UNIT VIII

UNIT VIII
TABLE OF CONTENTS

UNIT VIII: DIFFERENTIAL EQUATIONS

Expectation:	Students will solve differential equations graphically, numerically and analytically and will be able to apply this to real world situations.

OVERVIEW:
Students will be able to solve differential equations using slope fields, Euler's method and the method of separation of variables. Real world applications will include growth and decay problems and logistic models.

INDICATORS:

1. Verify that a solution satisfies a differential equation.

* 2. Sketch a solution of the differential equation given a figure of an equation's slope field.

* 3. Calculate an approximate numerical solution to a given differential equation using Euler's method.

4. Use the method of separation of variables to solve a differential equation.

5. Use and solve growth and decay problems involving differential equations.

* 6. Use and solve logistic differential equations in modeling.

* BC Calculus indicator only

Unit VIII: Differential Equations

Indicators/ Objectives	Foerster: Calculus Key Curriculum 1998	Foerster: Calculus: Instructor's Resource Book Key Curriculum 1998	Finney, et al: Calculus S F A W 1999	Guide Pages
1	310		303	VIII 1-2
2	326-8	7.4,7.6	305-306; 347	VIII 3-6
3	333-4	7.5	350-355	VIII 5-6
4	309-320		320-321	VIII 1-4, 6-12
5	309-312	7.2	330-337; 342-343	VIII 7-8, 11-12
6	337,468		343-346	VIII 9-10

Indicators/ Objectives	Finney, et al: Calculus 1994	Guide Pages
1	662	
2	335 - 336	
3	666 - 670	
4	663 - 665	
5	537 - 538	
6	539 - 543	

NAME Key
DATE ____________

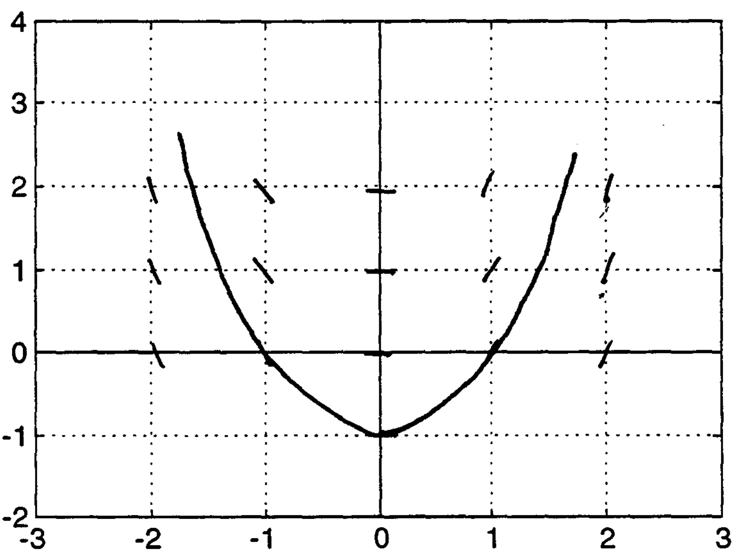
SLOPE FIELDS

In this exercise, you will learn to construct a geometric representation of the integral curves of a differential equation.

1. a) Complete the table by substituting the given values into the differential equation, $\frac{dy}{dx} = 2x$.

x	-2	-2	-2	-1	-1	-1	0	0	0	1	1	1	2	2	2
y	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2
$\frac{dy}{dx}$	-4	-4	-4	-2	-2	-2	0	0	0	2	2	2	4	4	4

b) Draw a very short segment of the tangent line at each point (from the table) on the grid below. [Remember that $\frac{dy}{dx}$ represents the slope of the line tangent to some function y at any given point on y]



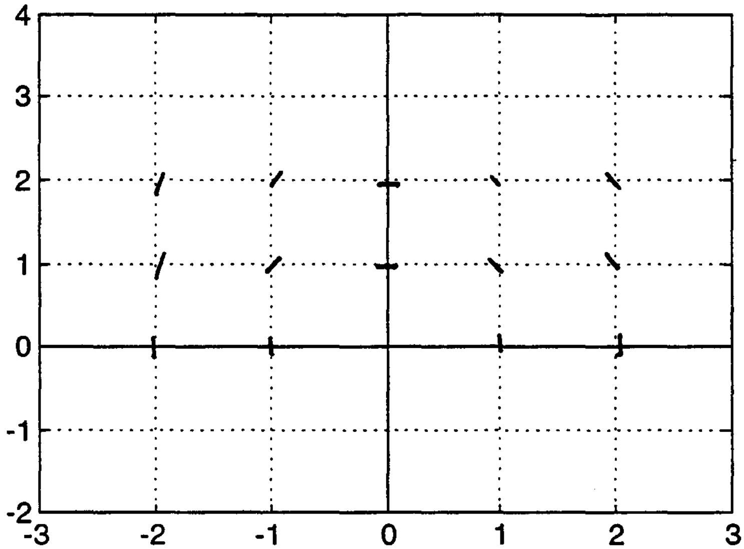
c) Separate the variables and integrate both sides of the equation in part a). How is your answer related to the graph?

$$\frac{dy}{dx} = 2x \qquad dy = 2x\,dx$$
$$y = x^2 + C$$

d) If $y(0) = -1$, sketch y on the grid. $\quad y = x^2 - 1$

2. a) Complete the table for the differential equation, $\frac{dy}{dx} = -\frac{x}{2y}$, and sketch the slope field on the grid.

x	-2	-2	-2	-1	-1	-1	0	0	0	1	1	1	2	2	2
y	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2
$\frac{dy}{dx}$	∞	1	$\frac{1}{2}$	∞	$\frac{1}{2}$	$\frac{1}{4}$	ind	0	0	∞	$-\frac{1}{2}$	$-\frac{1}{4}$	∞	-1	$-\frac{1}{2}$



b) Based on the slope field, what type of equation do you think the solution for the differential equation will be?

The solutions will be ellipses

c) Find a general solution for $\frac{dy}{dx} = -\frac{x}{2y}$.

$$y^2 = -\frac{x^2}{2} + C$$

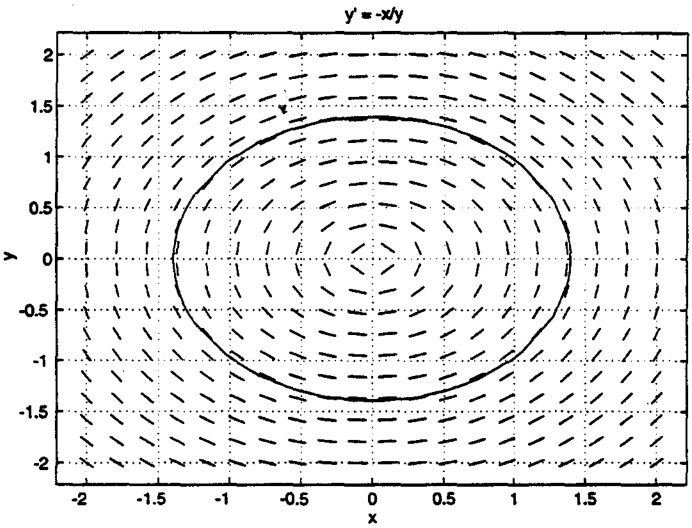
AP CALCULUS
UNIT VIII – ACTIVITY 2

NAME Key
DATE

USING SLOPE FIELDS TO SKETCH SOLUTIONS OF DIFFERENTIAL EQUATIONS

Solve each given differential equation then use the given slope field to sketch the indicated particular solution.

