UNIT VII

UNIT VII
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* UNIT VII: INFINITE SERIES

Expectation:	Students will create power series for elementary functions and explore the convergence or divergence of various series.

OVERVIEW: Power series are created using various techniques and convergence and divergence of series of constant terms are explored.

INDICATORS:

1. Construct a power series for functions recognized as a sum , $\frac{a}{1-r}$, of an infinite geometric series.

2. Construct MacLaurin series expansion for e^x, sin x, cos x, $\frac{1}{1-x}$.

3. Construct Taylor series expansion for functions centered at x = a.

4. Apply a power series expansion to approximate the value of a given function for a particular value of a variable.

5. Form new series from known series (use differentiation, integration, and other manipulations).

6. Determine Lagrange error bound for Taylor polynomials.

7. Demonstrate an understanding of the terms of a series as areas of rectangles and their relationship to improper integrals.

8. Determine whether a series of constant terms converges or diverges by using an appropriate test:
   a. n-th term
   b. geometric series
   c. p-series
   d. integral
   e. comparison
   f. ratio
   g. alternating series with error bound.

9. Apply the definition of absolute and conditional convergence to an alternating series.

10. Determine the radius and interval of convergence for a power series.

*BC Calculus indicators only

Unit VII: Infinite Series

Indicators/ Objectives	Foerster: Calculus Key Curriculum 1998	Foerster: Calculus: Instructor's Resource Book Key Curriculum 1998	Finney, et al: Calculus S F A W 1999	Guide Pages
1	599 - 603	12.3	457 - 461	VII 1-2
2	607; 616 - 617		471 - 477	
3	609 – 612; 615 - 616	12.4	462 – 465; 469 - 475	VII 4
4	617		473 – 475; 480	VII 4
5	610 – 612; 616 - 618		461 - 465	VII 1-2
6	643 - 646	12.8 a, b	482	VII 3-4
7	630-633	12-7a, 12-8b	496-497	
8	624; 630; 633 - 642	12.2; 12.6; 12.7 a, b	489 – 492; 496 -502; 508	
9	634 - 635		490 – 491; 502 - 505	
10	624 – 626; 636		457 – 465; 487; 493; 503 - 504	

Indicators/ Objectives	Finney, et al: Calculus 1994	Guide Pages
1		
2	763	
3	747 - 750	
4	747; 758 - 761	
5	737 - 741	
6	752 - 755	
7	708 - 710	
8	692 - 724	
9	724 - 729	
10	735 - 736	

AP CALCULUS
UNIT VII -- ACTIVITY I

NAME Key
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POWER SERIES

1. The power series representation of a function about a given center is unique, but the method by which such representation is obtained is not. For example, the power series representation for $f(x) = \frac{1}{1-x}$ about x=0 may be established by using any of the following methods:

(i). Recognize that $\frac{1}{1-x}$ is equivalent to $\frac{a}{1-r}$, the sum of an infinite geometric series with first term a and common ratio r. For $\frac{1}{1-x}$, $a = 1, r = x$, thus the geometric series, i.e. power series, representation of

$\frac{1}{1-x}$ *is* $1 + x + x^2 + x^3 + x^4 + \cdots; \quad |x| < 1.$

(ii). Use long division to produce the series;

$$\begin{array}{r} 1+x+\cdots \\ 1-x\,\big)\overline{1\qquad\qquad} \\ \underline{1-x} \\ x \\ \underline{x-x^2} \\ x^2\cdots \end{array}$$

(iii). Compute $f(0), f'(0), f''(0), \cdots$ *etc.* and substitute these values in the general form of the power series.

Exercises: In 1-3, use a method of your choice to write the power series expansion centered at 0 for the following functions.

1. $f(x) = \frac{1}{1-3x}$; 2. $f(x) = \frac{1}{1+2x}$; 3. $f(x) = \frac{10}{5x-1}$

4. Expand $\frac{1}{1+x}$ (a). in power of x,
(b). in power of (x-2).

$(Hint: \frac{1}{1+x} = \frac{1}{3+(x-2)}.$ *Let* $u = x - 2.)$

5. Use the results of 4(a) to write a series expansion for $\frac{1}{1+x^2}$.

AP CALCULUS
UNIT VII -- ACTIVITY I

NAME Key
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POWER SERIES

Exercises: In 1-3, use a method of your choice to write the power series expansion centered at 0 for the following functions. $n = 0, 1, 2, \ldots$

