UNIT II

UNIT II
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UNIT II: DERIVATIVES OF FUNCTIONS

Expectation:	The student will demonstrate an understanding of the concept of the derivative and compute derivatives of basic functions.

OVERVIEW:

The concept of the derivative should be approached graphically, numerically and analytically. Computation of the derivative of basic functions should include the chain rule and implicit differentiation.

INDICATORS:

1. Demonstrate an understanding of the geometrical, numerical and analytical concept of the derivative.

2. Interpret the derivative as an instantaneous rate of change.

3. Demonstrate an understanding of the derivative defined as the limit of a difference quotient including:
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \text{ and } \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

1. Demonstrate an understanding of the relationship between differentiability and continuity.

5. Demonstrate an understanding of the derivative as a slope of a curve at a point, including points at which there are vertical tangents and horizontal tangents.

6. Demonstrate knowledge of derivatives of basic functions, including polynomial, power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.

7. Demonstrate an understanding of the basic rules for the derivatives of sums, products and quotients of functions.

8. Determine f' and f'' using the chain rule.

9. Determine f' and f'' using implicit differentiation.

* 10. Determine f' and f'' of a function in parametric form.

* BC Calculus indicator only.

Indicators/ Objectives	Foerster: Calculus Key Curriculum 1998	Foerster: Calculus: Instructor's Resource Book Key Curriculum 1998	Finney, et al: Calculus S F A W 1999	Guide Pages
1	84-90	3.3	97-103	II 1 – 5, 6 - 9
2	3-13	1.1; 1.2	82-90	II 1 - 4
3	80-84	3.2; 3.4	95-97	II - 15
4	153-160	4.6	105-112	
5				
6	90-98; 105-106; 141-145; 145-153; 253-260; 272-284	3.6; 4.5	113; 118; 120; 134-141;157-163; 163-170	II – 10 II - 12
7	131-141	4.2; 4.3	112-121	II - 11
8	107-110	3.7	141-149	II - 11
11	168-172	4.8	149-157	II 13 - 14
10	160 – 167	4.7	144 – 145; 147	
Review				II - Review

Indicators/ Objectives	Finney, et al: Calculus 1994	Guide Pages
1-2	167 - 170; 179 - 183	
3	172, 175	
4	175	
5		
6	185 - 194, 214 - 218; 228 - 237; 536, 519 - 527	
7	185 - 1942221 - 226; 228 - 234	
8	221 - 226	
9	194, 221 - 226; 228 - 234	
10	801 - 803	

AP CALCULUS
UNIT II – ACTIVITY 1

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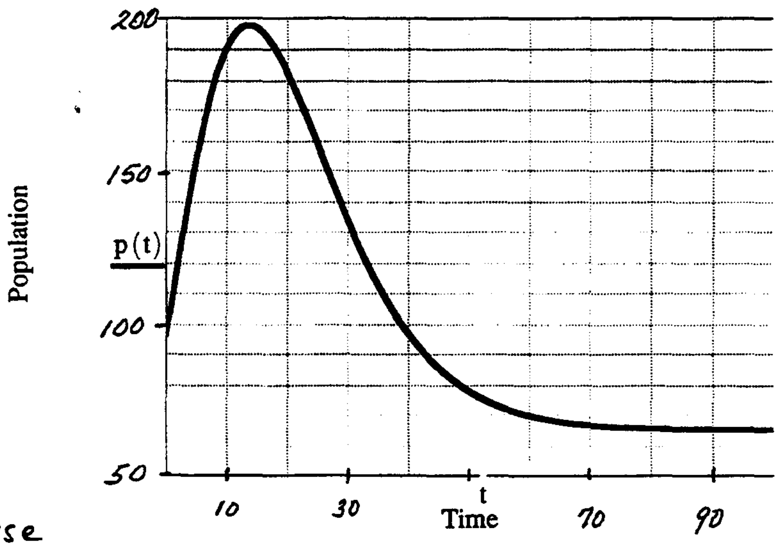
AVERAGE AND INSTANTANEOUS RATES OF CHANGE

1. The graph below shows the population of a small woodland mammal over a period of 100 days. The population increases from birth and then decreases as the mammals leave the protection of the nest.



Answers to these questions will vary

a) What is the average rate of growth during the first 10 days? During the first 30 days?

≈ 10 mammals/day
≈ 1 mammal /day

b) What is the rate of growth of the population at the 10th day and at the 30th day?

≈ 3 m/d
≈ −4 m/d

c) Starting with $t = 0$, when is the average rate of growth 0 animals/day? What is the significance of this?

About days 38 – 39. No. of births and deaths have balanced out

d) When is the instantaneous rate of growth 0 animals/day?

Around 14 and 100

e) Over what period of time is the rate of change in population increasing? The rate of change in population decreasing?

Rate of growth = $p'(t)$

Increasing when $p''(t) > 0$, $(30, 100)$

Decreasing when $p''(t) < 0$, $(0, 30)$

2. Given the following data for a function g.

x	2.4	2.6	2.8	3.0	3.2	3.4
g(x)	1.65	1.70	1.82	2.00	2.30	2.7

a) Find the average rate of change from x = 2.6 to x = 3.2.

$$\frac{\Delta g}{\Delta x} = \frac{2.30 - 1.70}{3.2 - 2.6} = 1.0$$

b) Approximate $g'(3)$.

$$g'(3) \approx \frac{g(3.2) - g(2.8)}{3.2 - 2.8} = \frac{2.30 - 1.82}{3.2 - 2.8} = 1.2$$

c) Find an equation that approximates the tangent line to g at the point (3, 2).

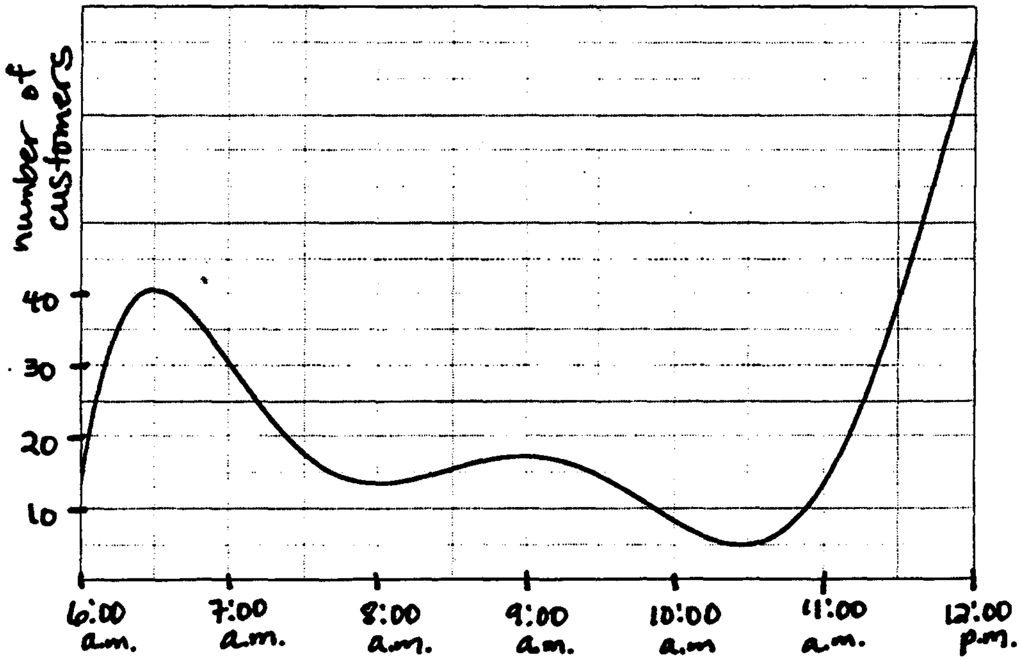
$y - 2 = 1.2(x - 3)$ or $y = 1.2x - 1.6$

d) Use the tangent line found in c) to estimate $g(3.1)$ and $g(2.9)$.

$g(2.9) \approx 1.88$

$g(3.1) \approx 2.12$

3. The graph below shows how the number of customers at Burger Queen varied over a recent Monday morning.



a) For each time interval below, find the change in the number of customers AND the average rate of change in the number of customers. Be sure to indicate the units of your answers.

Answers will vary

1) 6:00 am to 12:00 pm   ≈ 10 c/h
2) 7:30 am to 10:30 am   ≈ −4 c/h

b) Estimate the instantaneous rate of change of the number of customers at each of the following times. Be sure to include units.

1) 11:00 am   ≈ 30 c/h
2) 8:00 am   0
3) 7:30 am   ≈ −15 c/h

4. Your mom needs someone to give the dog a bath and someone to taste her brownies. Since you and your little brother both want to do the same chore, you decide the fair thing to do would be to flip a coin; loser has to taste the brownies. When the coin is flipped, the height of the coin, measured in feet, is modeled by $g(t) = -t^2 + 4t + 2$, where t is in seconds.

a) Find the change in the height of the coin between times t = 1 and t = 2 seconds.

1 ft

b) Find the average rate of change of the height of the coin between t = 1 and t = 2 seconds.

1 ft/sec

c) Find the average rate of change of the height of the coin from t = 1 and t = 1 + h seconds.

(2 - h) ft/sec

d) Find the instantaneous rate of change of the coin at time t = 1.

2 ft/sec

e) IF you were to graph g(t), what graphical significance would your answers to part b and d have? (You do not need to draw the graph.)

b: the slope of the secant line between (1, g(1)), (2, g(2))

d: the slope of the tangent line at t = 1.

AP CALCULUS
UNIT II ACTIVITY 2

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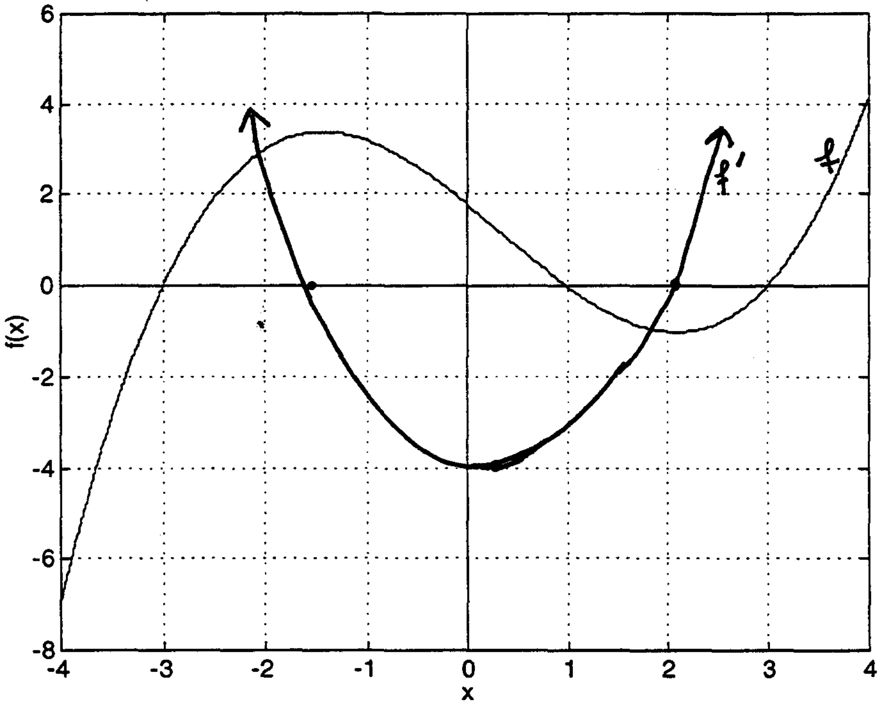
THE DERIVATIVE OF A FUNCTION



1. The graph of *f* is shown above.

   a. Use the graph to estimate $f'(-2)$ and $f'(1)$.