1. $\frac{dy}{dx} = -\frac{x}{y}$ sketch starting from the point (1, 1)



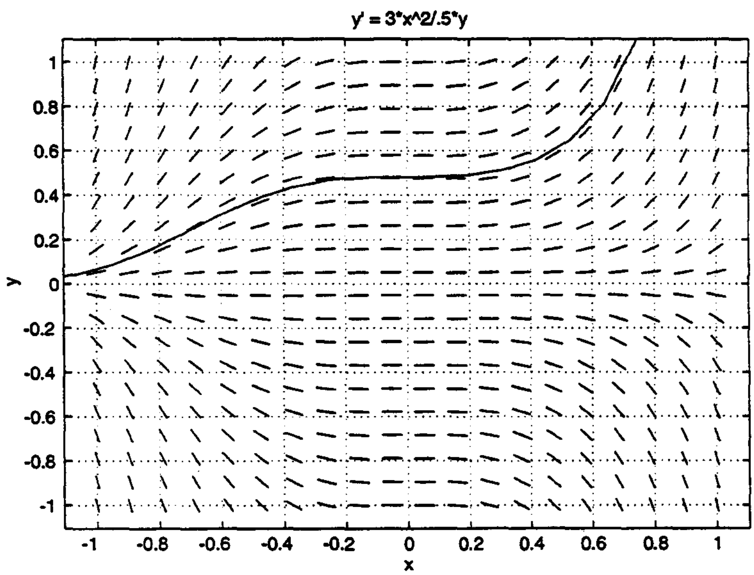
$y\,dy = -x\,dx$

$\frac{y^2}{2} = -\frac{x^2}{2} + \frac{c}{2}$

$x^2 + y^2 = c$

$y = \pm\sqrt{c - x^2}$

2. $\frac{dy}{dx} = \frac{3x^2}{.5y}$ sketch starting from the point (-.5, .4)



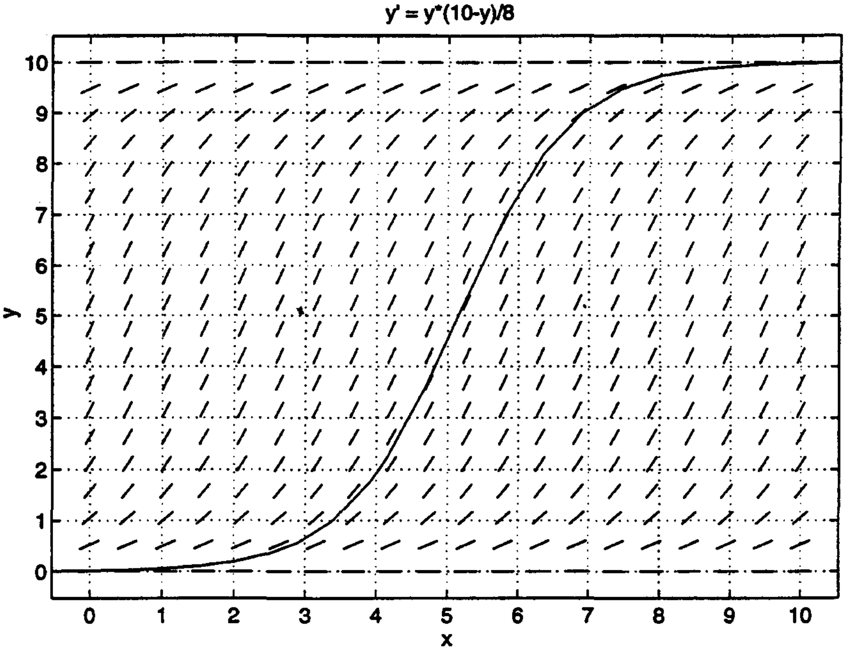
$.5y\,dy = 3x^2\,dx$

$y\,dy = 6x^2\,dx$

$\frac{y^2}{2} = 2x^3 + c_1$

$y = \pm\sqrt{4x^3 + c}$

3. $\frac{dy}{dx} = \frac{y(10-y)}{8}$ sketch starting from the point (4, 2)



$$\frac{dy}{y(10-y)} = \frac{1}{8}\,dx$$

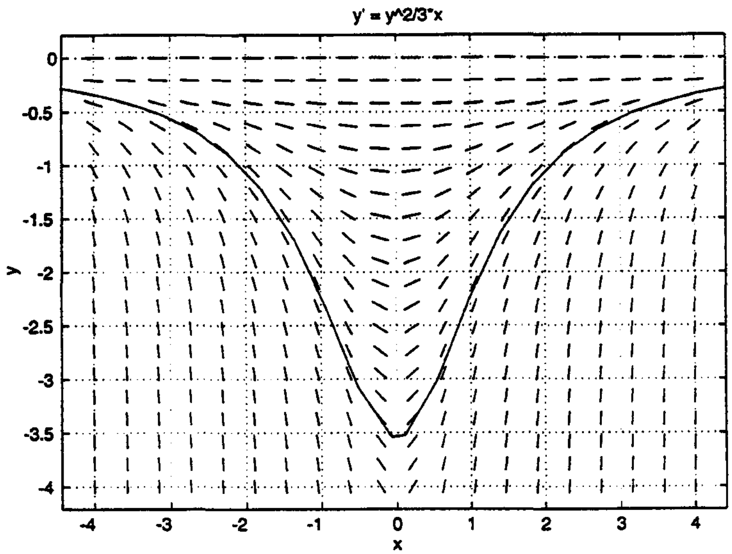
$$\left(\frac{1}{y} + \frac{1}{10-y}\right)dy = \frac{5}{4}\,dx$$

$$\ln\left|\frac{y}{10-y}\right| = \frac{5}{4}x + C_1$$

$$y = \frac{10C_2 e^{5/4\,x}}{1 + C_2 e^{5/4\,x}}$$

$$= \frac{10}{1 + Ce^{-5/4\,x}}$$

4. $\frac{dy}{dx} = \frac{y^2}{3x}$ sketch starting from the point (0, -3.5)



$$\frac{dy}{y^2} = \frac{1}{3}\,\frac{dx}{x}$$

$$-\frac{1}{y} = \frac{1}{3}\ln|x| + \frac{C_1}{3}$$

$$y = \frac{-3}{\ln|x| + C}$$

AP CALCULUS
UNIT VIII – ACTIVITY 3

NAME Key
DATE

EULER'S METHOD

In this activity, you will learn a numerical method for approximating solutions for differential equations.

1. The following problems refer to the differential equation $\frac{dy}{dx} = \frac{x}{y}$.

a. Calculate the slope at (0, 2) using the differential equation.

$\frac{dy}{dx} = 0$ at $(0, 2)$

b. If $dx = 0.5$, find dy ($dy = \frac{dy}{dx} \bullet dx$).

$dy = (0)(.5) = 0$

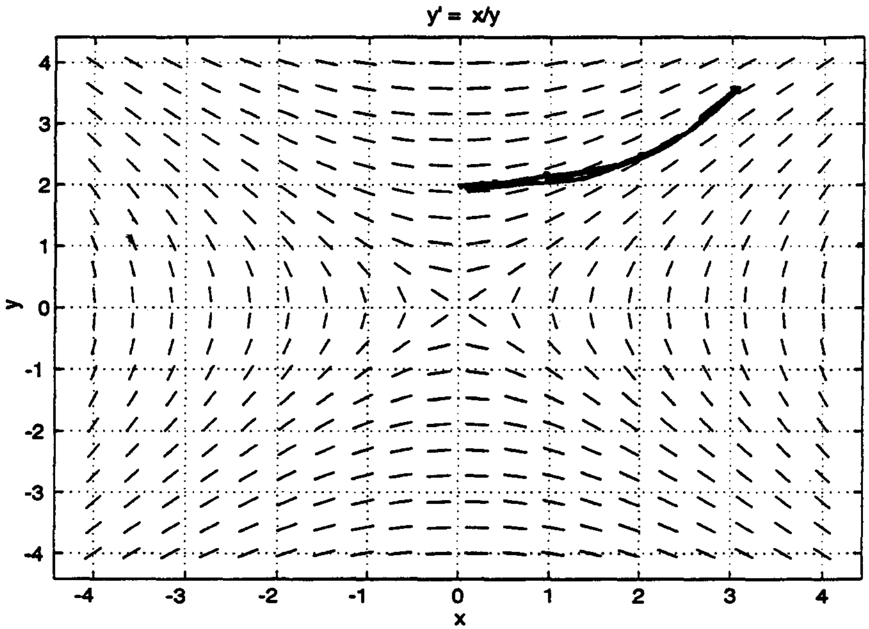
c. Find a new point by adding dx to x and dy to y. What are the coordinates of this point?