1. $f(x) = \dfrac{1}{1-3x}$ $a = 1,\ r = 3x$

$$f(x) = 1 + 3x + (3x)^2 + \cdots + (3x)^n + \cdots$$

2. $f(x) = \dfrac{1}{1+2x}$ $a = 1,\ r = -2x$

$$f(x) = 1 - 2x + 4x^2 - \cdots + (-2x)^n + \cdots$$

3. $f(x) = \dfrac{10}{5x-1} = \dfrac{-10}{1-5x}$ $a = -10,\ r = 5x,$

$$f(x) = -(10 + 50x + 250x^2 + \cdots 10(5x)^n + \cdots)$$

4. Expand $\dfrac{1}{1+x}$ (a). in power of x,

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots (-x)^n$$

(b). in power of (x-2).

$$\frac{1}{1+x} = \frac{1}{3+(x-2)} = \frac{1/3}{1+\frac{1}{3}(x-2)};\quad a = \tfrac{1}{3},\ r = -\tfrac{1}{3}(x-2)$$

so: $\dfrac{1}{1+x} = \dfrac{1}{3} - \dfrac{1}{9}(x-2) + \dfrac{1}{27}(x-2)^2 - \cdots + \left(-\dfrac{1}{3}(x-2)\right)^n + \cdots$

Use the results of 4(a) to write a series expansion for $\dfrac{1}{1+x^2}$.

$$1 - x^2 + x^4 - x^6 + \cdots + (-x^2)^n + \cdots$$

AP CALCULUS
UNIT VII – ACTIVITY 2

NAME______________
DATE ______________

POWER SERIES II

1. Write the first five non-zero terms and a general term of the power series expansion for $f(x)=\dfrac{1}{1-x}$. Use this series and calculus to represent the following functions as power series: $\dfrac{1}{1-x} = 1+x+x^2+x^3+\cdots+x^n+\cdots$

   a. $x\cdot f(x) \quad = x+x^2+x^3+\cdots+x^{n+1}+\cdots$

   b. $\dfrac{f(x)}{2x} \quad = \frac{1}{2}\left(\frac{1}{x}+1+x+x^2+\cdots+x^{n-1}+\cdots\right)$

   c. $f'(x) \quad = 1+2x+3x^2+\cdots+(n+1)x^n+\cdots$

   } using $n=0,1,\cdots$ –or simply: x^n, nx^{n-1}, respectively.

   d. $\int f(x)\,dx \quad = x+\frac{x^2}{2}+\frac{x^3}{3}+\cdots\frac{x^{n+1}}{n+1}+\cdots+C$

   $= C+x+\frac{x^2}{2}+\cdots+\frac{x^{n+1}}{n+1}+\cdots$

2. Find power series to represent the given functions. Show all your work. Whenever possible, include a general formula for the nth term.

   a. $\dfrac{x}{1-2x} = x\left(\frac{1}{1-2x}\right) = x\left(1+2x+4x^2+\cdots+(2x)^n+\cdots\right)$

   use: $a=1, r=2x$

   $= x+2x^2+4x^3+\cdots+2^nx^{n+1}+\cdots$

   b. $\dfrac{x}{1+x} = x\left(\frac{1}{1+x}\right) = x\left(1-x+x^2-\cdots+(-x)^n+\cdots\right)$

   $= x-x^2+x^3+\cdots(-1)^nx^{n+1}+\cdots$

   c. $\dfrac{x}{2+x} = \frac{x}{2}\left(\frac{1}{1+\frac{x}{2}}\right) = \frac{x}{2}\left(1-\frac{x}{2}+\frac{x^2}{4}+\cdots+\left(\frac{-x}{2}\right)^n+\cdots\right)$

   $a=1, r=-\frac{x}{2}$

   $= \frac{x}{2}-\frac{x^2}{2^2}+\frac{x^3}{2^3}+\cdots+(-1)^n\cdot\frac{x^{n+1}}{2^{n+1}}+\cdots$

3. Let $g(x)=\dfrac{1}{1+x}$. Use a series expansion for g(x) to find a series expansion for $h(x)=\int g(x)\,dx$. What "familiar" function is h?

$g(x) = 1-x+x^2-x^3+\cdots+(-x)^n+\cdots$

$\int g(x)\,dx = C+x-\frac{x^2}{2}+\frac{x^3}{3}+\cdots+(-1)^{n+1}\frac{x^n}{n}+\cdots$

$\int g(x)\,dx = \int\frac{1}{1+x}\,dx = \ln|x+1|+C$

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ERROR ANALYSIS

1. The following problems relate to $g(x)=\sqrt{x}$.

a) Write the first <u>three</u> terms of the Taylor series for $g(x)$ about $x = 16$.