   $f'(-2) \approx 1.4$ $f'(1) \approx -1.6$

   Answers will vary

   b. On what interval is $f'(x)$ positive? Negative?

   pos. $[-4, -1.5)$ and $(2, 4]$
   neg. $(-1.5, 2)$

   c. Where does the graph of *f* have horizontal tangents?

   $x = -1.5$, $x = 2$

   d. Where does f' have a minimum value? Explain.

   $x \approx \frac{1}{4}$ f' decreases for $x < \frac{1}{4}$
   f' increases for $x > \frac{1}{4}$

   e. Sketch the graph of f' on the grid above.

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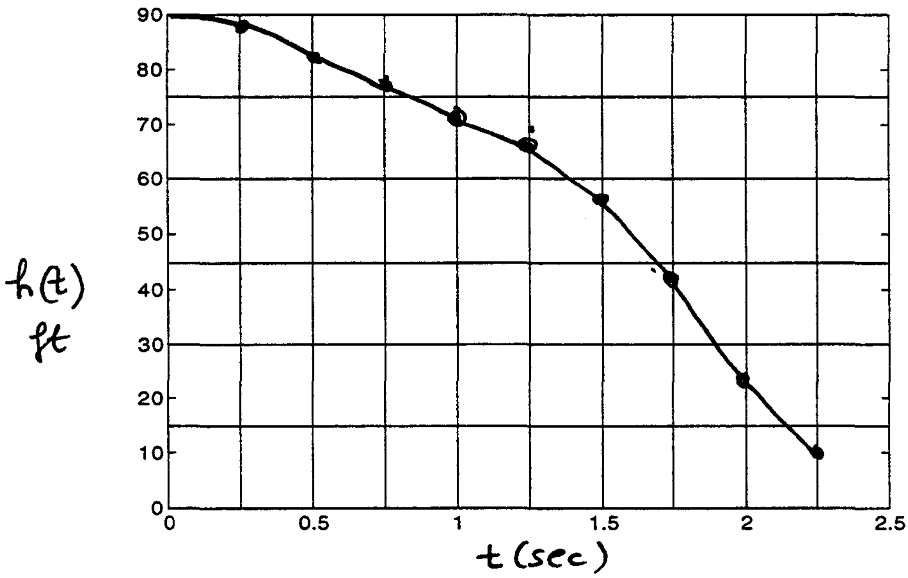
CONCEPTUALIZING THE DERIVATIVE THROUGH GRAPHICAL APPROACHES

The following table of values shows the height (in ft.) of a free-falling object after *t* seconds.

t	0	.25	.5	.75	1	1.25	1.5	1.75	2	2.25
h(t)	90	89	86	81	74	65	54	41	26	9

a). Plot these values on the grid below:



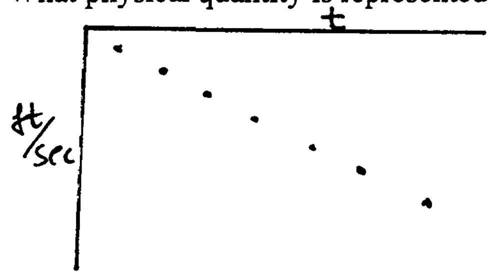
b). Connect the points with line segments.

c). Fill in the table below with the slope of the line segment at the given value of t.

t(sec)	.125	.375	.625	.875	1.125	1.375	1.625	1.875	2.125
m(ft/sec)	-4	-12	-20	-28	-36	-44	-52	-60	-68

d). What physical quantity do these slopes represent? The average rate of change in height at .25 second intervals.

e). Graph the ordered pairs from the table in part c) on your calculator. Sketch what you see. What physical quantity is represented by the slope of this graph?



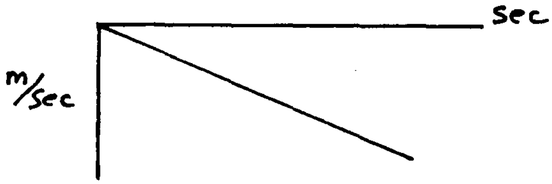
The rate of change of the slope

2. Suppose the equation $h(t) = -16t^2 + 90$ models the height of the object after t seconds.

   a). Use the definition of a derivative to find $h'(t)$.

   $$h'(t) = \lim_{h \to 0} \frac{[-16(t+h)^2 + 90] - [-16t^2 + 90]}{h} = -32t$$

   b). Sketch a graph of $h'(t)$ on the interval [0, 2.25].



   c) How does this graph compare with the graph in question 1, part c?

   The graphs look the same.

3. Since the velocity function (derivative) was found to be v(t) = -32t, we can now investigate the rate of change of the velocity. The following is the table of values for the velocity at various times.

T	0	.25	.5	.75	1	1.25	1.5	1.75	2	2.25
v(t)	0	-8	-16	-24	-32	-40	-48	-56	-64	-72

   a). What is the common term for rate of change of velocity?

   Acceleration

   b). What pattern exists in the change of velocity?

   The velocity decreases 8 units every ¼ sec

   c). Write an equation for the rate of change of velocity.

   $v'(t) = -32$ or $a(t) = -32$ ft/sec^2

   d). Is this equation the derivative of the velocity function? Explain.

   Yes, it is the instantaneous rate of the velocity.

4. What generalization can be made regarding the relationship between the three physical quantities discussed in this investigation (position, velocity, and acceleration)?

   $s(t) = f(t)$

   $v(t) = f'(t)$

   $a(t) = v'(t)$

AP CALCULUS
UNIT II – ACTIVITY 4

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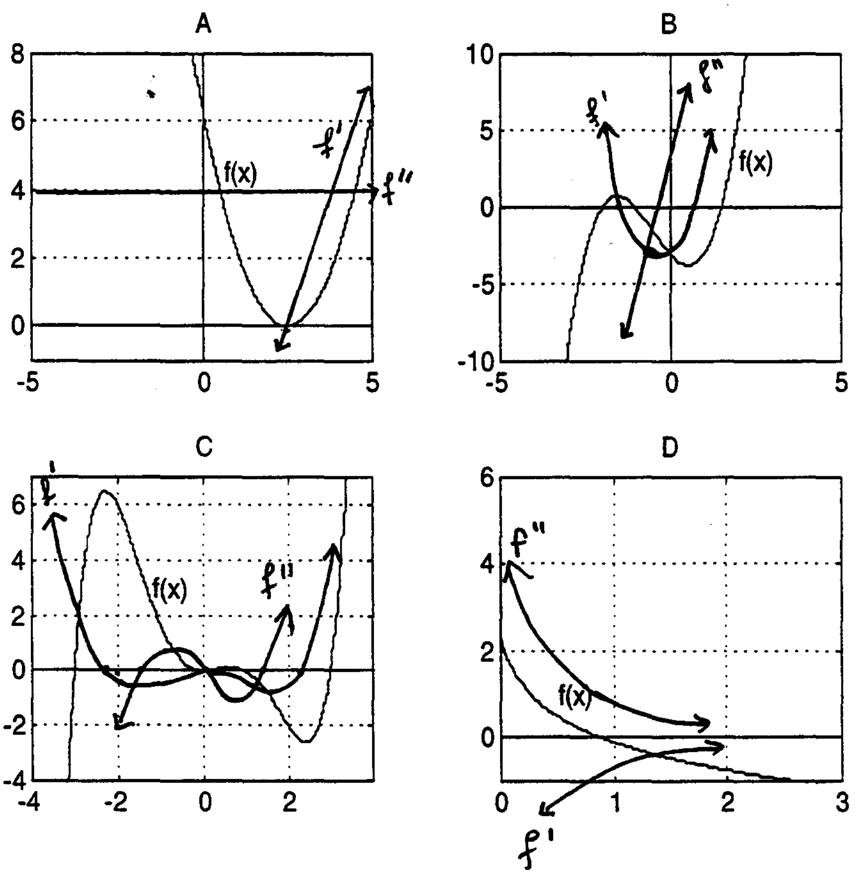
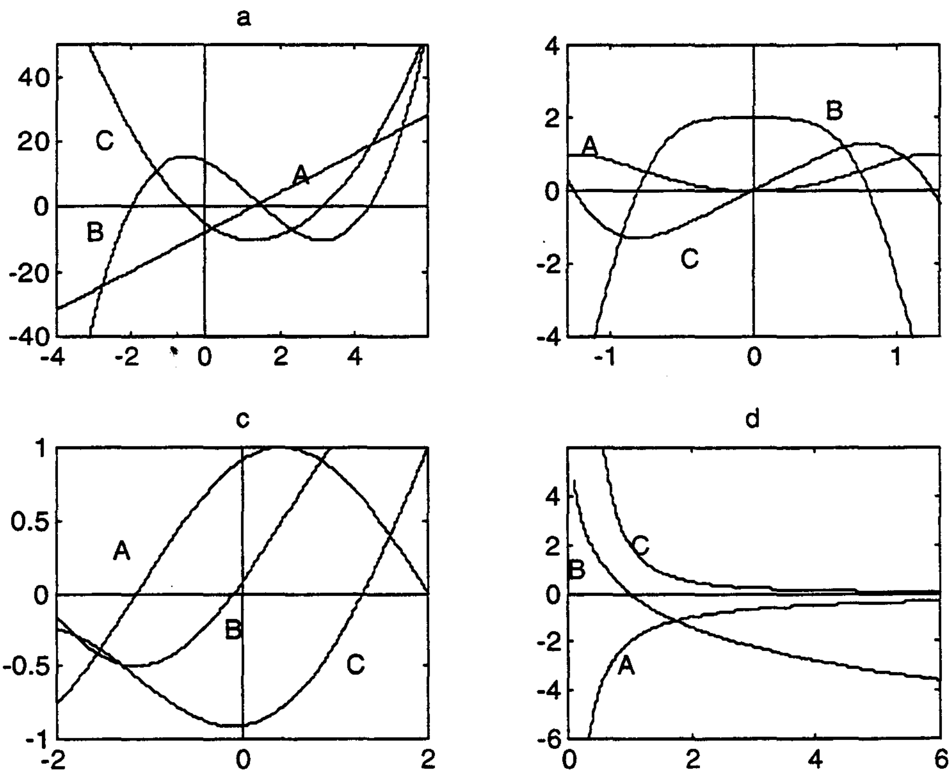
ANALYZING GRAPHS OF FUNCTIONS AND DERIVATIVES

1. Sketch the graphs of the first and second derivatives of the functions shown below.



2. Each of the figures below shows the sketches of f, f', f''. Identify each curve and justify your answer.



(a) B : f

C : f'

A : f''

(b) A : f

C : f'

B : f''

(c) C : f

B : f'

A : f''

(d) B : f

A : f'

C : f''

AP CALCULUS
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DERIVATIVE PRACTICE I

For problems 1-10, find the derivative of the given function. Use your own paper, and show your work. Remember that algebra and/or the use of a trigonometric identity can simplify some problems before you take the derivative.

