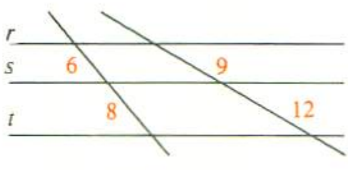
c.



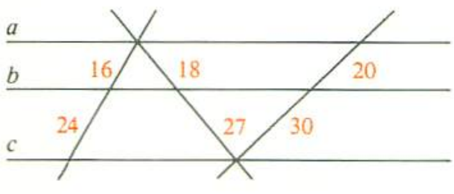
7. Explain why the expressions $3y$ and $5y$ can be used in Exercise 6(c).

8. **a.** State the converse of the corollary to the Triangle Proportionality Theorem.

   b. Is the converse true? (*Hint:* Can you draw a diagram with lengths like those shown below, but in which lines r, s, and t are not parallel?)



Ex. 8



Ex. 9

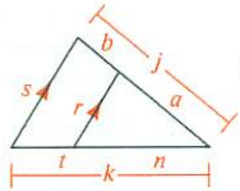
9. Must lines a, b, and c shown above be parallel? Explain.

Written Exercises

A **1.** Tell whether the proportion is correct.

a. $\frac{r}{s} = \frac{a}{b}$ b. $\frac{j}{a} = \frac{s}{r}$ c. $\frac{a}{b} = \frac{n}{t}$

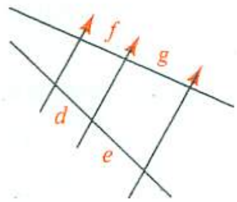
d. $\frac{t}{k} = \frac{a}{j}$ e. $\frac{r}{s} = \frac{n}{k}$ f. $\frac{b}{j} = \frac{t}{k}$



2. Tell whether the proportion is correct.

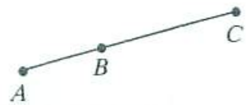
a. $\frac{d}{f} = \frac{g}{e}$ b. $\frac{f}{g} = \frac{e}{d}$

c. $\frac{g}{f} = \frac{e}{d}$ d. $\frac{d}{f} = \frac{e}{g}$

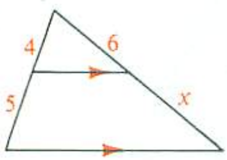


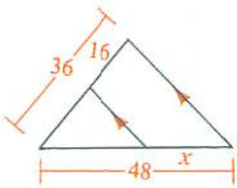
In Exercises 3-6, $\frac{AB}{BC} = \frac{3}{5}$. Copy and complete the table.

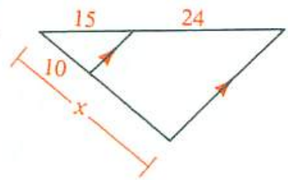
	3.	4.	5.	6.
AB	6	?	?	?
BC	?	25	?	?
AC	?	?	56	100

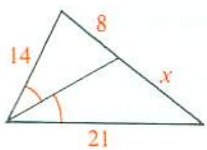


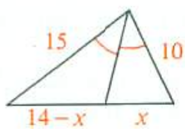
Find the value of *x*.

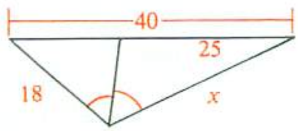
7.


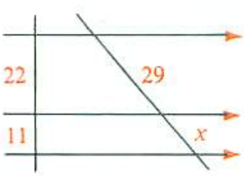
8.


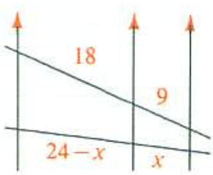
9.


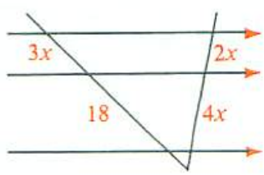
10.


11.


12.


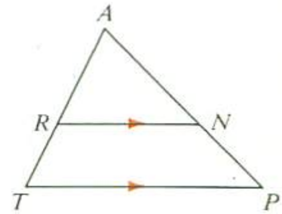
13.


14.


15.


Copy the table and fill in as many spaces *as possible.* It may help to draw a new sketch for each exercise and label lengths as you find them.