$(.5, 2)$

d. Complete the chart by repeating the process.

x ($dx = 0.5$)	y	$\frac{dy}{dx}$	dy
0	2	0	0
0.5	2	.25	.125
1.0	2.125	.47058...	.23529...
1.5	2.36029	.63551...	.31775...
2.0	2.67805..	.74681...	.37340...
2.5	3.05145...	.81928...	.40964...
3.0	3.46109...	.86677...	.43338

e. The slope field for $\frac{dy}{dx} = \frac{x}{y}$ is pictured below. Plot the points from the table on the slope field. How well do the points generated by Euler's Method fit the pattern? They follow the pattern closely.



f. Solve the differential equation algebraically and graph it. Did Euler's Method give you a good approximation for this solution?

$$y\,dy = x\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + \frac{c}{2}$$

$$y^2 - x^2 = c.$$

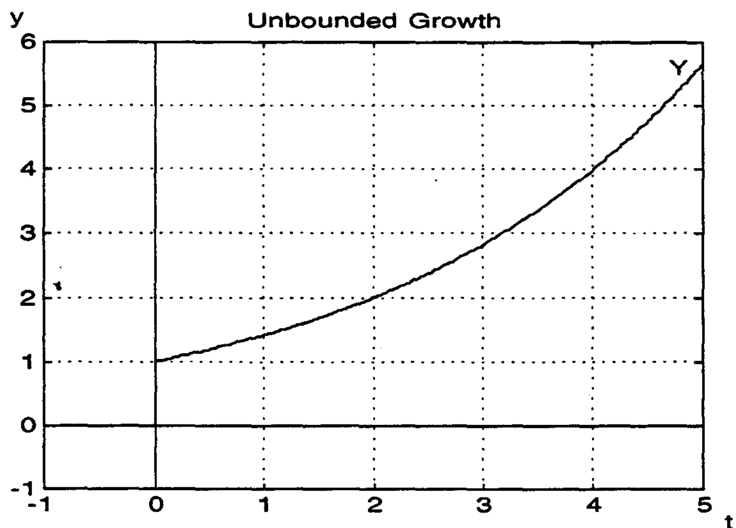
Yes.

AP CALCULUS
UNIT VIII – ACTIVITY 4

NAME Key.
DATE

GROWTH MODELS

UNBOUNDED GROWTH



Rate is proportional to the amount present. ($k > 0$)

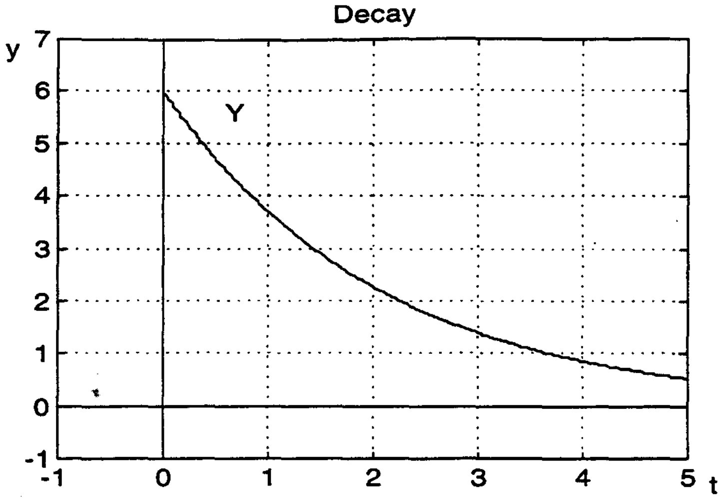
Equation: $\frac{dy}{dt} = ky$ Applications: Exponential growth; inflation; interest compounded continuously

Solution: $y = y_0 e^{kt}$
(y_0 is the initial amount)

1. The value of Chet's baseball card collection increases with age and its rate of appreciation at any time t is proportional to its value at that time. The value of the collection was \$25,000 ten years ago, and its present value is \$35,000.
   (a) Write a differential equation to model the value of the collection. $\frac{dy}{dt} = ky$
   (b) Solve the differential equation.
   (c) Chet will sell the collection when its value reaches \$50,000. When will he sell? (b) $y = 25{,}000\, e^{\frac{1}{10} \ln(35/25)\, t}$ (c) in 10.6 years ($t = 20.6$)

2. The mold grows on Phun Guy's sneakers in his gym locker at a rate proportional to the amount present. The initial weight of the mold is 3 grams and after two days it weighs 7 grams.
   (a) Write a differential equation to model the weight of the mold on the sneakers. $\frac{dy}{dt} = ky$
   (b) Solve the differential equation.
   (c) How much does it weigh after ten days?
   (b) $y = 3e^{\frac{1}{2}\ln(7/3)\, t}$ (c) 207.494 g.

DECAY



Rate is proportional to amount present. ($k < 0$)

Equation: $\frac{dy}{dt} = ky$ Applications: Radioactive decay; depletion of natural resources

Solution: $y = y_0 e^{-kt}$
(y_0 is the initial amount)

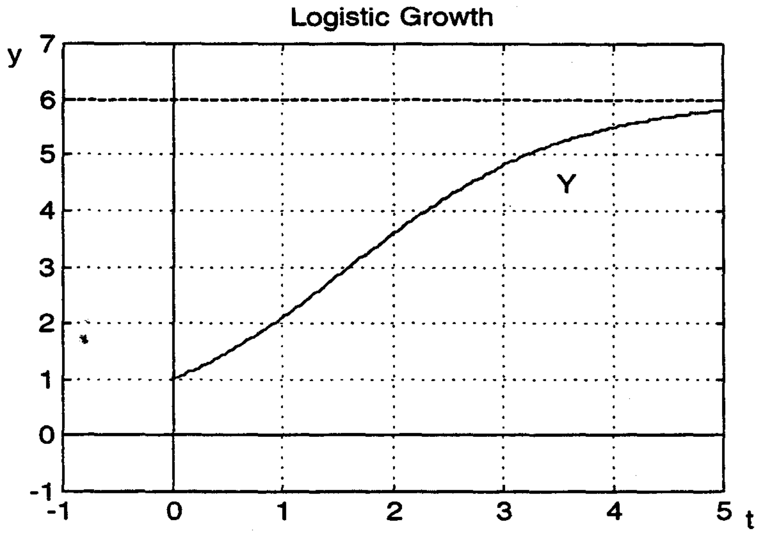
1. Oil is pumped continuously from a well at a rate proportional to the amount of oil left in the well. Initially there were 2 million barrels of oil in the well. Six years later there were 1.2 million barrels remain.
   (a) Write a differential equation to model the amount of oil in the well.
   (b) At what rate was the amount of oil in the well decreasing when there were 1.5 million barrels of oil remaining?
   (c) Solve the differential equation.
   (d) It will no longer be profitable to pump oil from the well when there are fewer than 100,000 barrels of oil remaining. When should pumping stop?

(a) $\frac{dy}{dt} = ky$
$k = -.085$
(b) $(-.085)(1.5) \approx -.127$ million barrels of oil
(c) $y = 2e^{-.0851t}$
~ 35 yrs.

2. For a constant temperature, the rate of change of barometric pressure, p, with respect to altitude, h, is proportional to p. The pressure is 30 inches of mercury at sea level and 29 inches of mercury at 1000 feet.
   (a) Write a differential equation to model the barometric pressure.
   (b) Solve the differential equation.
   (c) At what altitude will the pressure be 25 inches of mercury?