1) $y=\frac{1}{2}x^4-5\sqrt{x}+\frac{3}{x^3}$ $y' = 2x^3 - \frac{5}{2\sqrt{x}} - \frac{9}{x^4}$

2) $y=\cos(3-5x)$ $y' = 5\sin(3-5x)$

3) $y=3\sec\frac{x}{2}$ $y' = \frac{3}{2}\sec\frac{x}{2}\tan\frac{x}{2}$

4) $f(x)=\frac{5x}{x^2-4}$ $y' = \frac{-5(x^2+4)}{(x^2-4)^2}$

5) $g(t)=\frac{3t^5-t^2+6}{t}$ $g'(t) = 12t^3 - 1 - \frac{6}{t^2}$

6) $y=\left(\frac{x^2-3}{5}\right)^3$ $y' = \frac{6x}{5}\left(\frac{x^2-3}{5}\right)^2$

7) $h(x)=\tan^4(\pi x)$ $h'(x) = 4\pi\tan^3(\pi x)\sec^2(\pi x)$

8) $p(\Theta)=\frac{1}{2\sin\Theta\cos\Theta}$ $p'(\Theta) = -2\csc\Theta\cot\Theta$ (since $\frac{1}{2\sin\Theta\cos\Theta} = \csc 2\Theta$)

9) $v(t)=\sqrt[3]{2t^4-1}$ $v'(t) = \frac{8t^3}{3}(2t^4-1)^{-2/3}$

10) $y=\sqrt{\cos^2x+3+\sin^2x}$ $y' = 0$ (since $y = \sqrt{4} = 2$)

11) Find $\frac{d^2y}{dx^2}$ if $y=\frac{4-x}{4+x}$ $y' = \frac{-8}{(x+4)^2}$, $y'' = \frac{16}{(x+4)^3}$

12) The table below gives the values of two functions and their derivatives at x = 1 and x = 4.

x	f(x)	g(x)	f'(x)	g'(x)
1	2	4	π	-1
4	6	2/3	-5	3

Use the information in the table to find $h'(x)$ at the given value of x.

a) $h(x) = f(x) - 2g(x)$ at $x = 1$

$h'(x) = f'(x) - 2g'(x)$; $h'(1) = \pi + 2$

b) $h(x) = f(x) \bullet g(x)$ at $x = 4$

$h'(x) = f(x) \cdot g'(x) + f'(x) \cdot g(x)$; $h'(4) = \frac{44}{3}$

c) $h(x) = \frac{f(x)}{g(x)}$ at $x = 4$

$h'(x) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$; $h'(4) = -48$

d) $h(x) = f(g(x))$ at $x = 1$

$h'(x) = f'(g(x)) \cdot g'(x)$; $h'(1) = 5$

e) $h(x) = \frac{1}{g(x)}$ at $x = 4$

$h'(x) = -\frac{g'(x)}{[g(x)]^2}$; $h'(4) = -\frac{27}{4}$

f) $h(x) = \sqrt{f(x) - g^2(x)}$ at $x = 4$

$h'(x) = \frac{1}{2}\left(f(x) - g^2(x)\right)^{-1/2}\left[f'(x) - 2g(x)g'(x)\right]$;

$h'(4) = \frac{-27\sqrt{2}}{20}$

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DERIVATIVE PRACTICE II

Find the derivative of each of the following functions, where *a, b,* and *k* are constants. Use your own paper, and show your work. If possible, simplify the expression before taking the derivative.

1) $f(x) = 2^{-x}$ $\quad f'(x) = -2^{-x}\ln 2$

2) $p(t) = e^{\sin 3t}$ $\quad p'(t) = 3\cos 3t\, e^{\sin t}$

3) $y = \cos(\arctan \pi x)$ $\quad y' = \dfrac{-\pi \sin(\arctan \pi x)}{1 + \pi^2 x^2}$

4) $f(x) = e^{3x}(x^2 + 7^x)$ $\quad f'(x) = e^{3x}(2x + 7^x \ln 7) + 3e^{3x}(x^2 + 7^x)$

5) $y = \cos^{-1}(5x^2)$ $\quad y' = \dfrac{-10x}{\sqrt{1 - 25x^4}}$

6) $h(x) = \ln(\sec(x^3))$ $\quad h'(x) = 3x^2 \cdot \tan(x^3)$

7) $g(x) = e^{\ln(ax)}$ $= ax;\quad g'(x) = a$

8) $f(x) = \sin^{-1}(\sin 5x)$ $\quad f'(x) = 5$ (Actually, ± 5, depending on $5x$)

9) $y = \ln\left(\dfrac{e^{kx}}{b}\right)$ $= kx - \ln b;\quad y' = k.$

10) $f(x) = \ln\sqrt{x^3 - 4}$ $= \frac{1}{2}\ln(x^3 - 4);\quad f'(x) = \dfrac{3x^2}{2(x^3 - 4)}$

11) $f(x) = \log_5(\tan x)$ $\quad f'(x) = \dfrac{1}{(\ln 5)(\sin x \cos x)}$

12) $y = \dfrac{1}{8}\log_2 x^4$ $\quad y' = \dfrac{1}{x \ln 4}$ (provided $x > 0$; $\ln x^4 = 4 \ln|x|$)

13) $p(t) = \tan^{-1}\left(\dfrac{3}{t}\right)$ $\quad p'(t) = \dfrac{-3}{t^2 + 9}$

14) $y = x^{\ln x}$ $\quad y' = \dfrac{2 \cdot x^{\ln x} \cdot \ln x}{x}$

15) $y = \dfrac{e^t - e^{-t}}{e^t + e^{-t}}$ $\quad y' = \dfrac{4}{(e^t + e^{-t})^2}$

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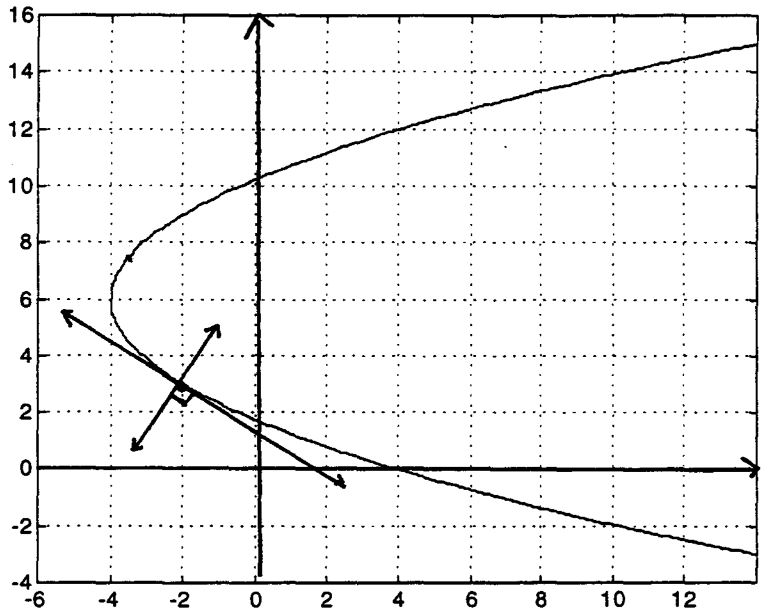
GRAPHICAL INTERPRETATION OF IMPLICIT DIFFERENTIATION

1. The graph of $9x - 2y^2 + 24y = 36$ is shown below.



a) Find an expression for $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{9}{4y-24}$$

b) Find an equation of the tangent to the curve at (-2,3). Sketch the tangent line on the graph above.

$$y - 3 = -\frac{3}{4}(x+2) \quad \text{or} \quad y = -\frac{3}{4}x + \frac{3}{2}$$

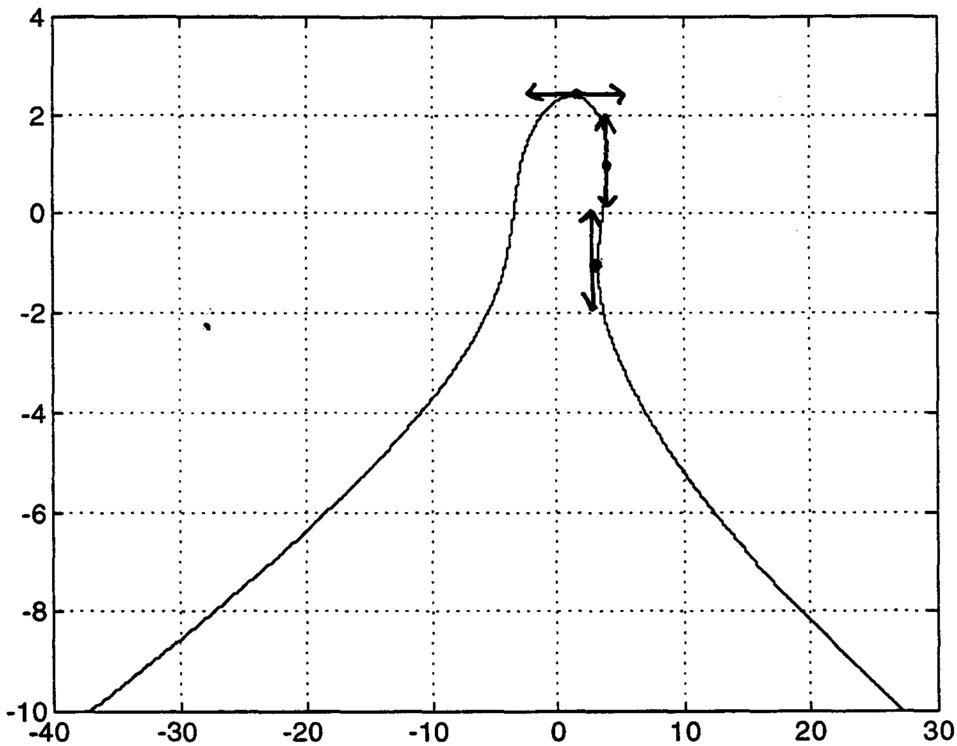
c) Find an equation of the line normal to the curve at the point (-2,3) and sketch it on the graph.

$$y - 3 = \frac{4}{3}(x+2) \quad \text{or} \quad y = \frac{4}{3}x + \frac{17}{3}$$

d) Are there any points on the graph where the tangent is horizontal? Vertical? Justify your answer.

No horizontal tangent, since $\frac{dy}{dx} \neq 0$ $(9 \neq 0)$
Vertical tangent when $4y - 24 = 0$, or $y = 6$, and $x = -4$. $\therefore$ the point is $(-4,6)$.

2. Consider the implicit relation $x^2 - xy + y^3 = 12$ graphed below.



a) Find a general equation for $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{y-2x}{3y^2-x}$$

b) Find an equation of the horizontal tangent(s) of the curve. Horizontal tangent when $y - 2x = 0$ and $3y^2 - x \neq 0$.

$y = 2x$: $x^2 - 2x^2 + 8x^3 = 12$ (Calculator): $x = 1.1879$

$y \approx 3.76$

c) Find an equation of the vertical tangent(s) of the curve. Tangents are vertical when $3y^2 - x = 0$ and $y - 2x \neq 0$.

$x = 3y^2$: $9y^4 - 3y^3 + y^3 = 12$ (Calculator): $y = -1.02304$ or $y = 1.1347451$

$x \approx 3.140$

$x \approx 3.863$

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EXTENSIONS OF IMPLICIT DIFFERENTIATION

I. For the following relations, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

1. $y^2 - 3xy + 2x = 12$

$2yy' - 3xy' - 3y + 2x = 0 \Rightarrow y' = \frac{3y-2}{2y-3x}$

$y'' = \frac{(2y-3x)3y' - (3y-2)(2y'-3)}{(2y-3x)^2} =$

$= \frac{(2y-3x)\cdot 3\frac{3y-2}{2y-3x} - (3y-2)\left(2\cdot\frac{3y-2}{2y-3x} - 3\right)}{(2y-3x)^2}$

$= \frac{(2y-3x)(9y-6) - (3y-2)(9x-4)}{(2y-3x)^3}$

2. $\cos y - 3x + 2 = 0$

$-\sin y \cdot y' - 3 = 0, \quad y' = -\frac{3}{\sin y} = -3\csc y$

$y'' = 3\csc y \cot y \cdot y' = -9\csc^2 y \cot y.$

II. For the following relations, evaluate $\frac{d^2y}{dx^2}$ for the given value of x. Also find equations for the lines tangent and normal to the curve at that point.