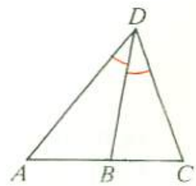
	AR	RT	AT	AN	NP	AP	RN	TP
B 16.	6	4	?	9	?	?	?	15
17.	?	?	?	?	6	16	?	?
18.	18	?	?	?	?	?	30	40
19.	12	?	20	?	?	30	15	?
20.	18	?	?	26	?	?	24	36
21.	?	?	33	24	20	?	?	50



22. Prove the corollary to the Triangle Proportionality Theorem.

23. Prove the Triangle Angle-Bisector Theorem.

Complete.



24. $AD = 21$, $DC = 14$, $AC = 25$, $AB = \underline{\ ?\ }$

25. $AC = 60$, $CD = 30$, $AD = 50$, $BC = \underline{\ ?\ }$

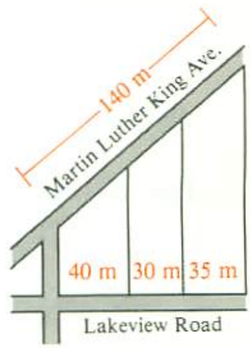
26. $AB = 27$, $BC = x$, $CD = \frac{4}{3}x$, $AD = x$, $AC = \underline{\ ?\ }$

27. $AB = 2x - 12$, $BC = x$, $CD = x + 5$, $AD = 2x - 4$, $AC = \underline{\ ?\ }$

28. Three lots with parallel side boundaries extend from the avenue to the road as shown. Find, to the nearest tenth of a meter, the frontages of the lots on Martin Luther King Avenue.

29. The lengths of the sides of $\triangle ABC$ are $BC = 12$, $CA = 13$, and $AB = 14$. If M is the midpoint of $\overline{CA}$, and P is the point where $\overline{CA}$ is cut by the bisector of $\angle B$, find MP.

30. Prove: If a line bisects both an angle of a triangle and the opposite side, then the triangle is isosceles.



Ex. 28

C 31. Discover and prove a theorem, about planes and transversals, suggested by the corollary to the Triangle Proportionality Theorem.

32. Prove that there cannot be a triangle in which the trisectors of an angle also trisect the opposite side.

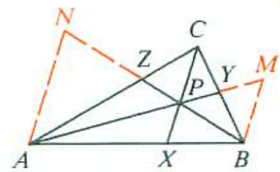
33. Can there exist a $\triangle ROS$ in which the trisectors of $\angle O$ intersect $\overline{RS}$ at D and E, with $RD = 1$, $DE = 2$, and $ES = 4$? Explain.

34. Angle E of $\triangle ZEN$ is obtuse. The bisector of $\angle E$ intersects $\overline{ZN}$ at X. J and K lie on $\overline{ZE}$ and $\overline{NE}$ with $ZJ = ZX$ and $NK = NX$. Discover and prove something about quadrilateral $ZNKJ$.

★ **35.** In $\triangle RST$, U lies on $\overline{TS}$ with $TU:US = 2:3$. M is the midpoint of $\overline{RU}$. $\overrightarrow{TM}$ intersects $\overline{RS}$ in V. Find the ratio $RV:VS$.

★ **36.** Prove *Ceva's Theorem:* If P is any point inside $\triangle ABC$, then $\frac{AX}{XB} \cdot \frac{BY}{YC} \cdot \frac{CZ}{ZA} = 1$.

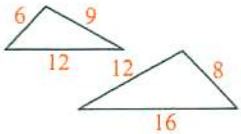
(*Hint:* Draw lines parallel to $\overline{CX}$ through A and B. Apply the Triangle Proportionality Theorem to $\triangle ABM$. Show that $\triangle APN \sim \triangle MPB$, $\triangle BYM \sim \triangle CYP$, and $\triangle CZP \sim \triangle AZN$.)



Self-Test 2

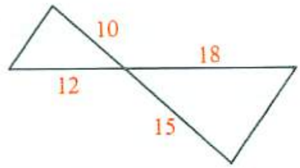
State the postulate or theorem you can use to prove that two triangles are similar.

1.

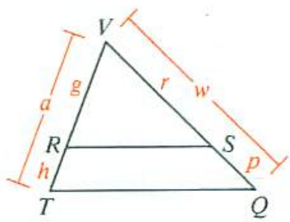


2.

3.



In the figure, it is given that $\overline{RS} \parallel \overline{TQ}$. Complete the proportion.