$\frac{dp}{dh} = kp = 3.39 \times 10^{-5} p$
$p = 30e^{-3.39 \times 10^{-5} h}$
5378.059 ft.

LOGISITIC GROWTH



Rate is jointly proportional to the amount present and to the difference between the amount present and a fixed amount. $(k > 0)$

Equation: $\frac{dy}{dt} = ky(B-y)$ Application: long-term population growth; spread of a disease or rumor

Solution: $y = \frac{B}{1 + Ae^{-Bkt}}$

When t = 0, $y = \frac{B}{1+A}$

B represents the upper limit of the population (carrying capacity)

Note: When solving logistic growth equations, keep the constant on the same side of the equation as *dt*.

1. In a company of 1000 people, a rumor is started by the 20 members of the accounting department and spreads logistically. After one day, 50 people have heard the rumor.

a) Set up a differential equation to model the spread of the rumor.
b) Solve the differential equation.
c) What is the maximum number of people who can hear the rumor?
d) How many people have heard the rumor after 6 days ?

a) $\frac{dR}{dt} = kR(1000-R)$

b) $R = \frac{1000}{1+49e^{-\ln(49/19)t}}$

c) 1000 people

d) 857 people

2. A population of a town in a new suburban area grows according to the logistics model $\frac{dy}{dt} = \frac{y(10-y)}{15}$ measured in thousands of people per year. At $t = 0$ years the size of the population is 2,000 people.

(a) What is the maximum size of the population? 10,000 people
(b) Solve the differential equation.
(c) When is the population growing most rapidly?

(b) $\frac{dy}{y(10-y)} = \frac{1}{15}dt$ (leaving the constant, here $\frac{1}{15}$, with the simpler expression, here dt, generally helps simplify the work)

$\frac{A}{y} + \frac{B}{10-y} = \frac{1}{y(10-y)}$

$A = B = \frac{1}{10}$

$\int\left(\frac{1}{y} + \frac{1}{10-y}\right)dy = \frac{10}{15}\int dt$

$\ln\left|\frac{y}{10-y}\right| = \frac{2}{3}t + C \text{ ; } C = \ln\frac{1}{4}$

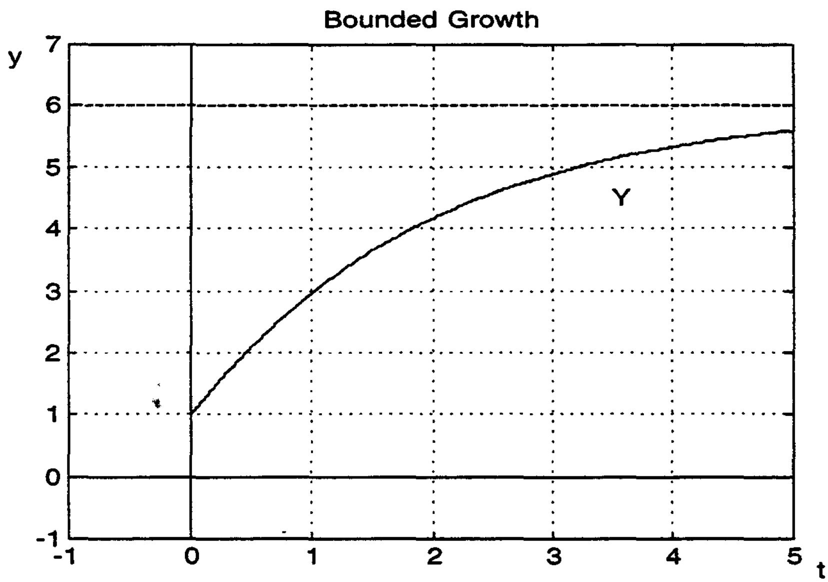
$y = \frac{10}{1+4e^{-2/3 t}}$

(c) max. of $\frac{dy}{dt}$ when $y = 5$:

$\frac{5}{10-5} = \frac{1}{4}e^{2/3 t}$

$t \approx 2.079$ yrs

BOUNDED GROWTH



Rate is proportional to the difference between the amount present and a fixed limit. ($k > 0$)

Equation: $\frac{dy}{dt} = k(B - y)$

Solution: $y = B - Ae^{-kt}$

Applications: learning curve; Newton's Law of Cooling

1. At Acme Inc., a new employee performs a job more efficiently each day according to $\frac{dy}{dt} = k(80 - y)$ where y is the number of units produced per day. The employee produces 20 units the first day and 50 units on the tenth day.
   a) Express y as a function of t.
   b) How many units per day can the employee eventually be expected to produce?
   c) How many units is the employee producing after 60 days?

$y = 80 - 60e^{-\frac{\ln 2}{10}t}$

$\lim_{t \to \infty} y = 80$ units per day

79 units

2. Relief supplies are air-dropped from a plane with velocity, $v(t)$, feet per second. The velocity of the supplies after being dropped are modeled by the differential equation $\frac{dv}{dt} = -3v - 27$ where $v(0) = -30$.

a) Express v in terms of t.

b) Terminal velocity is defined as $\lim_{t\to\infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

a) $\frac{dv}{v+9} = -3\,dt$

$\ln|v+9| = -3t + C \qquad C = \ln 21$

$v = 21e^{-3t} - 9$

b) $\lim_{t\to\infty} (21e^{-3t} - 9) = -9$ ft/sec.

ADDITIONAL EXAMPLES may be found at the following websites

1. http://www/math.duke.edu/modules2/materials/diffeq/index.html

UNIT VIII
STUDENT WORKSHEETS

NAME ____________
DATE ____________

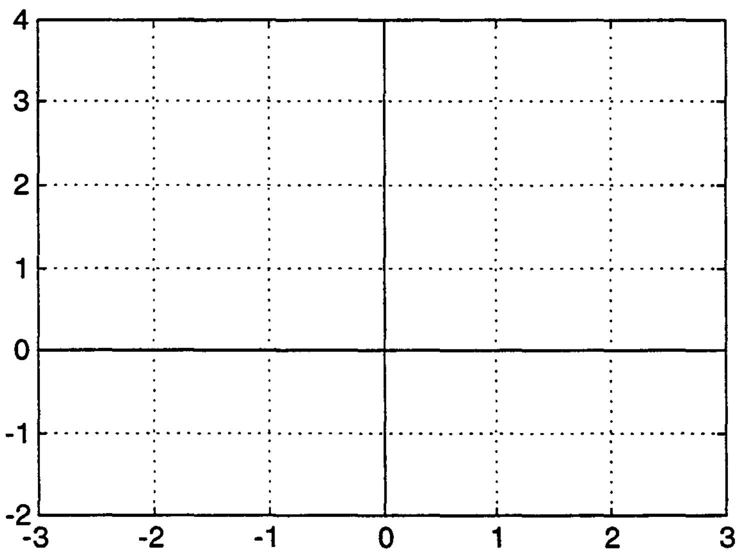
SLOPE FIELDS

In this exercise, you will learn to construct a geometric representation of the integral curves of a differential equation.

1. a) Complete the table by substituting the given values into the differential equation, $\frac{dy}{dx} = 2x$.

x	-2	-2	-2	-1	-1	-1	0	0	0	1	1	1	2	2	2
y	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2
$\frac{dy}{dx}$															

b) Draw a very short segment of the tangent line at each point (from the table) on the grid below. [Remember that $\frac{dy}{dx}$ represents the slope of the line tangent to some function y at any given point on y]

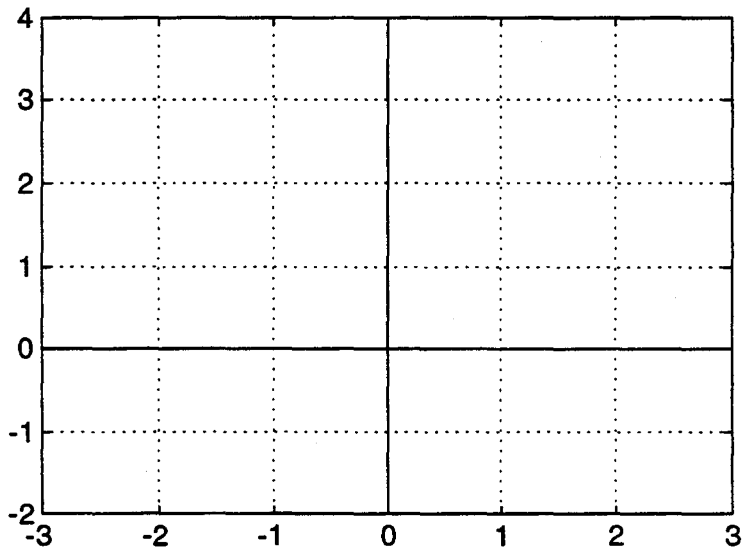


c) Separate the variables and integrate both sides of the equation in part a). How is your answer related to the graph?

d) If $y(0) = -1$, sketch y on the grid.