1. $e^{2y} - 4x^2 - 3y = 0$; *evaluate for* $x = \frac{1}{2}$

$2e^{2y} \cdot y' - 8x - 3y' = 0$, $\quad y' = \frac{8x}{2e^{2y} - 3}$; when $x = \frac{1}{2}$, $y = 0$

$y'(\frac{1}{2}, 0) = -4$

T: $y = -4(x - \frac{1}{2})$ $\quad$ N: $y = \frac{1}{4}(x - \frac{1}{2})$

$$y'' = \frac{(2e^{2y} - 3)8 - 8x(4e^{2y} \cdot y')}{(2e^{2y} - 3)^2} \Rightarrow y'' = \frac{(2-3)8 - 4(4(-4))}{1^2}$$

$$= 56$$

2. $3x^2y - 6y + 9 = 0$; *evaluate for* $x = 2$.

$6xy + 3x^2y' - 6y' = 0$, $\quad y' = \frac{-6xy}{-6 + 3x^2} = \frac{6xy}{3x^2 - 6}$

When $x = 2$, $y = -\frac{3}{2}$ $\quad y'(2, -\frac{3}{2}) = 3$

T: $y + \frac{3}{2} = 3(x - 2)$ $\quad$ N: $y + \frac{3}{2} = -\frac{1}{3}(x - 2)$

$$y'' = \frac{(3x^2 - 6)(6y + 6xy') - 6xy(6x)}{(3x^2 - 6)^2}; \quad y'' = 10.5$$

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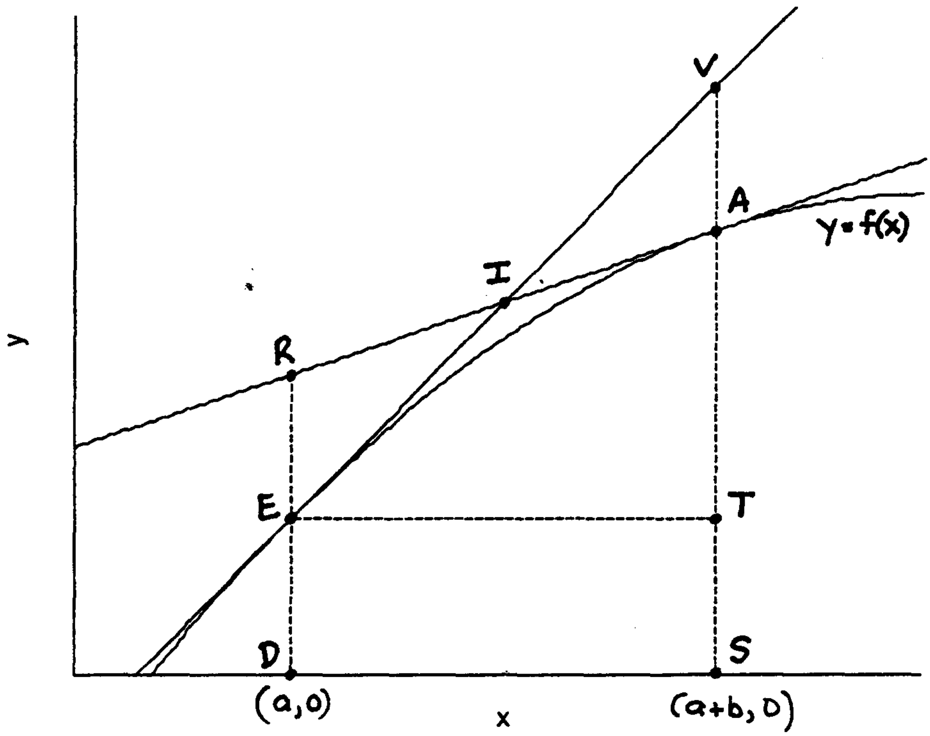
GEOMETRIC REPRESENTATION OF DERIVATIVES

Each quantity below can be represented in the diagram. Use the letters in the diagram to represent each quantity, and tell if it is a length, a slope or an area.



Note: Point D has coordinates $(a, 0)$ and point S has coordinates $(a+b, 0)$.

1. $f(a+b)$ = AS or f(a+b)

1. $f'(a+b)$ slope of AR or $\frac{AT - RE}{ET}$

3. $f'(a)b$ Since $f'(a) = \frac{TV}{ET} = \frac{TV}{(a+b)-a} = \frac{TV}{b}$, $f'(a) \cdot b = TV$

4. $f(a)b$ area of rectangle DETS

5. $f(a+b) - f(a)$ = AT

6. $\frac{f(a+b) - f(a)}{b}$ Slope of AE or $\frac{AT}{ET}$

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RULES OF DIFFERENTIATION

I. Find f' in terms of g'.
(a is constant)

1. $f(x)=g[x+g(a)]$

   $g'(x+g(a))$

2. $f(x)=g[x\bullet g(a)]$

   $g'(x\cdot g(a))\cdot g(a)$

3. $f(x)=g[x+g(x)]$

   $g'(x+g(x))(1+g'(x))$

4. $f(x)=g(x)\bullet(x-a)$

   $g'(x)(x-a)+g(x)$

5. $f(x)=g^2(ax)$

   $2g(ax)\cdot g'(ax)\cdot a$

6. $f(x)=g(a)\bullet(x-a)$

   $g(a)$

7. $f(x)=x\bullet g[x\bullet g(a)]$

   $g(x\cdot g(a))+x\cdot g'(x\cdot g(a))\cdot g(a)$

8. $f(x+3)=g(x^2)$

   $f'(x)=f'(x+3)=2x\,g'(x^2)$

II. Let $f(x)=x^3$. Find each of the following: $f'(x)=3x^2$

1. $f'(9)$, $f'(25)$ 243, 1875

2. $f'(3^2)$, $f'(5^2)$ " "

3. $f'(a^2)$, $f'(x^2)$ $3a^4$, $3x^4$

4. Let $g(x)=f(x^2)$. Find $f'(x)$, $g'(x)$, $f'(x^2)$, $g'(x^2)$

   $g'(x)=2x\,f'(x^2)$

   Since $f'(x)=3x^2$,
   $g'(x)=2x\,f'(x^2)=6x^5$
   $f'(x^2)=3x^4$
   $g'(x^2)=6x^{10}$

AP CALCULUS
UNIT II – ACTIVITY 11

NAME Key
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LIMITS AND THE DEFINTION OF DERIVATIVE

I. Evaluate each limit.

1. $\lim_{h \to 0} \frac{8(x+h)-8x}{h}$ 8   ($f(x)=8x$, $f'(x)=8$)

2. $\lim_{h \to 0} \frac{[4(x+h)^3+7(x+h)]-[4x^3+7x]}{h}$ $12x^2+7$ ($f(x)=4x^3+7x$, $f'(x)=12x^2+7$)

3. $\lim_{\Delta x \to 0} \frac{3|x+\Delta x|-3|x|}{\Delta x}$ *at* $x=5$ 3

4. $\lim_{x \to 6} \frac{f(x)-f(6)}{x-6}$ *if* $f(x)=-2x^2+x$ $-4x+1$ at $x=6$: -23

II. Each limit expression represents $f'(c)$. Identify f and c, and find the limit.

1. $\lim_{h \to 0} \frac{3(2+h)^4-48}{h} = 96$

$f(x)=3x^4$; $c=2$

2. $\lim_{x \to 7} \frac{\sqrt{2x-5}-3}{x-7} = 3$

$f(x)=\sqrt{2x-5}$, $c=7$

3. $\lim_{n \to \frac{-\pi}{2}} \frac{\sin n+1}{n+\frac{\pi}{2}} = 0$

$f(x)=\sin x$, $c=-\frac{\pi}{2}$

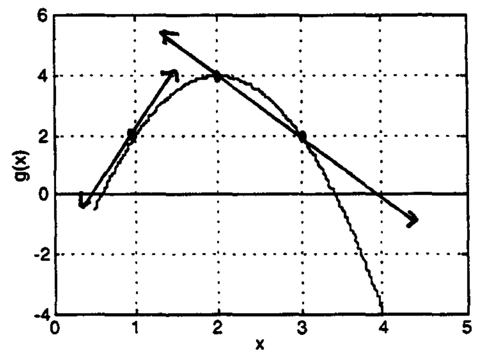
4. $\lim_{h \to 0} \frac{3e^{-h}-3}{h} = -3$

$f(x)=3e^{-x}$, $c=0$

III. On the graph of g, draw lines with the given slopes. How is each line related to g ?

1. $\frac{g(3)-g(2)}{3-2}$ slope of secant line

2. $\lim_{h \to 0} \frac{g(1+h)-g(1)}{h}$ slope of tangent at $x=1$



AP CALCULUS

NAME Key

DATE

DERIVATIVES REVIEW

For problems 1-15, find the derivative of the given function, where *a*, *b* and *k* are constants. Use your own paper, and show your work. Remember that sometimes you can simplify the expression before you take the derivative.

1) $f(x)=\dfrac{5}{(b^2-x^2)^2}$ $\dfrac{20x}{(b^2-x^2)^3}$

2) $y=xe^{\tan x}$ $e^{\tan x}(x\sec^2 x+1)$

3) $g(x)=\arctan(3x^2+1)$ $\dfrac{6x}{9x^4+6x^2+2}$

4) $p(t)=a^{t^3-t}$ $a^{t^3-t}(3t^2-1)\ln a$

5) $y=\dfrac{\sin(5-x)}{x^2}$ $\dfrac{-x\cos(5-x)-2\sin(5-x)}{x^3}$

6) $y=\ln\left(\cos\left(\dfrac{t}{k}\right)\right)$ $-\dfrac{1}{k}\tan\dfrac{t}{k}$

7) $h(x)=\dfrac{1}{\cos^2 x}-\sqrt{\cos^2 \pi x+\sin^2 \pi x}$ $2\tan x\sec^2 x$

8) $y=\dfrac{x^3}{8}(2\ln x-1)$ $\dfrac{x^2}{8}(6\ln x-1)$

9) $y(t)=\left[\cos(t^2+3)\right]^{100}$ $-200t\cos^{99}(t^2+3)\sin(t^2+3)$

10) $g(\theta)=\dfrac{\theta}{\csc^2\theta}$ $\sin\theta(2\theta\cos\theta+\sin\theta)$

11) $y(t)=\ln e^{(at^2-b)}$ $2at$

12) $f(x)=\log_2\sqrt{\sin x}$ $\dfrac{\cot x}{\ln 4}$

13) $y=\dfrac{1}{9}\sin^{-1}(e^{3x})$ $\dfrac{e^{3x}}{3\sqrt{1-e^{6x}}}$

14) $y=(\tan x)^x$ $(\tan x)^x\left(\dfrac{x\sec^2 x}{\tan x}+\ln\tan x\right)$

15) $y=(x+1)^{\sin x}$ $(x+1)^{\sin x}\left(\dfrac{\sin x}{x+1}+\cos x\ln(x+1)\right)$

For problems 16-18, find $\dfrac{dy}{dx}$.