4. $\frac{g}{h} = \frac{?}{p}$

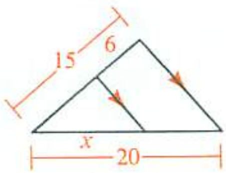
5. $\frac{a}{h} = \frac{w}{?}$

6. $\frac{r}{g} = \frac{p}{?}$

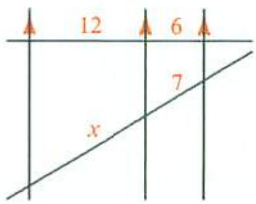
7. $\frac{h}{p} = \frac{?}{w}$

Find the value of x.

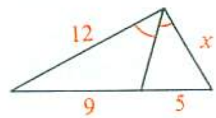
8.



9.



10.

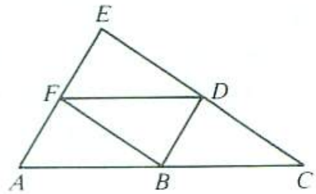


Challenge

Given: $\overline{FD} \parallel \overline{AC}$; $\overline{BD} \parallel \overline{AE}$; $\overline{FB} \parallel \overline{EC}$

Show that B, D, and F are midpoints of $\overline{AC}$, $\overline{CE}$, and $\overline{EA}$.

(*Hint:* If $\frac{AB}{BC} = \frac{BC}{AB}$, then $(AB)^2 = (BC)^2$, and $AB = BC$.)



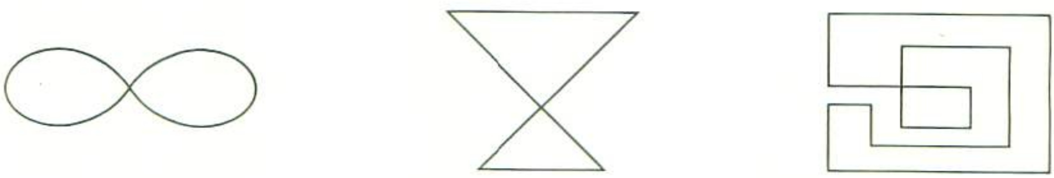
EXTRA

Topology

In the geometry we have been studying, our interest has been in congruent figures and similar figures, that is, figures with the same size and shape or at least the same shape. If we were studying the branch of geometry called *topology*, we would be interested in properties of figures that are even more basic than size and shape. For example, imagine taking a rubber band and stretching it into all kinds of figures.

These figures have different sizes and shapes, but they still have something in common: Each one can be turned into any of the others by stretching and bending the rubber band. In topology figures are classified according to this kind of family resemblance. Figures that can be stretched, bent, or molded into the same shape without cutting or puncturing belong to the same family and are called *topologically equivalent*. Thus circles, squares, and triangles are equivalent. Likewise the straight line segment and wiggly curves below are equivalent.

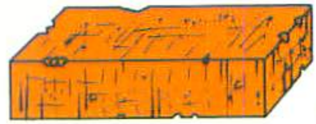
Notice that to make one of these figures out of the rubber band you would have to cut the band, so these two-ended curves are not equivalent to the closed curves in the first illustration. Suppose that in the following plane figures the lines are joined where they cross. Then these figures belong to a third family. They are equivalent to each other but not to any of the figures above.



One of the goals of topology is to identify and describe the different families of equivalent figures. A person who studies topology (called a *topologist*) may be interested in classifying solid figures as well as figures in a plane. For example, the topologist would consider an orange, a teaspoon, and a brick equivalent to each other.

Orange

Teaspoon

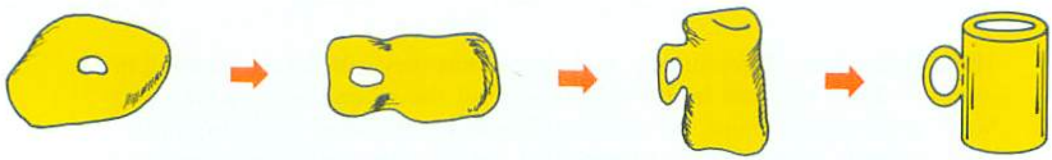


Brick

In fact, a doughnut is topologically equivalent to a coffee cup. (See the diagrams below.) For this reason, a topologist has been humorously described as a mathematician who can't tell the difference between a doughnut and a coffee cup!

Think of the objects as made of modeling clay.

Push thumb into clay to make room for coffee.



Exercises

In each exercise tell which figure is *not* topologically equivalent to the rest. Exercises 1 and 2 show plane figures.