$g(x) = x^{1/2}$, $g'(x) = \frac{1}{2}x^{-1/2}$, $g''(x) = -\frac{1}{4}x^{-3/2}$, $g'''(x) = \frac{3}{8}x^{-5/2}$

$g(x) \approx g(16) + g'(16)(x-16) + g''(16)\frac{(x-16)^2}{2!}$

$= 4 + \frac{x-16}{8} - \frac{(x-16)^2}{128}$

b) Use this series to approximate $\sqrt{19}$.

$\sqrt{19} \approx 4 + \frac{3}{8} - \frac{9}{128} \approx 4.305$ First unused term:

$g'''(16)\frac{(x-16)^3}{3!} = \frac{3}{8\cdot 4^5}\cdot\frac{3^3}{3!} \approx .00165$

c) Give an expression for the maximum value for the error of your approximation.

E ≤ |first unused term|

The series is alternating after the first term, so use the Alternating Series Remainder Estimation Theorem. (See (b))

d) Find a range of values for which the maximum error is less than .0003.

We need: $\left|\frac{3}{8\cdot 4^5}\frac{(x-16)^3}{3!}\right| < 3\times10^{-4}$

$|x-16| < 1.2997$

2. The function f has derivatives of all orders for all real numbers x. Assume that $f(1)=-2$, $f'(1)=6$, $f''(1)=-4$, and $f'''(1)=6$.

a) Write the third-degree Taylor polynomial for g about $x=1$.

$$T_3 = -2 + 6(x-1) - 2(x-1)^2 + (x-1)^3$$

b) Use your answer from part a) to approximate $g\,(0.5)$.

$$g(.5) \approx .625$$

c) If $\left|g^{(4)}(1)\right| \le 2$ over the closed interval $[0.5, 1]$, explain why $g(0.5) \ne -4$.

The error in using 3 terms to approximate $g(.5)$ is no greater than the fourth term.

The error is $\le \left|\dfrac{g^4(1)\ (.5)^4}{4!}\right| = .00521$

-4 is outside the interval $.625 \pm .00521$

UNIT VII
STUDENT WORKSHEETS

NAME________________
DATE ________________

POWER SERIES

Exercises: In 1-3, use a method of your choice to write the power series expansion centered at 0 for the following functions.

1. $f(x) = \dfrac{1}{1-3x}$

2. $f(x) = \dfrac{1}{1+2x}$

3. $f(x) = \dfrac{10}{5x-1}$

4. Expand $\dfrac{1}{1+x}$ (a). in power of x,

   (b). in power of (x-2).

   Use the results of 4(a) to write a series expansion for $\dfrac{1}{1+x^2}$.

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POWER SERIES II

1. Write the first five non-zero terms and a general term of the Maclaurin series expansion for $f(x) = \dfrac{1}{1-x}$. Use the results to find infinite series representation for each of the following functions.

   a. $x \cdot f(x)$

   b. $\dfrac{f(x)}{2x}$

   c. $f'(x)$

   d. $\int f(x)dx$

2. Find power series to represent the given functions. Show all your work. Whenever possible, include a general formula for the nth term.

   a. $\dfrac{x}{1-2x}$

   b. $\dfrac{x}{1+x}$

   c. $\dfrac{x}{2+x}$

3. Let $g(x) = \dfrac{1}{1+x}$. Use a series expansion for g(x) to find a series expansion for $h(x) = \int g(x)dx$. What "familiar" function is h?

NAME ____________________
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ERROR ANALYSIS

1. The following problems relate to $g(x) = \sqrt{x}$.

a) Write the first three terms of the Taylor series for $g(x)$ about $x = 16$.

b) Use this series to approximate $\sqrt{19}$.

c) Give an expression for the maximum value for the error of your approximation.

d) Find a range of values for which the maximum error is less than .0003.

2. The function f has derivatives of all orders for all real numbers x. Assume that $f(1) = -2$, $f'(1) = 6$, $f''(1) = -4$, and $f'''(1) = 6$.

a) Write the third-degree Taylor polynomial for g about $x = 1$.

b) Use your answer from part a) to approximate g (0.5).

c) If $\left|g^{(4)}(1)\right| \leq 2$ over the closed interval $[0.5, 1]$, explain why $g(0.5) \neq -4$.