16) $x^3-4x^2y+y^2=17$ $\dfrac{-3x^2+8xy}{-4x^2+2y}$

17) $\cos(xy)=x-2y$ $\dfrac{1+y\sin(xy)}{2-x\sin xy}$

18) $x=5-3\cos t$, $y=6t+3\sin t$ $\dfrac{2+\cos t}{\sin t}$

For problems 19 and 20, find $\dfrac{d^2y}{dx^2}$. X

19) $x^2-y^2=8$ $\dfrac{x^2-y^2}{y^3}$ or $\dfrac{8}{y^3}$

20) $x=2t+3$, $y=t^4$ $3t^2$

UNIT II

STUDENT WORKSHEETS

NAME_______________

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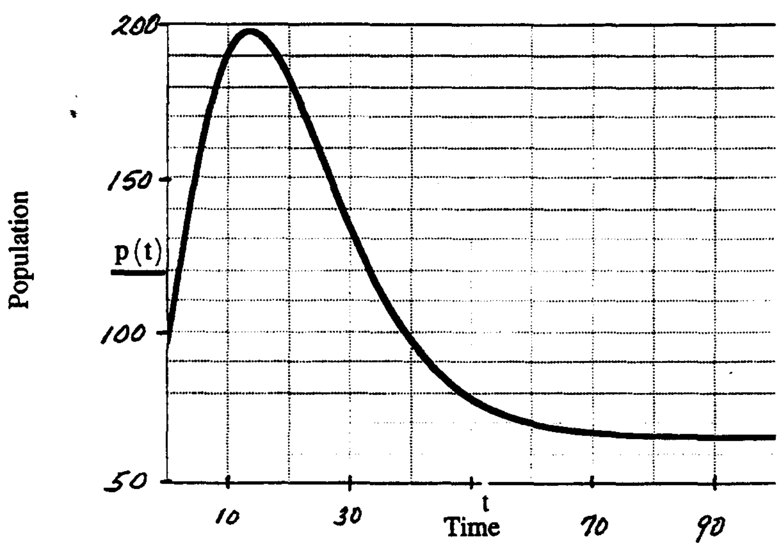
AVERAGE AND INSTANTANEOUS RATES OF CHANGE

1. The graph below shows the population of a small woodland mammal over a period of 100 days. The population increases from birth and then decreases as the mammals leave the protection of the nest.



a) What is the average rate of growth during the first 10 days? During the first 30 days?

b) What is the rate of growth of the population at the 10th day and at the 30th day?

c) Starting with $t = 0$, when is the average rate of growth 0 animals/day? What is the significance of this?

d) When is the instantaneous rate of growth 0 animals/day?

e) Over what period of time is the rate of change in population increasing? The rate of change in population decreasing?

2. Given the following data for a function g.

x	2.4	2.6	2.8	3.0	3.2	3.4
g(x)	1.65	1.70	1.82	2.00	2.30	2.7

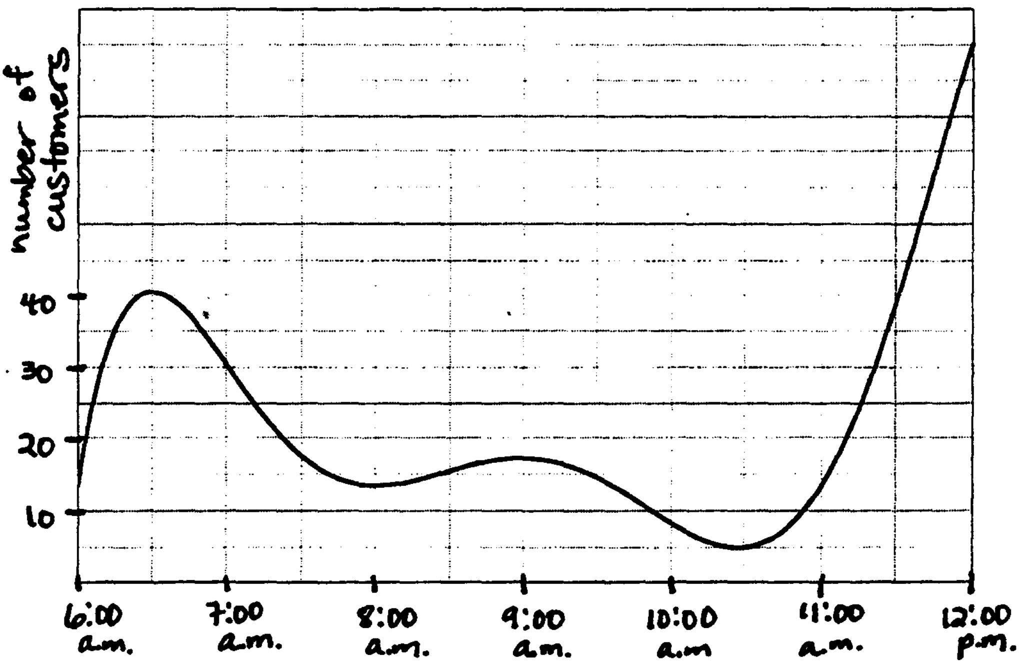
a) Find the average rate of change from x = 2.6 to x = 3.2.

b) Approximate $g'(3)$.

c) Find an equation that approximates the tangent line to g at the point (3, 2).

d) Use the tangent line found in c) to estimate $g(3.1)$ and $g(2.9)$.

3. The graph below shows how the number of customers at Burger Queen varied over a recent Monday morning.



a) For each time interval below, find the change in the number of customers AND the average rate of change in the number of customers. Be sure to indicate the units of your answers.

1) 6:00 am to 12:00 pm
2) 7:30 am to 10:30 am

b) Estimate the instantaneous rate of change of the number of customers at each of the following times. Be sure to include units.

1) 11:00 am
2) 8:00 am
3) 7:30 am

4. Your mom needs someone to give the dog a bath and someone to taste her brownies. Since you and your little brother both want to do the same chore, you decide the fair thing to do would be to flip a coin; loser has to taste the brownies. When the coin is flipped, the height of the coin, measured in feet, is modeled by $g(t) = -t^2 + 4t + 2$, where t is in seconds.

a) Find the change in the height of the coin between times t = 1 and t = 2 seconds.

b) Find the average rate of change of the height of the coin between t = 1 and t = 2 seconds.

c) Find the average rate of change of the height of the coin from t = 1 and t = 1 + h seconds.

d) Find the instantaneous rate of change of the coin at time t = 1.

e) IF you were to graph g(t), what graphical significance would your answers to part b and d have? (You do not need to draw the graph.)

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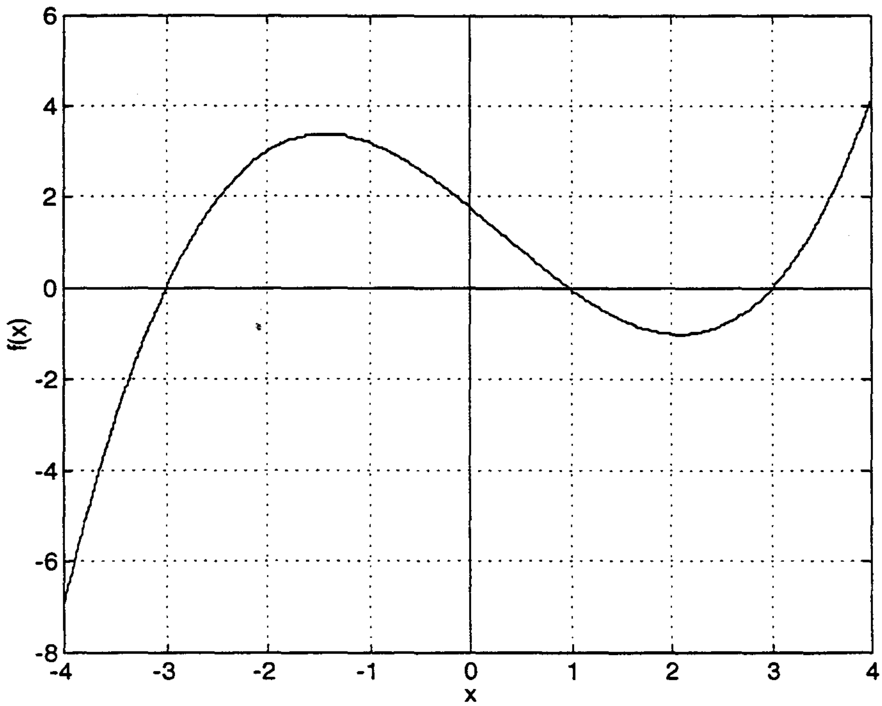
THE DERIVATIVE OF A FUNCTION



1. The graph of f is shown above.

   a. Use the graph to estimate $f'(-2)$ and $f'(1)$.

   b. On what interval is $f'(x)$ positive? Negative?

   c. Where does the graph of f have horizontal tangents?

   d. Where does f' have a minimum value? Explain.

   e. Sketch the graph of f' on the grid above.

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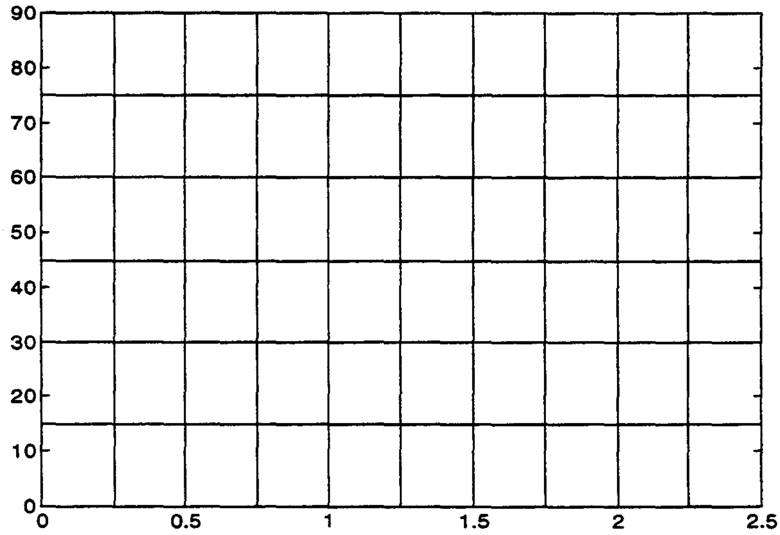
CONCEPTUALIZING THE DERIVATIVE THROUGH GRAPHICAL APPROACHES

The following table of values shows the height (in ft.) of a free-falling object after *t* seconds.

t	0	.25	.5	.75	1	1.25	1.5	1.75	2	2.25
h(t)	90	89	86	81	74	65	54	41	26	9

a). Plot these values on the grid below:



b). Connect the points with line segments.

c). Fill in the table below with the slope of the line segment at the given value of t.

t(sec)	.125	.375	.625	.875	1.125	1.375	1.625	1.875	2.125
m(ft/sec)									

d). What physical quantity do these slopes represent?

e). Graph the ordered pairs from the table in part c) on your calculator. Sketch what you see. What physical quantity is represented by the slope of this graph?

2. Suppose the equation $h(t) = -16t^2 + 90$ models the height of the object after t seconds.

a). Use the definition of a derivative to find $h'(t)$.

b). Sketch a graph of $h'(t)$ on the interval [0, 2.25].

c) How does this graph compare with the graph in question 1, part c?

3. Since the velocity function (derivative) was found to be v(t) = -32t, we can now investigate the rate of change of the velocity. The following is the table of values for the velocity at various times.

T	0	.25	.5	.75	1	1.25	1.5	1.75	2	2.25
v(t)	0	-8	-16	-24	-32	-40	-48	-56	-64	-72

a). What is the common term for rate of change of velocity?

b). What pattern exists in the change of velocity?

c). Write an equation for the rate of change of velocity.

d). Is this equation the derivative of the velocity function? Explain.

4. What generalization can be made regarding the relationship between the three physical quantities discussed in this investigation (position, velocity, and acceleration)?