1. a.  **b.**  **c.**  **d.** 

2. a.  **b.**  **c.**  **d.** 

3. a. solid ball **b.** hollow ball **c.** crayon **d.** comb

4. a. saucer **b.** car key **c.** coffee cup **d.** wedding ring

5. a. hammer **b.** screwdriver **c.** thimble **d.** sewing needle

6. Group the block numbers shown into three groups such that the numbers in each group are topologically equivalent to each other.

7. Make a series of drawings showing that the items in each pair are topologically equivalent to each other.
a. a drinking glass and a dollar bill
b. a tack and a paper clip

Chapter Summary

1. The ratio of a to b is the quotient $\frac{a}{b}$ (b cannot be 0). The ratio $\frac{a}{b}$ can also be written $a:b$.
2. A proportion is an equation, such as $\frac{a}{b} = \frac{c}{d}$, stating that two ratios are equal.
3. The properties of proportions (see page 209) are used to change proportions into equivalent equations. For example, the product of the extremes equals the product of the means.
4. Similar figures have the same shape. Two polygons are similar if and only if corresponding angles are congruent and corresponding sides are in proportion.
5. Ways to prove two triangles similar:
   AA Similarity Postulate   SAS Similarity Theorem   SSS Similarity Theorem
6. Ways to show that segments are proportional:
   a. Corresponding sides of similar polygons are in proportion.
   b. If a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides proportionally.
   c. If three parallel lines intersect two transversals, they divide the transversals proportionally.
   d. If a ray bisects an angle of a triangle, then it divides the opposite side into segments proportional to the other two sides.

Chapter Review

Write the ratio in simplest form.

1. 15:25   2. 6:12:9   3. $\frac{16xy}{24x^2}$ **5-1**

4. The measures of the angles of a triangle are in the ratio 4:4:7. Find the three measures.

Is the equation equivalent to the proportion $\frac{30 - x}{x} = \frac{8}{7}$?

5. $7x = 8(30 - x)$ 6. $\frac{x}{30 - x} = \frac{7}{8}$ **5-2**

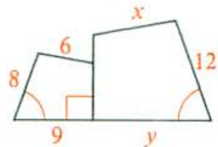
7. $8x = 210 - 7x$ 8. $\frac{30}{x} = \frac{15}{7}$

9. If $\triangle ABC \sim \triangle NJT$, then $\angle B \cong$ _?_. 5-3

10. If quad. $DEFG \sim$ quad. $PQRS$, then $\frac{FG}{RS} = \frac{GD}{?}$.

11. $\triangle ABC \sim \triangle JET$, and the scale factor of $\triangle ABC$ to $\triangle JET$ is $\frac{5}{3}$. If $BC = 20$, then $ET =$ _?_.

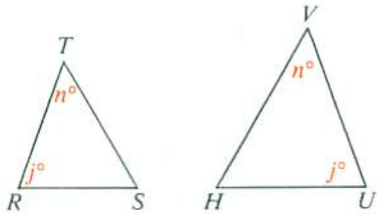
12. The quadrilaterals are similar. Find the values of x and y.



13. a. $\triangle RTS \sim$ _?_ 5-4
b. What postulate or theorem justifies the statement in part (a)?

14. $\frac{RT}{?} = \frac{TS}{?} = \frac{RS}{?}$

15. Suppose you wanted to prove
$$RS \cdot UV = RT \cdot UH.$$
You would first use similar triangles to show that $\frac{RS}{?} = \frac{?}{?}$.



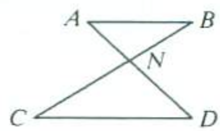
If two triangles shown can be proved similar, state the similarity. If not, write *no*.

16. $\angle A \cong \angle D$

17. $\angle B \cong \angle D$

18. $CN = 16$, $ND = 14$, $BN = 7$, $AN = 8$

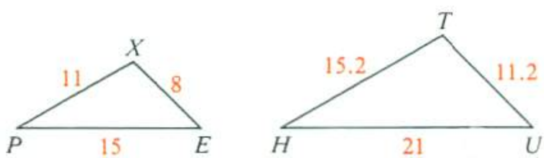
19. $AN = 7$, $AB = 6$, $DN = 14$, $DC = 12$



5-5

Exs. 16–19

20.