2. a) Complete the table for the differential equation, $\frac{dy}{dx} = -\frac{x}{2y}$, and sketch the slope field on the grid.

x	-2	-2	-2	-1	-1	-1	0	0	0	1	1	1	2	2	2
y	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2
$\frac{dy}{dx}$															



b) Based on the slope field, what type of equation do you think the solution for the differential equation will be?

c) Find a general solution for $\frac{dy}{dx} = -\frac{x}{2y}$.

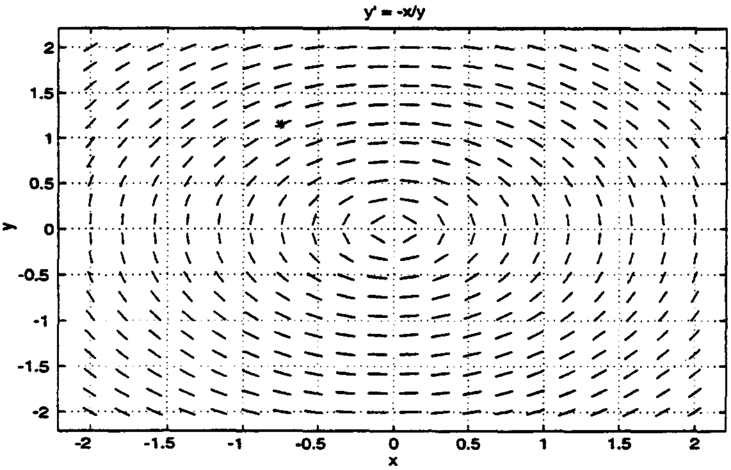
AP CALCULUS
UNIT VIII – ACTIVITY 2

NAME ____________________
DATE ____________________

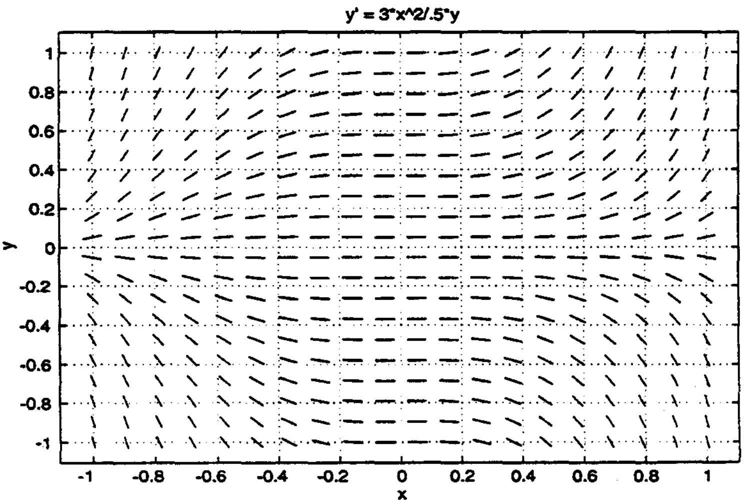
USING SLOPE FIELDS TO SKETCH SOLUTIONS OF DIFFERENTIAL EQUATIONS

Solve each given differential equation then use the given slope field to sketch the indicated particular solution.

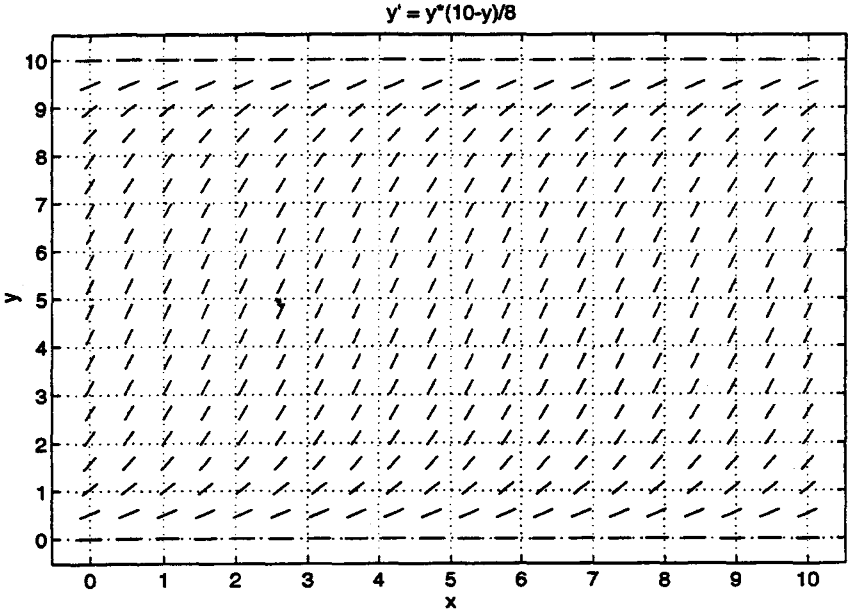
1. $\frac{dy}{dx} = -\frac{x}{y}$ sketch starting from the point (1, 1)



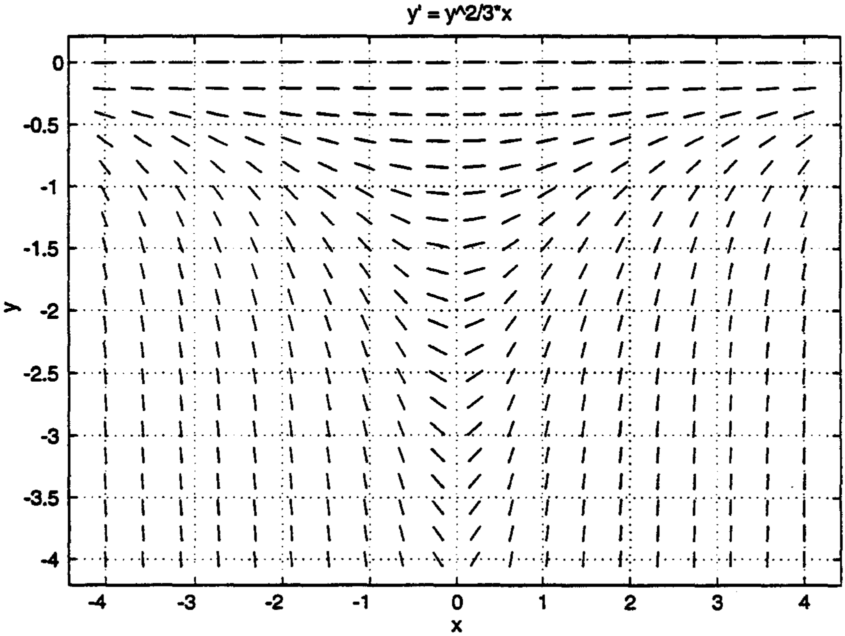
2. $\frac{dy}{dx} = \frac{3x^2}{.5y}$ sketch starting from the point (-.5, .4)



3. $\frac{dy}{dx} = \frac{y(10-y)}{8}$ sketch starting from the point (4, 2)



4. $\frac{dy}{dx} = \frac{y^2}{3x}$ sketch starting from the point (0, -3.5)



NAME ____________________
DATE ____________________

EULER'S METHOD

In this activity, you will learn a numerical method for approximating solutions for differential equations.