AP CALCULUS
UNIT II – ACTIVITY 4

NAME________________
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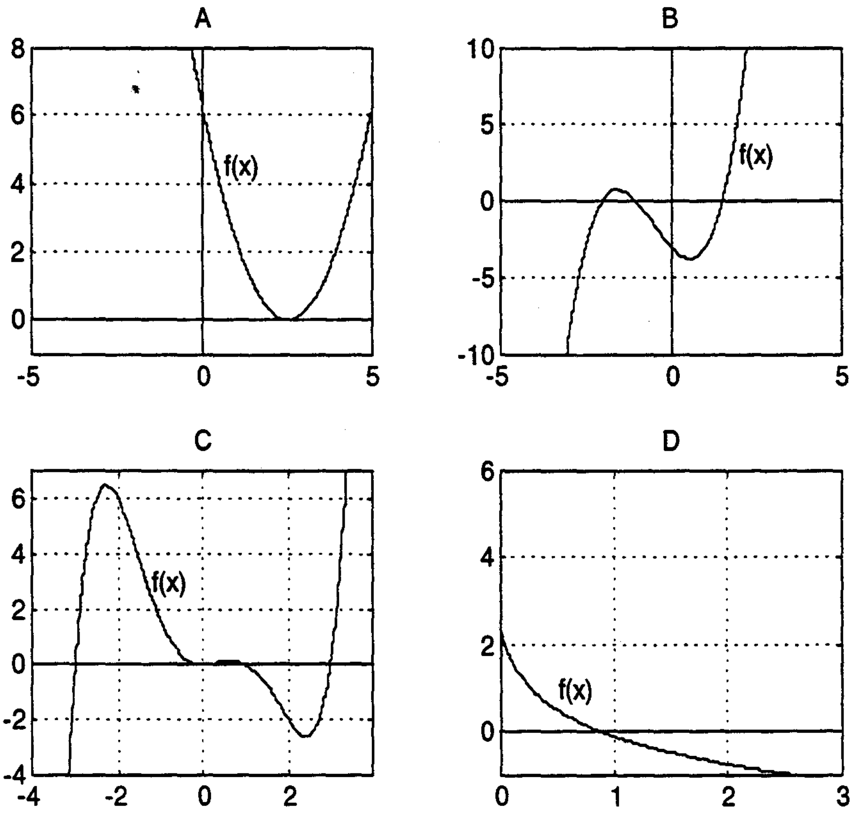
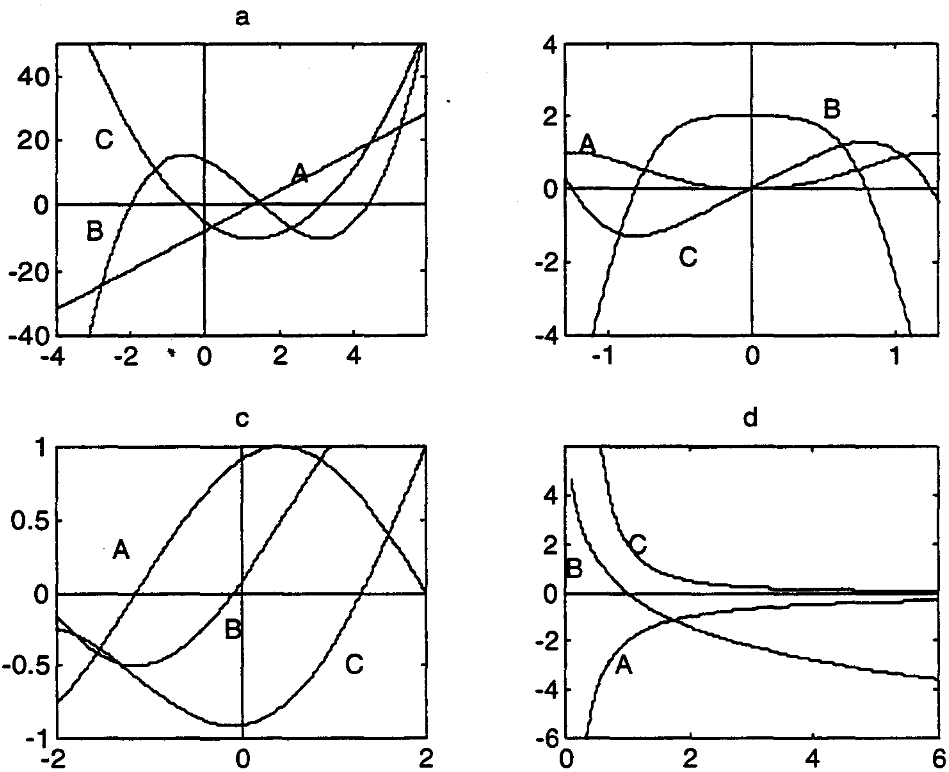
ANALYZING GRAPHS OF FUNCTIONS AND DERIVATIVES

1. Sketch the graphs of the first and second derivatives of the functions shown below.



2. Each of the figures below shows the sketches of f, f', f''. Identify each curve and justify your answer.



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DERIVATIVE PRACTICE I

For problems 1-10, find the derivative of the given function. Use your own paper, and show your work. Remember that algebra and/or the use of a trigonometric identity can simplify some problems before you take the derivative.

1) $y = \frac{1}{2}x^4 - 5\sqrt{x} + \frac{3}{x^3}$

2) $y = \cos(3 - 5x)$

3) $y = 3\sec\frac{x}{2}$

4) $f(x) = \frac{5x}{x^2 - 4}$

5) $g(t) = \frac{3t^5 - t^2 + 6}{t}$

6) $y = \left(\frac{x^2 - 3}{5}\right)^3$

7) $h(x) = \tan^4(\pi x)$

8) $p(\Theta) = \frac{1}{2\sin\Theta\cos\Theta}$

9) $v(t) = \sqrt[3]{2t^4 - 1}$

10) $y = \sqrt{\cos^2 x + 3 + \sin^2 x}$

11) Find $\frac{d^2y}{dx^2}$ if $y = \frac{4 - x}{4 + x}$

12) The table below gives the values of two functions and their derivatives at x = 1 and x = 4.

x	f(x)	g(x)	f'(x)	g'(x)
1	2	4	π	-1
4	6	2/3	-5	3

Use the information in the table to find $h'(x)$ at the given value of x.

a) $h(x) = f(x) - 2g(x)$ at $x = 1$

b) $h(x) = f(x) \bullet g(x)$ at $x = 4$

c) $h(x) = \dfrac{f(x)}{g(x)}$ at $x = 4$

d) $h(x) = f(g(x))$ at $x = 1$

e) $h(x) = \dfrac{1}{g(x)}$ at $x = 4$

f) $h(x) = \sqrt{f(x) - g^2(x)}$ at $x = 4$

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DERIVATIVE PRACTICE II

Find the derivative of each of the following functions, where *a, b,* and *k* are constants. Use your own paper, and show your work. If possible, simplify the expression before taking the derivative.

1) $f(x) = 2^{-x}$

2) $p(t) = e^{\sin 3t}$

3) $y = \cos(\arctan \pi x)$

4) $f(x) = e^{3x}(x^2 + 7^x)$

5) $y = \cos^{-1}\left(5x^2\right)$

6) $h(x) = \ln(\sec(x^3))$

7) $g(x) = e^{\ln(ax)}$

8) $f(x) = \sin^{-1}(\sin 5x)$

9) $y = \ln\left(\dfrac{e^{kx}}{b}\right)$

10) $f(x) = \ln\sqrt{x^3 - 4}$

11) $f(x) = \log_5(\tan x)$

12) $y = \dfrac{1}{8}\log_2 x^4$

13) $p(t) = \tan^{-1}\left(\dfrac{3}{t}\right)$

14) $y = x^{\ln x}$

15) $y = \dfrac{e^t - e^{-t}}{e^t + e^{-t}}$

NAME ______________________
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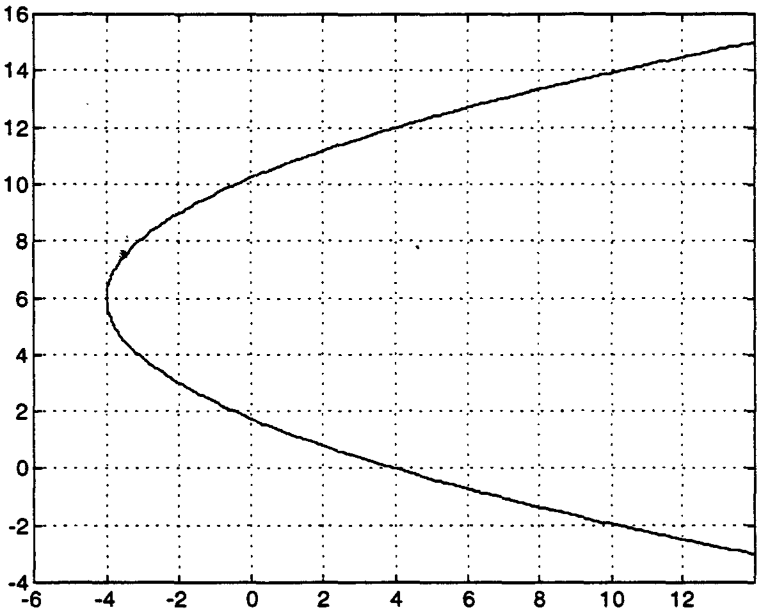
GRAPHICAL INTERPRETATION OF IMPLICIT DIFFERENTIATION

1. The graph of $9x - 2y^2 + 24y = 36$ is shown below.



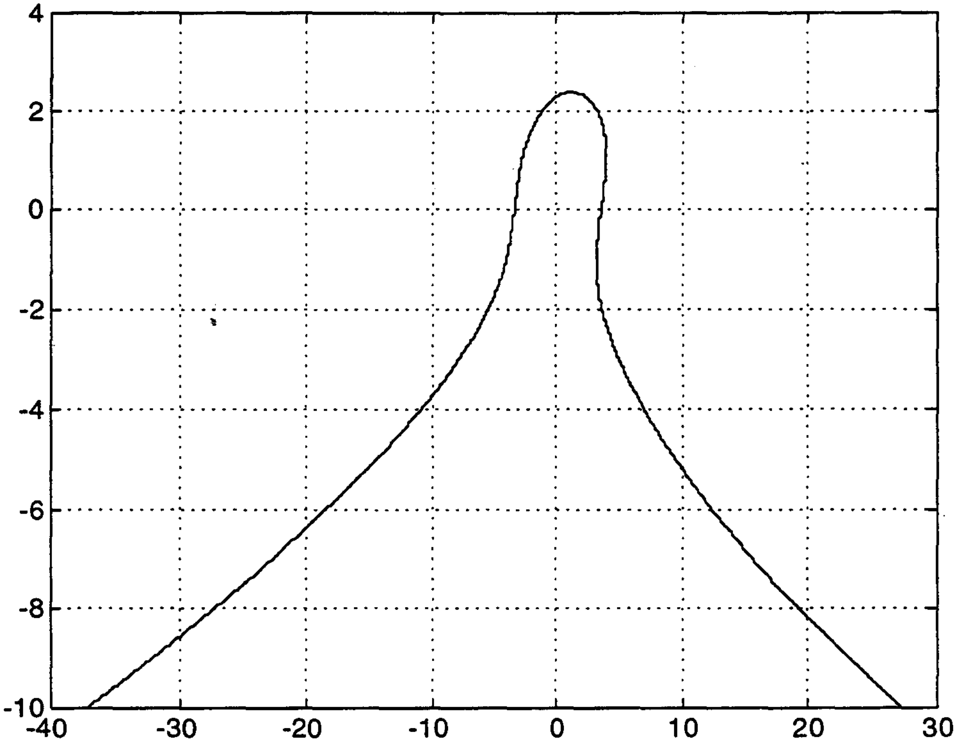
a) Find an expression for $\frac{dy}{dx}$.

b) Find an equation of the tangent to the curve at (-2,3). Sketch the tangent line on the graph above.

c) Find an equation of the line normal to the curve at the point (-2,3) and sketch it on the graph.

d) Are there any points on the graph where the tangent is horizontal? Vertical? Justify your answer.

2. Consider the implicit relation $x^2 - xy + y^3 = 12$ graphed below.



a) Find a general equation for $\frac{dy}{dx}$.

b) Find an equation of the horizontal tangent(s) of the curve.

c) Find an equation of the vertical tangent(s) of the curve.

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UNIT II – ACTIVITY 8

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EXTENSIONS OF IMPLICIT DIFFERENTIATION

I. For the following relations, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

1. $y^2 - 3xy + 2x = 12$

2. $\cos y - 3x + 2 = 0$

II. For the following relations, evaluate $\frac{d^2y}{dx^2}$ for the given value of x. Also find equations for the lines tangent and normal to the curve at that point.