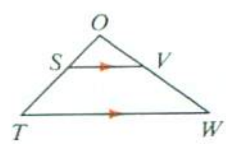


21. Which proportion is *incorrect*? 5-6

(1) $\frac{OS}{ST} = \frac{OV}{VW}$ (2) $\frac{SV}{TW} = \frac{OS}{ST}$ (3) $\frac{OT}{OW} = \frac{OS}{OV}$

22. If $OS = 8$, $ST = 12$, and $OV = 10$, then $OW =$ _?_.

23. If $OS = 8$, $ST = 12$, and $OW = 24$, then $VW =$ _?_.



24. In $\triangle ABC$, the bisector of $\angle B$ meets $\overline{AC}$ at K. $AB = 18$, $BC = 24$, and $AC = 28$. Find AK.

Chapter Test

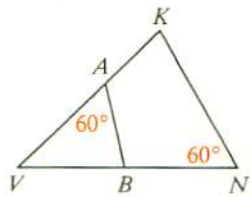
1. Two sides of a rectangle have the lengths 20 and 32. Find, in simplest form, the ratio of:
   a. the length of the shorter side to the length of the longer side
   b. the perimeter to the length of the longer side

2. If quad. $ABCD \sim$ quad. $THUS$, then: **a.** $\angle U \cong$ __?__ **b.** $\frac{BC}{HU} = \frac{AD}{?}$

3. If $x:y:z = 4:6:9$ and $z = 45$, then $x =$ __?__ and $y =$ __?__.

4. If $\frac{8}{9} = \frac{x}{15}$, then $x =$ __?__.

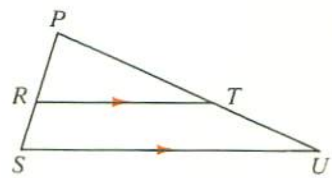
5. If $\frac{a}{b} = \frac{c}{10}$, then $\frac{a+b}{?} = \frac{?}{10}$.



6. What postulate or theorem justifies the statement $\triangle AVB \sim \triangle NVK$?

7. $\frac{AB}{NK} = \frac{VA}{?}$

8. The scale factor of $\triangle AVB$ to $\triangle NVK$ is $\frac{5}{8}$. If $VA = 2.5$ and $VB = 1.7$, then $VN =$ __?__.

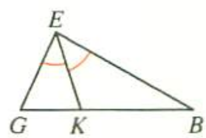


9. If $PR = 10$, $RS = 6$, and $PT = 15$, then $TU =$ __?__.

10. If $PT = 32$, $PU = 48$, and $RS = 10$, then $PR =$ __?__.

11. If $PR = 14$, $RS = 7$, and $RT = 26$, then $SU =$ __?__.

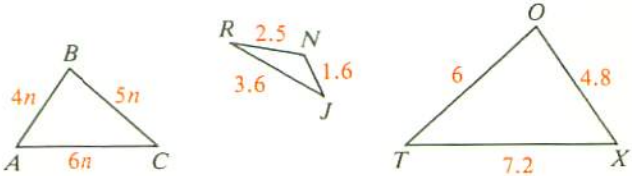
In $\triangle GEB$, the bisector of $\angle E$ meets $\overline{GB}$ at K.



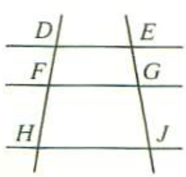
12. If $GK = 5$, $KB = 8$, and $GE = 7$, then $EB =$ __?__.

13. If $GE = 14$, $EB = 21$, and $GB = 30$, then $GK =$ __?__.

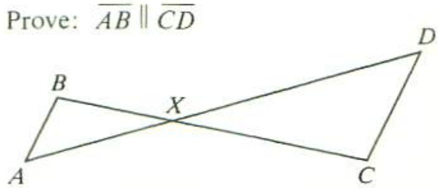
14. Given the triangles shown, state a similarity:
    $\triangle$ __?__ $\sim \triangle$ __?__.



15. Given: $\overleftrightarrow{DE} \parallel \overleftrightarrow{FG} \parallel \overleftrightarrow{HJ}$
    Prove: $DF \cdot GJ = FH \cdot EG$

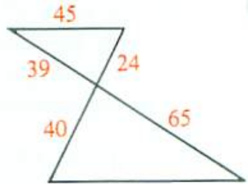


16. Given: $BX = 6$; $AX = 8$; $CX = 9$; $DX = 12$
    Prove: $\overline{AB} \parallel \overline{CD}$



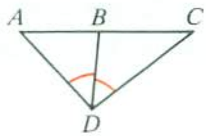
Mixed Review

1. In $\triangle XYZ$, $\angle Y \cong \angle Z$. If $XY = 5x + 3$, $YZ = 4x + 2$, and $XZ = 10x - 32$, find the lengths of $\overline{XY}$, $\overline{YZ}$, and $\overline{XZ}$.

