

1. a. geometric:  $t_n = 27\left(\frac{2}{3}\right)^{n-1}$       b. arithmetic:  $t_n = 4 + (n-1)7 = 7n - 3$   
 c. neither;  $t_n = n^2 + 1$
2.  $t_1 = -4$ ;  $t_2 = 2(-4) + 3 = -5$ ;  $t_3 = 2(-5) + 3 = -7$ ;  $t_4 = 2(-7) + 3 = -11$
3. a.  $x - 10 = \frac{x}{3} - x$ ;  $\frac{5}{3}x = 10$ ;  $x = 6$       b.  $\frac{x}{10} = \frac{\frac{x}{5}}{x}$ ;  $\frac{x}{10} = \frac{1}{3}$ ;  $x = \frac{10}{3}$
4.  $t_{25} = t_1 + (25-1)d = 23 + 24(-3) = -49$ ;  $S_{25} = \frac{25(23 + (-49))}{2} = 25(-13) = -325$
5.  $t_n = \frac{1}{4}\left(\frac{1}{2}\right)^{n-1} = 2^{n-3}$ ;  $S_n = \frac{\frac{1}{4}(1 - (\frac{1}{2})^n)}{1 - \frac{1}{2}} = \frac{1}{2}(1 - (\frac{1}{2})^n) = \frac{1}{2} - (\frac{1}{2})^{n+1}$
6.  $603 = 9 + (n-1)6$ ;  $594 = (n-1)6$ ;  $99 = n-1$ ;  $n = 100$ ;  
 $S_{100} = \frac{100(9 + 603)}{2} = 30,600$
7.  $S_{50} = 2 + 4 + 6 + \dots + 2 \cdot 50 = \frac{50(2 + 100)}{2} = 2550$
8. a.  $\lim_{n \rightarrow \infty} \frac{4n-1}{7n} = \lim_{n \rightarrow \infty} \frac{4 - \frac{1}{n}}{7} = \frac{4}{7}$       b.  $\lim_{n \rightarrow \infty} \frac{2n^3 - 3n}{5n^2 + 1} = \lim_{n \rightarrow \infty} \frac{2n - \frac{3}{n}}{5 + \frac{1}{n^2}} = \infty$   
 c.  $\lim_{n \rightarrow \infty} (0.75)^n = 0$  because  $\lim_{n \rightarrow \infty} r^n = 0$  if  $|r| < 1$ .
9.  $t_1 = 16$ ;  $r = \frac{-12}{16} = -\frac{3}{4}$ ;  $S = \frac{16}{1 - (-\frac{3}{4})} = \frac{16}{\frac{7}{4}} = \frac{64}{7}$
10.  $r = \frac{3x}{2}$ ;  $|r| < 1$ ;  $|\frac{3x}{2}| < 1$ ;  $-1 < \frac{3x}{2} < 1$ ;  $-\frac{2}{3} < x < \frac{2}{3}$

11. a.  $\sum_{j=0}^3 (-1)^j(3j-1) = (-1)^0(3 \cdot 0 - 1) + (-1)^1(3 \cdot 1 - 1) + (-1)^2(3 \cdot 2 - 1) + (-1)^3(3 \cdot 3 - 1) = -1 - 2 + 5 - 8 = -6$   
 b.  $\sum_{k=1}^{\infty} \left(\frac{1}{5}\right)^k = \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \dots = \frac{\frac{1}{5}}{1 - \frac{1}{5}} = \frac{\frac{1}{5}}{\frac{4}{5}} = \frac{1}{4}$
12.  $\sum_{k=1}^{\infty} \frac{2k}{2k+1}$
13. a. When  $n$  is large,  $\frac{n}{2n^2+5} \approx \frac{1}{2n}$ . Since  $\sum_{n=1}^{\infty} \frac{1}{n}$  diverges, so do  $\sum_{n=1}^{\infty} \frac{1}{2n}$  and  $\sum_{n=1}^{\infty} \frac{n}{2n^2+5}$ .  
 b.  $\sum_{n=2}^{\infty} \frac{1}{(n-1)(n-1)} = \sum_{n=2}^{\infty} \frac{1}{n^2-1}$ ; when  $n$  is large,  $\frac{1}{n^2-1} \approx \frac{1}{n^2}$ ; since  $\sum_{n=2}^{\infty} \frac{1}{n^2}$  converges, so does  $\sum_{n=2}^{\infty} \frac{1}{(n-1)(n-1)}$ .
14.  $\tan^{-1} 2x = 2x - \frac{(2x)^3}{3} + \frac{(2x)^5}{5} - \frac{(2x)^7}{7} + \dots$ ,  $-\frac{1}{2} < x < 1$
15. (1) For  $n = 1$ ,  $1^2 = 1 = \frac{1(2)(2 \cdot 1 - 1)}{6}$ , so the statement is true when  $n = 1$ .  
 (2) Assume that  $1^2 + 2^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6}$  and prove the statement is true for  $n = k+1$ :  $1^2 + 2^2 + \dots + k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + \frac{6(k+1)^2}{6} = \frac{(k+1)(2k^2+7k+6)}{6} = \frac{(k+1)(k+2)(2k+3)}{6} = \frac{(k+1)[(k+1)+1][2(k+1)+1]}{6}$