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MULTIPLE CHOICE PRACTICE

A CALCULATOR MAY NOT BE USED ON THE FOLLOWING QUESTIONS

1. The Maclaurin series for sin x is $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots$ If f is a function such that $f'(x)=\sin(x^2)$, then the coefficient of x^9 in the Maclaurin series for f(x) is

$A)\ \frac{1}{9!}$ $B)\ \frac{1}{12}$ $C)\ \frac{10}{9}$ $D)\ \frac{1}{6}$ $E)\ \frac{2}{9!}$

2. Which of the following series converge?

I. $\sum_{n=1}^{\infty}\frac{n}{n+2}$ II. $\sum_{n=0}^{\infty}\frac{\cos(n\pi)}{n+1}$ III. $\sum_{n=1}^{\infty}\frac{1}{n}$

A) None *B) III only* *C) II only* *D) I and II only* *E) I and III only*

3. Which of the following series is absolutely convergent?

I. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}$ II. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$ III. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$

A) None *B) III only* *C) II only* *D) I and II only* *E) I and III only*

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS IN THIS SECTION

4. For what integer p, p > 1, will both $\sum_{n=1}^{\infty}\frac{(-1)^{pn}}{n}$ and $\sum_{n=1}^{\infty}(\frac{p}{4})^n$ converge?

$A)\ 6$ $B)\ 5$ $C)\ 4$ $D)\ 3$ $E)\ 2$

5. The Taylor series for lnx, centered at x = 1 is $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. If x is within .6 of 1, then what is the maximum error?

$A)\ 0$ $B)\ .005$ $C)\ .006$ $D)\ .032$ $E)\ .064$

6. What are the values of x for which the series $\sum_{n=1}^{\infty}\frac{(x+3)^n}{\sqrt{2n}}$ converges?

$A)\ -4\le x<-2$ $B)\ -1\le x<1$ $C)\ x<-2$

$D)\ -3\le x<3$ $E)\ x<3$

NAME ____________________
DATE ____________________

MULTIPLE CHOICE PRACTICE

A CALCULATOR MAY NOT BE USED ON THE FOLLOWING QUESTIONS

1. The Maclaurin series for sin x is $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots$ If f is a function such that $f'(x)=\sin(x^2)$, then the coefficient of x^9 in the Maclaurin series for f(x) is

A) $\frac{1}{9!}$ B) $\frac{1}{12}$ C) $\frac{10}{9}$ D) $\frac{1}{6}$ E) $\frac{2}{9!}$

(B) $f(x) = \sin x^2 = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \cdots$

$f'(x) = 2x - \frac{6x^5}{3!} + \frac{10x^9}{5!} - \cdots$

2. Which of the following series converge?

I. $\sum_{n=1}^{\infty}\frac{n}{n+2}$ II. $\sum_{n=0}^{\infty}\frac{\cos(n\pi)}{n+1}$ III. $\sum_{n=1}^{\infty}\frac{1}{n}$

(C) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \cdots$

A) None *B) III only* *(C) II only* *D) I and II only* *E) I and III only*

3. Which of the following series is absolutely convergent?

I. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}$ II. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$ III. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$

(E)

A) None *B) III only* *C) II only* *D) I and II only* *E) I and III only*

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS IN THIS SECTION

4. For what integer p, p > 1, will both $\sum_{n=1}^{\infty}\frac{(-1)^{pn}}{n}$ and $\sum_{n=1}^{\infty}(\frac{p}{4})^n$ converge?

(D) The first one converges when pn is odd; the second when $|p|<4$.

A) 6 *B)* 5 *C)* 4 *D)* 3 *E)* 2

5. The Taylor series for lnx, centered at x = 1 is $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. If x is within .6 of 1, then what is the maximum error?

(D)

A) 0 *B)* .005 *C)* .006 *D)* .032 *E)* .064

6. What are the values of x for which the series $\sum_{n=1}^{\infty}\frac{(x+3)^n}{\sqrt{2n}}$ converges?

(A)

A) $-4\le x<-2$ *B)* $-1\le x<1$ *C)* $x<-2$

D) $-3\le x<3$ *E)* $x<3$

AP CALCULUS
UNIT VII

NAME Key
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MULTIPLE CHOICE PRACTICE

A CALCULATOR MAY NOT BE USED ON THE FOLLOWING QUESTIONS

1. The Maclaurin series for sin x is $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots$ If f is a function such that $f'(x)=\sin(x^2)$, then the coefficient of x^9 in the Maclaurin series for f(x) is

(B)