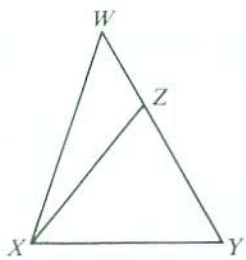


Ex. 2

2. a. The __?__ guarantees that the two triangles are __?__.
   b. Find the perimeter of the larger triangle.
3. The coordinates of points P and Q on a number line are -13 and 4. Find the coordinate of the midpoint of $\overline{PQ}$.
4. In $\triangle ABC$, $m\angle A : m\angle B : m\angle C = 3:3:4$.
   a. Is $\triangle ABC$ scalene, isosceles, or equilateral?
   b. Is $\triangle ABC$ acute, right, or obtuse?
   c. Name the longest side of $\triangle ABC$.
5. Describe the possible relationships between two planes.
6. To write an indirect proof, you assume temporarily that the __?__ is not true.
7. If two parallel lines are cut by a transversal, then __?__ angles are congruent, __?__ angles are congruent, and __?__ angles are supplementary.
8. Given: $RSTWYZ$ is a regular hexagon.
   Prove: $RSWY$ is a rectangle.
   (Begin by drawing a diagram.)
9. If $AB = x - 5$, $BC = x - 2$, $CD = x + 4$, and $DA = x$, find the value of x.



10. Use an indirect proof to show that no triangle has sides of length x, y, and $x + y$.
11. Find the sum of the measures of the angles of an octagon.
12. Given: $\angle WXY \cong \angle XZY$
    Prove: $(XY)^2 = WY \cdot ZY$



13. The measures of three consecutive angles of a quadrilateral are 58, 122, and 58. Must the diagonals
    a. be perpendicular?
    b. bisect each other?
    c. be congruent?

Algebra Review

Simplify.

Example **a.** $\sqrt{56} = \sqrt{4 \cdot 14} = \sqrt{4} \cdot \sqrt{14} = 2\sqrt{14}$

b. $\sqrt{8} \cdot \sqrt{6} = \sqrt{8 \cdot 6} = \sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3}$

c. $\sqrt{\frac{5}{6}} = \frac{\sqrt{5}}{\sqrt{6}} = \frac{\sqrt{5}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{30}}{\sqrt{36}} = \frac{\sqrt{30}}{6}$, or $\frac{1}{6}\sqrt{30}$

d. $\frac{\sqrt{55}}{\sqrt{22}} = \sqrt{\frac{55}{22}} = \sqrt{\frac{5}{2}} = \frac{\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10}}{2}$

e. $\left(\frac{3\sqrt{6}}{2}\right)^2 = \frac{3\sqrt{6}}{2} \cdot \frac{3\sqrt{6}}{2} = \frac{9 \cdot 6}{4} = \frac{27}{2}$

1. $\sqrt{81}$ **2.** $\sqrt{0}$ **3.** $\sqrt{24}$ **4.** $\sqrt{13^2}$

5. $(\sqrt{7})^2$ **6.** $\sqrt{600}$ **7.** $\sqrt{245}$ **8.** $\frac{1}{\sqrt{5}}$

9. $\frac{12}{\sqrt{2}}$ **10.** $\sqrt{\frac{2}{3}}$ **11.** $\sqrt{4} \cdot \sqrt{7}$ **12.** $\sqrt{8} \cdot \sqrt{15}$

13. $\frac{\sqrt{45}}{\sqrt{5}}$ **14.** $\frac{\sqrt{12}}{\sqrt{24}}$ **15.** $\frac{\sqrt{5}}{\sqrt{3}}$ **16.** $\frac{\sqrt{21}}{\sqrt{18}}$

17. $\frac{12}{\sqrt{15}}$ **18.** $\sqrt{\frac{80}{25}}$ **19.** $\sqrt{\frac{25}{80}}$ **20.** $3\sqrt{27}$

21. $\frac{1}{2}\sqrt{121}$ **22.** $\frac{4\sqrt{125}}{5}$ **23.** $\frac{12}{5\sqrt{6}}$ **24.** $\frac{15\sqrt{2}}{\sqrt{5}}$

25. $(9\sqrt{2})^2$ **26.** $\left(\frac{\sqrt{10}}{2}\right)^2$ **27.** $5(2\sqrt{3})^2$ **28.** $\frac{3}{4}(3\sqrt{8})^2$

Solve for x. Assume that x represents a positive number.