1. The following problems refer to the differential equation $\frac{dy}{dx} = \frac{x}{y}$.

   a. Calculate the slope at (0, 2) using the differential equation.

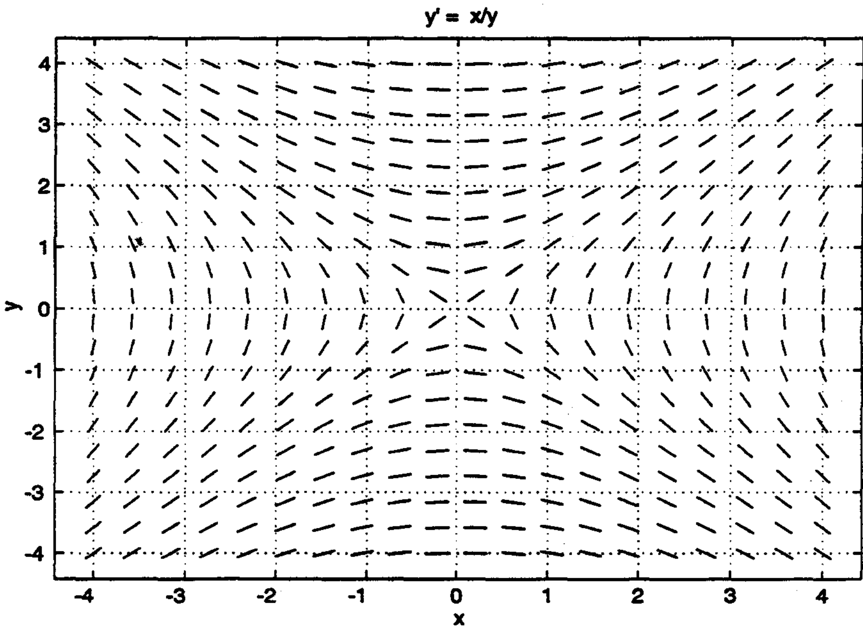
   b. If $dx = 0.5$, find dy ($dy = \frac{dy}{dx} \bullet dx$).

   c. Find a new point by adding dx to x and dy to y. What are the coordinates of this point?

   d. Complete the chart by repeating the process.

x ($dx = 0.5$)	y	$\frac{dy}{dx}$	dy
0	2		
0.5			
1.0			
1.5			
2.0			
2.5			
3.0			

e. The slope field for $\frac{dy}{dx} = \frac{x}{y}$ is pictured below. Plot the points from the table on the slope field. How well do the points generated by Euler's Method fit the pattern?

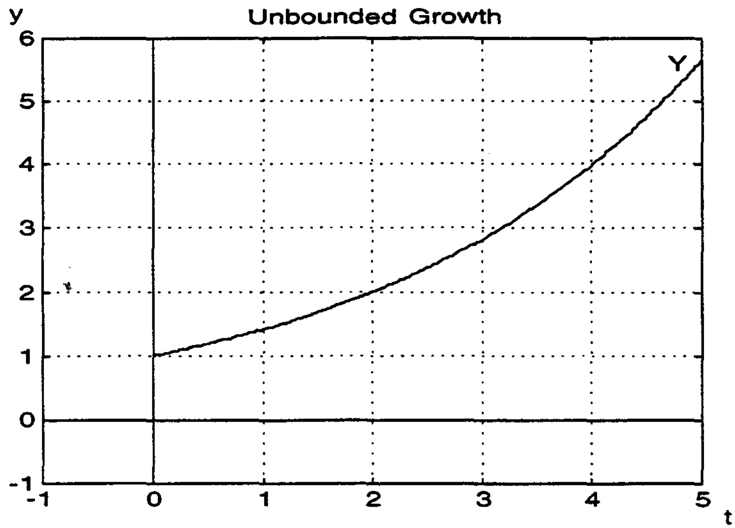


f. Solve the differential equation algebraically and graph it. Did Euler's Method give you a good approximation for this solution?

NAME ____________________
DATE ____________________

GROWTH MODELS

UNBOUNDED GROWTH



Rate is proportional to the amount present. ($k > 0$)

Equation: $\dfrac{dy}{dt} = ky$

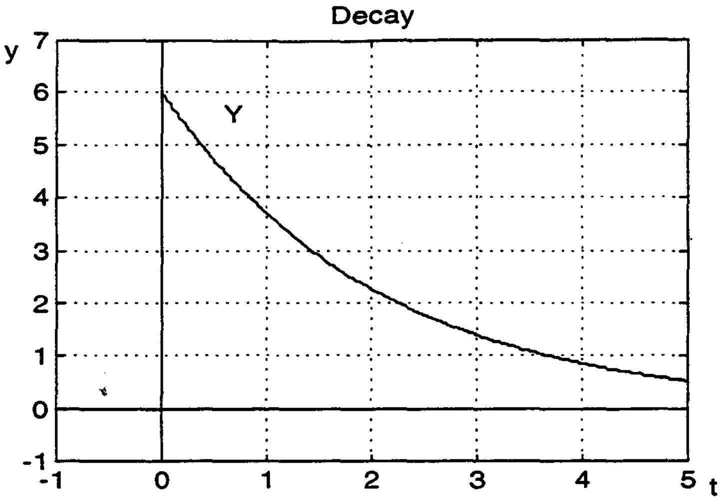
Applications: Exponential growth; inflation; interest compounded continuously

Solution: $y = y_0\, e^{kt}$
(y_0 is the initial amount)

1. The value of Chet's baseball card collection increases with age and its rate of appreciation at any time t is proportional to its value at that time. The value of the collection was \$25,000 ten years ago, and its present value is \$35,000.
   (a) Write a differential equation to model the value of the collection.
   (b) Solve the differential equation.
   (c) Chet will sell the collection when its value reaches \$50,000. When will he sell?

2. The mold grows on Phun Guy's sneakers in his gym locker at a rate proportional to the amount present. The initial weight of the mold is 3 grams and after two days it weighs 7 grams.
   (a) Write a differential equation to model the weight of the mold on the sneakers.
   (b) Solve the differential equation.
   (c) How much does it weigh after ten days?

DECAY



Rate is proportional to amount present. ($k < 0$)

Equation: $\frac{dy}{dt} = ky$

Applications: Radioactive decay; depletion of natural resources

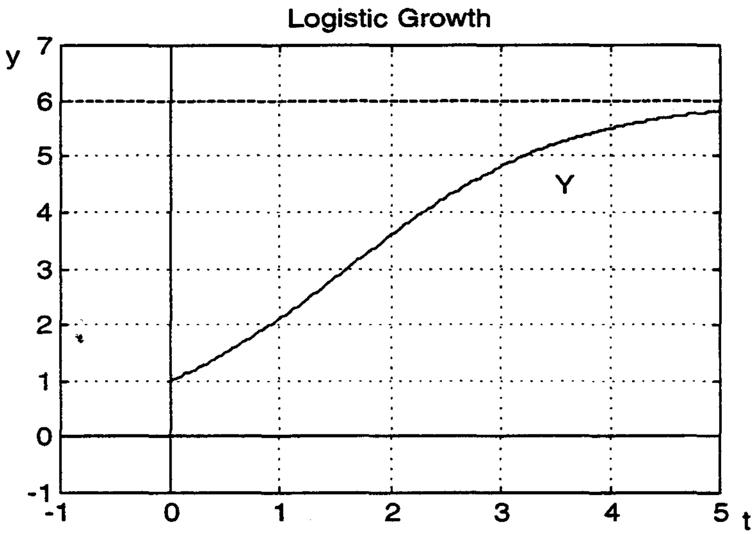
Solution: $y = y_0 e^{-kt}$

(y_0 is the initial amount)

1. Oil is pumped continuously from a well at a rate proportional to the amount of oil left in the well. Initially there were 2 million barrels of oil in the well. Six years later there were 1.2 million barrels remain.
   (a) Write a differential equation to model the amount of oil in the well.
   (b) At what rate was the amount of oil in the well decreasing when there were 1.5 million barrels of oil remaining?
   (c) Solve the differential equation.
   (d) It will no longer be profitable to pump oil from the well when there are fewer than 100,000 barrels of oil remaining. When should pumping stop?