1. $e^{2y} - 4x^2 - 3y = 0;$ *evaluate for* $x = \frac{1}{2}$

2. $3x^2y - 6y + 9 = 0;$ *evaluate for* $x = 2.$

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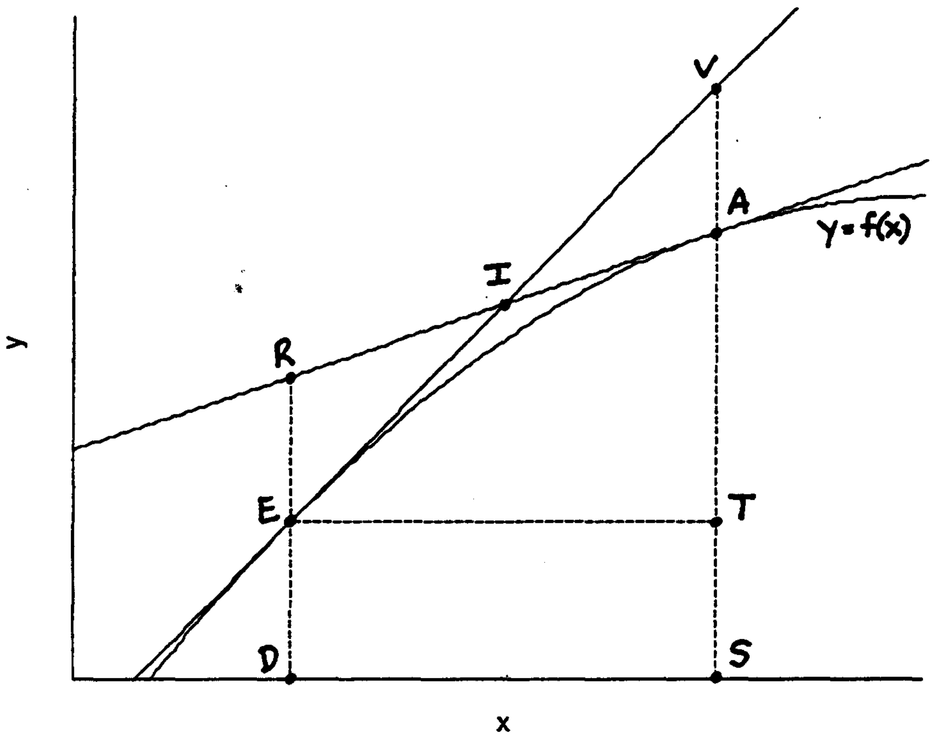
GEOMETRIC REPRESENTATION OF DERIVATIVES

Each quantity below can be represented in the diagram. Use the letters in the diagram to represent each quantity, and tell if it is a length, a slope or an area.



Note: Point D has coordinates $(a, 0)$ and point S has coordinates $(a+b, 0)$.

1. $f(a+b)$

1. $f'(a+b)$

3. $f'(a)b$

4. $f(a)b$

5. $f(a+b)-f(a)$

6. $\dfrac{f(a+b)-f(a)}{b}$

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RULES OF DIFFERENTIATION

I. Find f' in terms of g'.
(a is constant)

1. $f(x) = g[x + g(a)]$

2. $f(x) = g[x \bullet g(a)]$

3. $f(x) = g[x + g(x)]$

4. $f(x) = g(x) \bullet (x - a)$

5. $f(x) = g^2(ax)$

6. $f(x) = g(a) \bullet (x - a)$

7. $f(x) = x \bullet g[x \bullet g(a)]$

8. $f(x+3) = g(x^2)$

II. Let $f(x) = x^3$. Find each of the following:

1. $f'(9)$, $f'(25)$

2. $f'(3^2)$, $f'(5^2)$

3. $f'(a^2)$, $f'(x^2)$

4. Let $g(x) = f(x^2)$. Find $f'(x)$, $g'(x)$, $f'(x^2)$, $g'(x^2)$

NAME________________________
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RULES OF DIFFERENTIATION

I. Find f' in terms of g'.
(a is constant)

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1. $f'(9)$, $f'(25)$

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3. $f'(a^2)$, $f'(x^2)$

4. Let $g(x) = f(x^2)$. Find $f'(x)$, $g'(x)$, $f'(x^2)$, $g'(x^2)$

NAME________________
DATE________________

LIMITS AND THE DEFINTION OF DERIVATIVE

I. Evaluate each limit.

1. $\lim\limits_{h\to 0}\dfrac{8(x+h)-8x}{h}$

2. $\lim\limits_{h\to 0}\dfrac{\left[4(x+h)^3+7(x+h)\right]-\left[4x^3+7x\right]}{h}$

3. $\lim\limits_{\Delta x\to 0}\dfrac{3|x+\Delta x|-3|x|}{\Delta x}$ at $x=5$

4. $\lim\limits_{x\to 6}\dfrac{f(x)-f(6)}{x-6}$ if $f(x)=-2x^2+x$

II. Each limit expression represents $f'(c)$. Identify f and c, and find the limit.

1. $\lim\limits_{h\to 0}\dfrac{3(2+h)^4-48}{h}$

2. $\lim\limits_{x\to 7}\dfrac{\sqrt{2x-5}-3}{x-7}$

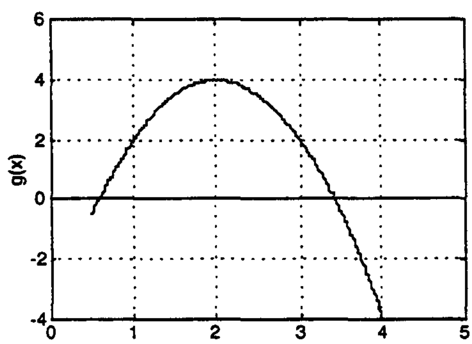
3. $\lim\limits_{n\to \frac{-\pi}{2}}\dfrac{\sin n+1}{n+\frac{\pi}{2}}$

4. $\lim\limits_{h\to 0}\dfrac{3e^{-h}-3}{h}$

III. On the graph of g, draw lines with the given slopes. How is each line related to g ?

1. $\dfrac{g(3)-g(2)}{3-2}$

2. $\lim\limits_{h\to 0}\dfrac{g(1+h)-g(1)}{h}$



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DERIVATIVES REVIEW

For problems 1-15, find the derivative of the given function, where *a, b* and *k* are constants. Use your own paper, and show your work. Remember that sometimes you can simplify the expression before you take the derivative.

1) $f(x)=\dfrac{5}{\left(b^2-x^2\right)^2}$

2) $y=xe^{\tan x}$

3) $g(x)=\arctan\left(3x^2+1\right)$

4) $p(t)=a^{t^3-t}$

5) $y=\dfrac{\sin(5-x)}{x^2}$

6) $y=\ln\left(\cos\left(\dfrac{t}{k}\right)\right)$

7) $h(x)=\dfrac{1}{\cos^2 x}-\sqrt{\cos^2 \pi x+\sin^2 \pi x}$

8) $y=\dfrac{x^3}{8}(2\ln x-1)$

9) $y(t)=\left[\cos\left(t^2+3\right)\right]^{100}$

10) $g(\theta)=\dfrac{\theta}{\csc^2\theta}$

11) $y(t)=\ln e^{\left(at^2-b\right)}$

12) $f(x)=\log_2\sqrt{\sin x}$

13) $y=\dfrac{1}{9}\sin^{-1}\left(e^{3x}\right)$

14) $y=(\tan x)^x$

15) $y=(x+1)^{\sin x}$

For problems 16-18, find $\dfrac{dy}{dx}$.

16) $x^3-4x^2y+y^2=17$

17) $\cos(xy)=x-2y$

18) $x=5-3\cos t,\quad y=6t+3\sin t$

For problems 19 and 20, find $\dfrac{d^2y}{dx^2}$.

19) $x^2-y^2=8$

20) $x=2t+3,\ y=t^4$

AP CALCULUS
UNIT II

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MULTIPLE CHOICE PRACTICE

Complete the following without using a calculator.

1. If $f(x) = \cos(e^{-x})$, then $f'(x) =$

A) $-e^{-x}\sin(e^{-x})$ B) $e^{-x}\sin(e^{-x})$ C) $-\sin(e^{-x})$ D) $-\sin(-e^{-x})$ E) $e^{-x}\sin(-e^{-x})$

2. If $x^3 + 2xy = 13$, then when x = 1, $\frac{dy}{dx} =$

A) $-\frac{17}{2}$ B) $-\frac{15}{2}$ C) -2 D) -1 E) $-\frac{1}{4}$

3. What is the instantaneous rate of change at of the function f given by $f(x) = \frac{x^2-5}{1-x}$ at x = 3?

A) -6 B) -2 C) 0 D) 2 E) 4

4. If $f(x) = \tan(2x)$, then $f'(\frac{\pi}{3}) =$

A) *undefined* B) -8 C) -4 D) 4 E) 8

5. The slope of the line tangent to the curve $y^3 + (xy-1)^2 = 0$ at $(-\frac{9}{4}, -4)$ is

A) 0 B) $\frac{1}{3}$ C) $\frac{32}{59}$ D) $\frac{8}{3}$ E) $\frac{16}{3}$

6. If p and q are twice differentiable and if $t(x) = p(q(x))$, then $t''(x) =$

A) $p'(q(x)) \bullet q'(x)$ B) $p''(q'(x)) \bullet (q'(x))^2 + p'(q(x)) \bullet q''(x)$
C) $p''(q(x)) \bullet (q'(x))^2 + p'(q(x)) \bullet q''(x)$ D) $p''(q(x)) \bullet (q'(x))^2 + p''(q(x)) \bullet q''(x)$
E) $p''(q'(x)) \bullet (q'(x))^2 + p''(q(x)) \bullet q''(x)$

7. If $\ln f(x) = 2x$, then $f'(x)$

A) $2e^{2x}$ B) $\frac{1}{x}$ C) $\frac{2x}{\ln x}$ D) e^{2x} E) $\frac{1}{2}e^{2x}$

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A graphing calculator may be required for some parts of these questions.

8. If $\dfrac{dy}{dx} = (1-2y)^2$, then $\dfrac{d^2y}{dx^2} =$

A) $-4(1-2y)(1-2y^2)$ B) $-4y(1-2y)^2$ C) $-4(1-2y)^3$ D) $4(1-2y)(1-2y^2)$ E) $-4y(1-2y)$

9. Let f be the function given by $f(x) = \ln 2x$ and let g be the function given by $g(x) = \dfrac{1}{2}x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

A) .541 B) .693 C) .841 D) .874 E) 1.100

10. Let f be the function given by $f(x) = \ln 2x$ and let g be the function given by $g(x) = \dfrac{1}{2}x^3$. Which of the following are x values where the tangents of the two graphs are perpendicular:

I. x = 0
II. x = - .667
III. x = -.874

A) I only B) II only C) I and II only
D) I, II, and III E) II and III only

11. Given $f(x) = e^{3x}$, $g(x) = 2x - 3$, and $h(x) = f(g(x))$, the value of $h'(2)$ is

A) 8.155 B) 16.310 C) 40.171 D) 60.257 E) 120.513

12. Let f be the function given by $e^{f(x)} = 3x + 1$, g be the function given by $2\ln g(x) = 4x^2 - 6$, and h be the function given by $h(x) = f(g(x))$. Find $h'(x)$.

A. $\dfrac{12xe^{2x^2-3}}{3e^{2x^2-3}+1}$ B. $\dfrac{xe^{2x^2-3}}{e^{2x^2-3}+\frac{1}{3}}$

C. $\dfrac{3e^{2x^2-3}}{3e^{2x^2-3}+1}$ D. $\dfrac{12e^{2x^2-3}}{3e^{2x^2-3}+1}$

E. $\dfrac{4xe^{2x^2-3}}{e^{2x^2-3}+1}$

AP CALCULUS
UNIT II

NAME Key
DATE

MULTIPLE CHOICE PRACTICE

Complete the following without using a calculator.