Example **a.** $\frac{x}{6} = \frac{3}{x}$ **b.** $\frac{144}{x} = \frac{x}{50}$ **c.** $x^2 + (3\sqrt{2})^2 = 9^2$

Solution

a. $x^2 = 6 \cdot 3$
$x = 18$
$x = \sqrt{18}$
$x = 3\sqrt{2}$

b. $x^2 = 144 \cdot 50$
$x = \sqrt{144} \cdot \sqrt{50}$
$x = 12 \cdot 5\sqrt{2}$
$x = 60\sqrt{2}$

c. $x^2 + 18 = 81$
$x = \sqrt{63}$
$x = \sqrt{9 \cdot 7}$
$x = 3\sqrt{7}$

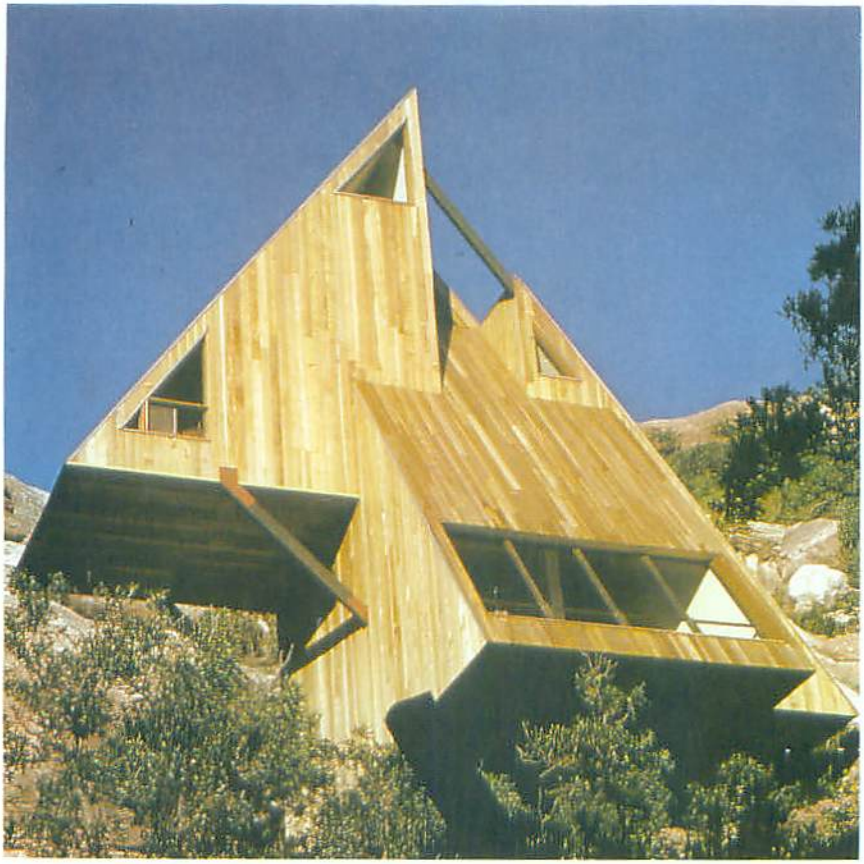
29. $\frac{2}{x} = \frac{x}{8}$ **30.** $\frac{1}{x} = \frac{x}{27}$ **31.** $\frac{x}{7} = \frac{5}{x}$

32. $\frac{x}{49} = \frac{100}{x}$ **33.** $\frac{x}{4} = \frac{32}{x}$ **34.** $\frac{30}{x} = \frac{x}{6}$

35. $x^2 = 3^2 + 4^2$ **36.** $x^2 + 3^2 = 4^2$ **37.** $5^2 + x^2 = 9^2$

38. $(4\sqrt{2})^2 + x^2 = (4\sqrt{3})^2$ **39.** $\frac{3x + 6}{x + 4} = \frac{x + 2}{x}$ **40.** $(2x)^2 + 15^2 = (3x)^2$

Right triangles are a prominent feature of the architectural design of this mountain cabin in Colorado. Properties of right triangles and an introduction to right-triangle trigonometry are the topics of this chapter.



Right Triangles

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