A) $\frac{1}{9!}$ B) $\frac{1}{12}$ C) $\frac{10}{9}$ D) $\frac{1}{6}$ E) $\frac{2}{9!}$

2. Which of the following series converge?

I. $\sum_{n=1}^{\infty}\frac{n}{n+2}$ II. $\sum_{n=0}^{\infty}\frac{\cos(n\pi)}{n+1}$ III. $\sum_{n=1}^{\infty}\frac{1}{n}$

(C)

A) None B) III only C) II only D) I and II only E) I and III only

3. Which of the following series is absolutely convergent?

I. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}$ II. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$ III. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$

(E)

A) None B) III only C) II only D) I and II only E) I and III only

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS IN THIS SECTION

4. For what integer p, p > 1, will both $\sum_{n=1}^{\infty}\frac{(-1)^{pn}}{n}$ and $\sum_{n=1}^{\infty}(\frac{p}{4})^n$ converge?

(D)

A) 6 B) 5 C) 4 D) 3 E) 2

5. The Taylor series for lnx, centered at x = 1 is $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. If x is within .6 of 1, then what is the maximum error?

(D)

A) 0 B) .005 C) .006 D) .032 E) .064

6. What are the values of x for which the series $\sum_{n=1}^{\infty}\frac{(x+3)^n}{\sqrt{2n}}$ converges?

A) $-4\le x<-2$ B) $-1\le x<1$ C) $x<-2$

(A)

D) $-3\le x<3$ E) $x<3$

Evaluate

1. $\sum_{k=1}^{4} \frac{1}{x}$

2. A geometric series converge, if and only if

3. Find $\int 6x^2\, dx$ if it equals 21 when x = 2

4. Is $f(x) = x^3 - 4x$ increasing or decreasing when $x = 1$?

5. Write 0.2 + 0.02 + 0.002 + … as a ratio of 2 integers.

Evaluate

1. $\sum_{k=1}^{4} \frac{1}{x}$

2. A geometric series converge if and only if

3. Find $\int 6x^2\, dx$ if it equals 21 when $x = 2$

4. Is $f(x) = x^3 - 4x$ increasing or decreasing when $x = 1$?

5. Write 0.2 + 0.02 + 0.002 + … as a ratio of 2 integers.

1. $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

$\frac{12+6+4+3}{12} = \frac{25}{12}$

2. $|\text{common ratio}| < 1$

3. $\frac{6x^3}{3} + c = 21$

$2(2)^3 + c = 21$

$c = 5$

$2x^3 + 5$

4. $f'(x) = 3x^2 - 4$

$f'(x) = -1 \quad < 1$

decreasing

5. $s = \frac{a}{1-r}$

$= \frac{.2}{1-.1} = \frac{2}{9}$

1. Evaluate: 5!

2. Use long division to express $\frac{1}{1-x}$ as a sum of 4 terms

3. $\frac{d}{dx}\int_0^{\frac{1}{x}} \cos 5t\, dt$

4. The radius of a snowball is decreasing at a rate of 2 mm/min. At what rate is the snowball melting when its radius is 30 mm?

5. Sketch the graph of $y = \ln x$

AP CALCULUS
UNIT VII – WARMUP SET 2

1. Evaluate: 5!

2. Use long division to express $\dfrac{1}{1-x}$ as a sum of 4 terms

3. $\dfrac{d}{dx}\displaystyle\int_0^{\frac{1}{x}} \cos 5x \, dx$

4. The radius of a snowball is decreasing at a rate of 2 mm/min. At what rate is the snowball melting when its radius is 30 mm?

5. Sketch the graph of $y = \ln x$

1.
$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

2.
$$1+x+x^2+\frac{x^3}{1-x}$$
$$1-x \;\big)\overline{1}$$
$$\underline{1-x}$$
$$+x$$
$$\underline{x-x^2}$$
$$x^2$$
$$\underline{x^2-x^3}$$
$$x^3$$

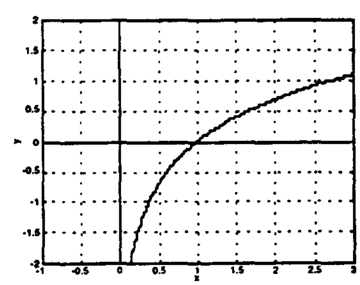
3.
$\cos\dfrac{5}{x}\left(-\dfrac{1}{x^2}\right)$

4. $V = \dfrac{4}{3}\pi r^3$

$\dfrac{dv}{dt} = 4\pi r^2 \dfrac{dr}{dt}$

$\dfrac{dv}{dt}\ 4\pi(900)(2)$

$\dfrac{dv}{dt} = 7200 \text{ mm}^3/\text{min}$

5.