2. For a constant temperature, the rate of change of barometric pressure, p, with respect to altitude, h, is proportional to p. The pressure is 30 inches of mercury at sea level and 29 inches of mercury at 1000 feet.
   (a) Write a differential equation to model the barometric pressure.
   (b) Solve the differential equation.
   (c) At what altitude will the pressure be 25 inches of mercury?

LOGISITIC GROWTH



Rate is jointly proportional to the amount present and to the difference between the amount present and a fixed amount. $(k > 0)$

Equation: $\frac{dy}{dt} = ky(B-y)$ Application: long-term population growth; spread of a disease or rumor

Solution: $y = \frac{B}{1 + Ae^{-Bkt}}$

When t = 0, $y = \frac{B}{1+A}$

B represents the upper limit of the population (carrying capacity)

<u>Note</u>: When solving logistic growth equations, keep the constant on the same side of the equation as dt.

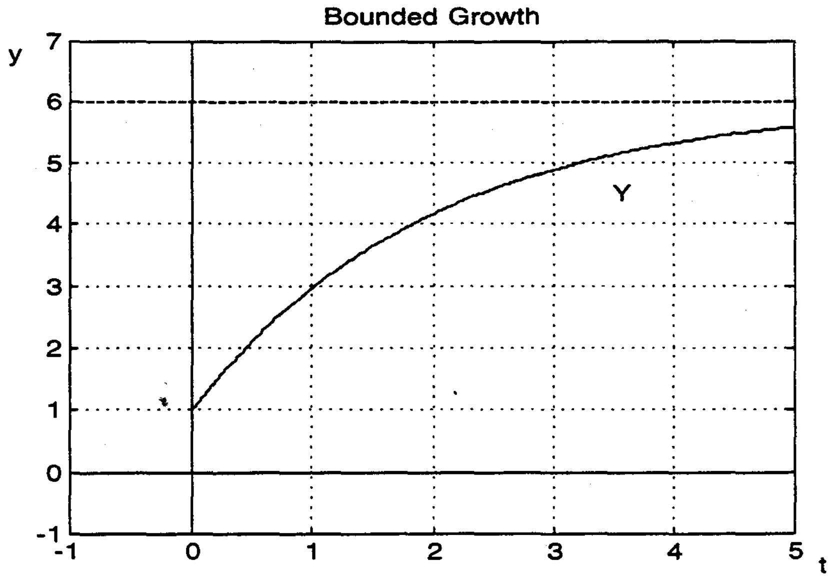
1. In a company of 1000 people, a rumor is started by the 20 members of the accounting department and spreads logistically. After one day, 50 people have heard the rumor.

a) Set up a differential equation to model the spread of the rumor.
b) Solve the differential equation.
c) What is the maximum number of people who can hear the rumor?
d) How many people have heard the rumor after 6 days ?

2. A population of a town in a new suburban area grows according to the logistics model $\frac{dy}{dt} = \frac{y(10-y)}{15}$ measured in thousands of people per year. At $t = 0$ years the size of the population is 2,000 people.

(a) What is the maximum size of the population?
(b) Solve the differential equation.
(c) When is the population growing most rapidly?

BOUNDED GROWTH



Rate is proportional to the difference between the amount present and a fixed limit. $(k > 0)$

Equation: $\frac{dy}{dt} = k(B - y)$

Applications: learning curve; Newton's Law of Cooling

Solution: $y = B - Ae^{-kt}$

1. At Acme Inc., a new employee performs a job more efficiently each day according to $\frac{dy}{dt} = k(80 - y)$ where y is the number of units produced per day. The employee produces 20 units the first day and 50 units on the tenth day.
   a) Express y as a function of t.
   b) How many units per day can the employee eventually be expected to produce?
   c) How many units is the employee producing after 60 days?

2. Relief supplies are air-dropped from a plane with velocity, $v(t)$, feet per second. The velocity of the supplies after being dropped are modeled by the differential equation $\frac{dv}{dt} = -3v - 27$ where $v(0) = -30$.

a) Express v in terms of t.

b) Terminal velocity is defined as $\lim_{t \to \infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

ADDITIONAL EXAMPLES may be found at the following websites

1. http://www/math.duke.edu/modules2/materials/diffeq/index.html

NAME____________________
DATE ____________________

MULTIPLE CHOICE PRACTICE

A CALCULATOR MAY NOT BE USED ON THE FOLLOWING QUESTIONS

1. A bird of prey spots its dinner on the ground and dives straight down toward the earth such that its downward velocity satisfies the differential equation $\frac{dv}{dt} = 2v - 12$, with initial condition v(0)=7. Which of the following is an expression for v in terms of t, where t is measured in seconds.

$A)\ v = e^{2t}$ $\quad B)\ v = e^{t}$ $\quad C)\ v = e^{2t} + 6$ $\quad D)\ v = e^{t} + 6$ $\quad E)\ v = e^{t} - 6$

2. If $\frac{dy}{dt} = ky$ and k is a positive even-numbered constant, then y could be

$A)\ y = -2e^{3t}$ $\quad B)\ y = -5e^{t}$ $\quad C)\ y = 2e^{-2t}$ $\quad D)\ y = 3e^{2t}$ $\quad E)\ y = 4e^{5t}$

3. For the differential equation given by $\frac{dy}{dx} = \frac{xy}{3}$, the particular solution with the initial condition f(0)=2 is

$A)\ y = 2e^{\frac{1}{6}x^2}$ $\quad B)\ y = 2e^{\frac{1}{2}x^2}$ $\quad C)\ y = 2e^{\frac{1}{3}x^2}$ $\quad D)\ y = 3e^{\frac{1}{6}x^2}$ $\quad E)\ y = 3e^{\frac{1}{2}x^2}$

4. Which of the following is a possible solution of the differential equation $\frac{dy}{dx} - 4xy = 0$?

$I.\ y = -3e^{x^2}$ $\quad II.\ 2e^{2x^2}$ $\quad III.\ -4e^{2x^2}$ $\quad IV.\ y = 5e^{2x}$

A) none $\quad$ *B) II only* $\quad$ *C) I and IV only* $\quad$ *D) II and III only* $\quad$ *E) II and IV only*

5. Given the differential equation $\frac{dy}{dx} = x^2$ and the initial condition y(0)=4, find the value of y(2).

$A)\ -\frac{4}{3}$ $\quad B)\ \frac{4}{3}$ $\quad C)\ \frac{8}{3}$ $\quad D)\ 4$ $\quad E)\ \frac{20}{3}$

6. Given f(x) is a function that satisfies the differential equation $\frac{dy}{dx} = y^2 - 5y + 2$ which of the following statements are true about the function's slope field?

I. All slope segments on any given vertical line are the same
II. All slope segments on any given horizontal line are the same
III. All slope segments on the line y = x are the same.

A) I only *B) II only* *C) I and II only* *D) II and III only* *E) I and III only*

AP CALCULUS
UNIT VIII

NAME Key
DATE

MULTIPLE CHOICE PRACTICE

A CALCULATOR MAY NOT BE USED ON THE FOLLOWING QUESTIONS

1. A bird of prey spots its dinner on the ground and dives straight down toward the earth such that its downward velocity satisfies the differential equation $\frac{dv}{dt} = 2v - 12$, with initial condition v(0)=7. Which of the following is an expression for v in terms of t, where t is measured in seconds.