1. If $f(x)=\cos(e^{-x})$, then $f'(x)=$

A) $-e^{-x}\sin(e^{-x})$ B) $e^{-x}\sin(e^{-x})$ C) $-\sin(e^{-x})$ D) $-\sin(-e^{-x})$ E) $e^{-x}\sin(-e^{-x})$

(B)

2. If $x^3+2xy=13$, then when x = 1, $\frac{dy}{dx}=$

A) $-\frac{17}{2}$ B) $-\frac{15}{2}$ C) -2 D) -1 E) $-\frac{1}{4}$

(B)

3. What is the instantaneous rate of change at of the function f given by $f(x)=\frac{x^2-5}{1-x}$ at x = 3?

(B) A) -6 B) -2 C) 0 D) 2 E) 4

4. If $f(x)=\tan(2x)$, then $f'(\frac{\pi}{3})=$

A) *undefined* B) -8 C) -4 D) 4 E) 8

(E)

5. The slope of the line tangent to the curve $y^3+(xy-1)^2=0$ at $(-\frac{9}{4},-4)$ is

A) 0 B) $\frac{1}{3}$ C) $\frac{32}{59}$ D) $\frac{8}{3}$ E) $\frac{16}{3}$

(E)

6. If p and q are twice differentiable and if $t(x)=p(q(x))$, then $t''(x)=$

A) $p'(q(x))\bullet q'(x)$ B) $p''(q'(x))\bullet(q'(x))^2+p'(q(x))\bullet q''(x)$

(C) C) $p''(q(x))\bullet(q'(x))^2+p'(q(x))\bullet q''(x)$ D) $p''(q(x))\bullet(q'(x))^2+p''(q(x))\bullet q''(x)$

E) $p''(q'(x))\bullet(q'(x))^2+p''(q(x))\bullet q''(x)$

7. If $\ln f(x)=2x$, then $f'(x)$

(A) A) $2e^{2x}$ B) $\frac{1}{x}$ C) $\frac{2x}{\ln x}$ D) e^{2x} E) $\frac{1}{2}e^{2x}$

Key

AP CALCULUS
UNIT II

A graphing calculator may be required for some parts of these questions.

8. If $\dfrac{dy}{dx}=(1-2y)^2$, then $\dfrac{d^2y}{dx^2}=$

(C)

A) $-4(1-2y)(1-2y^2)$ B) $-4y(1-2y)^2$ C) $-4(1-2y)^3$ D) $4(1-2y)(1-2y^2)$ E) $-4y(1-2y)$

9. Let f be the function given by $f(x)=\ln 2x$ and let g be the function given by $g(x)=\dfrac{1}{2}x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(D)

A) .541 B) .693 C) .841 D) .874 E) 1.100

10. Let f be the function given by $f(x)=\ln 2x$ and let g be the function given by $g(x)=\dfrac{1}{2}x^3$. Which of the following are x values where the tangents of the two graphs are perpendicular:

(C)

I. x = 0
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11. Given $f(x)=e^{3x}$, $g(x)=2x-3$, and $h(x)=f(g(x))$, the value of $h'(2)$ is

(E)

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12. Let f be the function given by $e^{f(x)}=3x+1$, g be the function given by $2\ln g(x)=4x^2-6$, and h be the function given by $h(x)=f(g(x))$. Find $h'(x)$.

A. $\dfrac{12xe^{2x^2-3}}{3e^{2x^2-3}+1}$ B. $\dfrac{xe^{2x^2-3}}{e^{2x^2-3}+\frac{1}{3}}$

(A)

C. $\dfrac{3e^{2x^2-3}}{3e^{2x^2-3}+1}$ D. $\dfrac{12e^{2x^2-3}}{3e^{2x^2-3}+1}$

E. $\dfrac{4xe^{2x^2-3}}{e^{2x^2-3}+1}$

1. Find the average rate of change of $f(x) = x^2 + x$ over the interval [2, 4]

2. At what point is the tangent to $f(x) = (x - 2)^2 + 3$ horizontal?

3. Evaluate: $\lim_{h \to 0} \frac{(3+h)^2 - 9}{h}$

4. If $f(x) = \begin{cases} 2x, & x \geq 3 \\ 4x - 1, & x < 3 \end{cases}$

   Find $\lim_{x \to 3^-} f(x)$

5. Sketch the graph, $y = \sin x$.

6. Evaluate: $\log 21 = \log 7 +$ ______

1. Find the average rate of change of $f(x) = x^2 + x$ over the interval $[2, 4]$

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   Find $\lim_{x \to 3^-} f(x)$

5. Sketch the graph, y = sin x.

6. Evaluate: log 21 = log 7 + ______

ANSWERS:

1. $\frac{f(4) - f(2)}{4 - 2}$

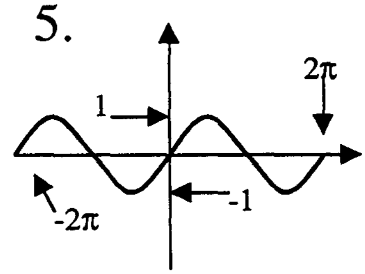
   $= 7$

2. At the vertex (2, 3)

3. 6

4. 11

5.



6. log 3

1. Find the slope of the line tangent to the graph of $y = x^2 + 3x - 5$ at its vertex.

2. Find the value for k which makes f(x) continuous.

$$f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2x + k, & x > 3 \end{cases}$$

3. Evaluate: $\lim_{x \to \infty} \dfrac{7x^2 + 3x - 2}{9x^2 - 5x + 3}$

4. Name two consecutive integers values of x which can be used to prove there is a positive root of $f(x) = x^2 + 2x - 7$.

5. Graph the following inequality on a number line: $|x - 3| < 5$

1. Find the slope of the line tangent to the graph of $y = x^2 + 3x - 5$ at its vertex.

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4. Name two consecutive integers values of x which can be used to prove there is a positive root of $f(x) = x^2 + 2x - 7$.

5. Graph the following inequality on a number line: $| x - 3 | < 5$

ANSWERS:

1. slope = 0

2. k = 2

3. $\dfrac{7}{9}$

2. 1 and 2 because f(1) = - 4 and f(2) =1

5.
$-5 < x - 3 < 5$
$-2 < x < 8$

1. Evaluate: $\lim_{x\to 0^-} f(x)$ *if* $f(x)=\frac{|x|}{x}$

2. Rewrite the following expression as a sum or difference: $\log\left(\frac{x}{y}\right)$

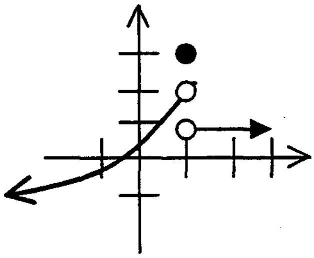
3. Given the graph below, evaluate the following values:

a) $\lim_{x\to 1^-} f(x)$

b) $\lim_{x\to 1^+} f(x)$

c) $\lim_{x\to 1} f(x)$

d) f(1)



4. What does the derivative of a function at x=a mean?

5. Graph $f(x)=\frac{x^2-2x-8}{3x-12}$ without the use of a calculator.

1. Evaluate: $\lim\limits_{x \to 0^-} f(x)$ *if* $f(x) = \dfrac{|x|}{x}$

2. Rewrite the following expression as a sum or difference: $\log\left(\dfrac{x}{y}\right)$

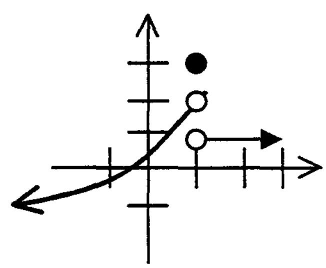
3. Given the graph below, evaluate the following values:

a) $\lim\limits_{x \to 1^-} f(x)$

b) $\lim\limits_{x \to 1^+} f(x)$

c) $\lim\limits_{x \to 1} f(x)$

d) f(1)



4. What does the derivative of a function at x=a mean?

5. Graph $f(x) = \dfrac{x^2 - 2x - 8}{3x - 12}$ without the use of a calculator.

ANSWERS:

1. –1

2. log x – log y

3.

a) 2

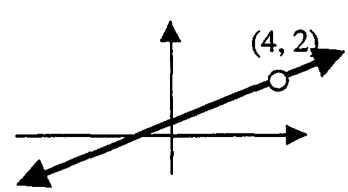
b) 1

c) DNE

d) 3

4. **The instantaneous rate of change of f(x) with respect to x at x = a.**

5. $\dfrac{(x-4)(x+2)}{3(x-4)} = \dfrac{x+2}{3}$, where $x \neq 4$



1. The instantaneous rate of change of a function is called the __________ of the function.

2. Evaluate: $\lim_{x \to -\infty} e^{3x}$

3. If $f(x) = e^{5x}$,
   then $f'(x) =$ ______________

4. Sketch a graph with a cusp at the point (2, 3).

5. Write the following expression as the log of a single argument:

$$\log 24 - \log 8 + \log 2$$

ANSWERS:

1. The instantaneous rate of change of a function is called the ________ of the function.

1. derivative

2. Evaluate: $\lim_{x \to -\infty} e^{3x}$

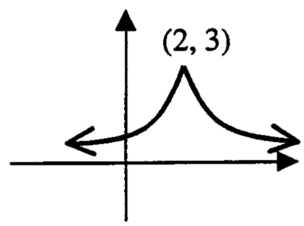
2. 0

3. If $f(x) = e^{5x}$, then $f'(x) =$ ____________

3. $5e^{5x}$

4. Sketch a graph with a cusp at the point (2, 3).

4. Answers vary.



5. Write the following expression as the log of a single argument:

$$\log 24 - \log 8 + \log 2$$

5. log 6

1. What is the graphical interpretation of the derivative of a point?

2. Which of the following limits can be used to determine the derivative of g(x)?

a) $\lim_{h \to 0} \frac{g(x+h)-g(h)}{h}$

b) $\lim_{h \to 0} \frac{g(x+h)-g(x)}{h}$

c) $\lim_{h \to 0} \frac{g(x+h)-g(x-h)}{2h}$

3. If $f(x) = \sin^3(5x)$, then $f'(x) =$ ______________

4. Simplify: $(3x + 5)^2$

5. If $f(x) = \arctan(e^x)$, find $f'(x)$.

ANSWERS:

1. What is the graphical interpretation of the derivative of a point?

   Answer 1. slope of the line tangent at that point

2. Which of the following limits can be used to determine the derivative of g(x)?

   a) $\lim_{h \to 0} \frac{g(x+h)-g(h)}{h}$

   b) $\lim_{h \to 0} \frac{g(x+h)-g(x)}{h}$

   c) $\lim_{h \to 0} \frac{g(x+h)-g(x-h)}{2h}$

   Answer 2. b and c

3. If $f(x) = \sin^3(5x)$, then $f'(x) =$ ______________

   Answer 3. 15sin5xcos5x or $15\sin^2(5x)$

4. Simplify: $(3x + 5)^2$

   Answer 4. $9x^2 + 30x + 25$

5. If $f(x) = \arctan(e^x)$, find $f'(x)$.

   Answer 5. $\frac{e^x}{1+e^{2x}}$

UNIT II INTERNET RESOURCES

http://www.math.odu.edu/cbii/calcanim/ and http://www.math.psu.edu/dna/graphics.html

Both sites have animated demonstrations of the derivative by way of local linearity.

http://www.netsrq.com/~hahn/calculus.html, http://www.barzilai.org/archive, http://archives.math.utk.edu.visual.calculus/ and http://www.hofstra.edu/~matscw/RealWorld/index.html

Sites feature some tutorials, but mostly have good drill and quiz resources for either in-class practice or at-home practice.