1. Evaluate: 5!

2. Use long division to express $\frac{1}{1-x}$ as a sum of 4 terms

3. $\frac{d}{dx}\int_0^{\frac{1}{x}} \cos 5x \, dx$

4. The radius of a snowball is decreasing at a rate of 2 mm/min. At what rate is the snowball melting when its radius is 30 mm?

5. Sketch the graph of $y = \ln x$

1. If $h(x) = \int f(x)\,dx$, then $\int_a^b f(x)\,dx = h(b) - h(a)$ is a statement of ________.

2. $f(x) = \dfrac{\tan^{-1} x}{e^x}$

$f'(x) =$ ______

3. Simplify: $\dfrac{(m+1)!}{(m-1)!}$

4. $\lim_{x \to 0} (1+x)^{\frac{1}{2}}$

5. Evaluate: $\dfrac{4!\,6!}{5!}$

1.

The fundamental Theorem of Calculus

2.

$$\frac{e^x\left(\dfrac{1}{1+x^2}\right) - \tan^{-1} x \cdot e^x}{e^{2x}}$$

3.

$$\frac{(m+1)(m)(m-1)!}{(m-1)} = (m+1)(m)$$

4. 1

5.

$$\frac{4!(6)5!}{5!} = 4 \cdot 3 \cdot 2 \cdot 6 = 144$$

1. If $F(x) = \int_{2}^{x} t + 1\, dt$,

then $F'(8) =$

2. $\frac{d}{dx} \int_{a}^{x} g(t)\, dt = g(x)$ is a statement of ______________

3. $\sum_{m=0}^{\infty} x^n$ is a ____________ series

4. $\lim_{x \to \frac{\pi}{3}} \frac{\sin x - \frac{\sqrt{3}}{2}}{x - \frac{\pi}{3}}$

5. $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots =$

*

1. If $F(x)=\int_{2}^{x} t+1\,dt$, then $F'(8)=$

$F'(x)=\sqrt{x+1}$
$F'(8)=\sqrt{9}=3$

2. $\frac{d}{dx}\int_{a}^{x} g(t)\,dt = g(x)$ is a statement of ______________

The fundamental Theorem of Calculus

3. $\sum_{m=0}^{\infty} x^n$ is a ___________ series

Geometric

4. $\lim_{x\to\frac{\pi}{3}} \frac{\sin x-\frac{\sqrt{3}}{2}}{x-\frac{\pi}{3}}$

$\lim_{x\to\frac{\pi}{3}} \frac{\sin x-\sin\frac{\pi}{3}}{x-\frac{\pi}{3}}$
$f(x)=\sin x$
$f'(x)=\cos x$
$f'\left(\frac{\pi}{3}\right)=\cos\frac{\pi}{3}=\frac{1}{2}$

5. $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots=$
*

e^x

1. What is the sum of the series $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \ldots +$

2. $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$

3. If the interval of convergence
* is $-1 < x < 7$, then the radius of convergence is ________

4. $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots + =$
*

5. The first three terms of the
* Taylor series for cos 2x are ________.

1. What is the sum of the series $2+\frac{2}{3}+\frac{2}{9}+\frac{2}{27}+\ldots+$

$$s=\frac{A}{1-r}$$

$$s=\frac{2}{1-\frac{1}{3}}=\frac{2}{\frac{2}{3}}=3$$

2. $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

4

3. If the interval of convergence
* is $-1 < x < 7$, then the radius of convergence is ________

sin x

4. $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots+=$
*

e

5. The first three terms of the
* Taylor series for cos 2x are ________.

$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}$$

$$\cos 2x = 1-2x^2+\frac{(2x)}{4!}$$

$$=1-2x^2+\frac{2x^4}{3}$$

UNIT VII INTERNET RESOURCES

http://www.barzilai.org/archive, http://archives.math.utk.edu.visual.calculus/ and http://www.hofstra.edu/~matscw/RealWorld/index.html

Sites feature some tutorials, but mostly have good drill and quiz resources for either in-class practice or at-home practice.

http://www.math.odu.edu/cbii/calcanim/

Site has a nice animated feature showing the convergence or divergence of infinite series.

http://www.math.odu/edu/~bogacki/citat/

Site has a good tutorial on infinite series which gives the student problems on convergence / divergence of series and then follows up with explanations of the solutions.