$\frac{dv}{v-6} = 2dt$

$\ln|v-6| = 2t + C$

(C) $A)\ v = e^{2t}$ $B)\ v = e^{t}$ $C)\ v = e^{2t} + 6$ $D)\ v = e^{t} + 6$ $E)\ v = e^{t} - 6$

2. If $\frac{dy}{dt} = ky$ and k is a positive even-numbered constant, then y could be

(D) $A)\ y = -2e^{3t}$ $B)\ y = -5e^{t}$ $C)\ y = 2e^{-2t}$ $D)\ y = 3e^{2t}$ $E)\ y = 4e^{5t}$

3. For the differential equation given by $\frac{dy}{dx} = \frac{xy}{3}$, the particular solution with the initial condition f(0)=2 is

(A) $A)\ y = 2e^{\frac{1}{6}x^2}$ $B)\ y = 2e^{\frac{1}{2}x^2}$ $C)\ y = 2e^{\frac{1}{3}x^2}$ $D)\ y = 3e^{\frac{1}{6}x^2}$ $E)\ y = 3e^{\frac{1}{2}x^2}$

4. Which of the following is a possible solution of the differential equation $\frac{dy}{dx} - 4xy = 0$?

$\frac{dy}{y} = 4x\,dx$

$\ln|y| = 2x^2 + C_1$

$y = \pm Ce^{2x^2}$

$I.\ y = -3e^{x^2}$ $II.\ 2e^{2x^2}$ $III.\ -4e^{2x^2}$ $IV.\ y = 5e^{2x}$

(D) A) *none* B) *II only* C) *I and IV only* D) *II and III only* E) *II and IV only*

5. Given the differential equation $\frac{dy}{dx} = x^2$ and the initial condition y(0)=4, find the value of y(2).

(E) $A)\ -\frac{4}{3}$ $B)\ \frac{4}{3}$ $C)\ \frac{8}{3}$ $D)\ 4$ $E)\ \frac{20}{3}$

6. Given f(x) is a function that satisfies the differential equation $\frac{dy}{dx} = y^2 - 5y + 2$ which of the following statements are true about the function's slope field?

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(B)

A) I only *B) II only* *C) I and II only* *D) II and III only* *E) I and III only*

1. Differentiate implicitly:

$$x^2y^3 = x + y$$

2. Solve for x:

$$\ln(x + 5) = 7$$

3. Solve for x:

$$1.75^x = 3.1$$

4. f(x) = ln (x-5)

   $f^{-1}(x) = ?$

5. $y = \cot(\frac{1}{x})$, $\frac{dy}{dx} = ?$

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$f^{-1}(x) = ?$

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ANSWERS

1.
$$x^2(3y^2y') + y^3(2x) = 1 + y'$$
$$y' = \frac{1-2xy^3}{3x^2y^2-1}$$

2.
$$e^7 = x + 5$$
$$x = e^7 - 5$$

3.
$$\ln 1.75^x = \ln 3.1$$
$$x \ln 1.75 = \ln 3.1$$
$$x = \frac{\ln 3.1}{\ln 1.75}$$

4.
$$e^y = e^{\ln(x-5)} = x - 5$$
$$e^y + 5 = x$$
$$f^{-1}(x) = e^x + 5$$

5.
$$\frac{dy}{dx} = -\csc^2(\frac{1}{x})(-\frac{1}{x^2})$$
$$= \frac{\csc^2(\frac{1}{x})}{x^2}$$

1. Sketch

$$y = e^x$$

2. Given: $x = e^{2t}$, $y = \sin t$

Find $\dfrac{dy}{dx}$ at $t = \dfrac{\pi}{3}$

3. Write the definition of a derivative of f(x) at x = a

4. Simplify:

$$2e^{\ln \frac{5}{7} t}$$

5. If $\dfrac{dh}{dt}$ is directly proportional to t, then $\dfrac{dh}{dt} = ?$

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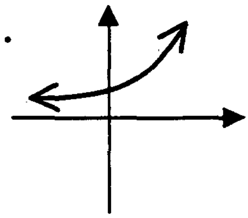
ANSWERS

1. 

2.

$$\frac{dx}{dt} = 2e^{2t}$$

$$\frac{dy}{dt} = \cos t$$

$$\frac{dy}{dx} = \frac{\frac{1}{2}}{2e^{\frac{2}{3}\pi}}$$

3.

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

4.

$$2(\tfrac{5}{7})t = \frac{10}{7}t$$

5. kt

1. $f(x) = \tan x + c$ is the __________ solution of a differential equation

2. $y = \tan x + 3$ is a(n) __________ solution of a differential equation

3. If $y = ce^{3x}$ and y = 8 when x = 0, then c =?

4. $\lim_{x \to .5} \dfrac{\arcsin x - \dfrac{\pi}{6}}{x - .5}$

*5. If $F(x) = \int_{2}^{\ln x} arc\cot t \, dt$, find F′(x).

	ANSWERS
1. $f(x) = \tan x + c$ is the ________ solution of a differential equation	1. General
2. $y = \tan x + 3$ is a(n) ________ solution of a differential equation	2. Particular
3. If $y = ce^{3x}$ and y = 8 when x = 0, then c =?	3. c =8
4. $\lim_{x\to .5} \dfrac{\arcsin x - \frac{\pi}{6}}{x - .5}$	4. $f(x) = \arcsin x$ $f'(x) = \dfrac{1}{\sqrt{1-x^2}}$ $f'(.5) = \dfrac{1}{\sqrt{1-\frac{3}{4}}} = \dfrac{2}{\sqrt{3}}$
*5. If $F(x) = \int_2^{\ln x} arc\cot t\, dt$, find F′(x).	5. $\dfrac{1}{x} arc\cot(\ln x)$

1. The height of a cylinder is twice its radius. The cylinder is sinking into mud at a rate of 3 mm/min. How fast is its volume disappearing when it is 4 mm into the mud?

2. $\int x^7 dx$

3. $\int 7^x dx$

4.

If $F(x) = \int f(x)dx,$

then $\int_3^7 f(x)dx =$

5. Find the slope of the line normal to the tangent at x=6 if $f(x) = x^2$.

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5. Find the slope of the line normal to the tangent at x=6 if f(x) = x^2.

ANSWERS

1. $V = \pi r^2 h,\ h = 2r$

$v = \pi \frac{h^3}{4}$

$\frac{dv}{dt} = \frac{3}{4}\pi h^2 \frac{dh}{dt}$

$\frac{dv}{dt} = \frac{3}{4}\pi(16)(3)$

$= 36\pi\ mm^3/\min$

2.

$\frac{x^8}{8} + c$

3.

$\frac{7^x}{\ln 7} + c$

4.

F (7)-F (3)

5.

$f'(x) = 2x$

$f'(6) = 12$

$m = -\frac{1}{12}$

1. If $\int_1^3 f(x) = 4$ and $\int_1^8 f(x) = 12$
then $\int_3^8 f(x) =$ ________

2.
$$xy + y^2 = 0,\ y \neq 0$$
$$y'(2,-2) = ?$$

3. If $\frac{dy}{dx} = k$, then y = ______

4. If $\frac{dy}{dx} = kx$, then y = ______

5. If $\frac{dy}{dx} = ky$, then y = ______

1. If $\int_1^3 f(x) = 4$ and $\int_1^8 f(x) = 12$ then $\int_3^8 f(x) =$ ________

2. $xy + y^2 = 0,\ y \neq 0$

$y'(2,-2) = ?$

3. If $\dfrac{dy}{dx} = k$, then y = ______

4. If $\dfrac{dy}{dx} = kx$, then y = ______

5. If $\dfrac{dy}{dx} = ky$, then y = _______

ANSWERS

1.

8

2.

$xy' + y(1) + 2yy' = 0$

$2y' - 2 - 4y' = 0$

$-2y' = 2$

$y' = -1$

3.

$dy = kdx$

$y = kx + c$

4.

$dy = kdx$

$y = \dfrac{kx^2}{2} + c$

5.

$y = Ce^{kx}$