UNIT V

UNIT V
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UNIT V: APPLICATIONS OF THE DEFINITE INTEGRAL

Expectation:	The students will use definite integrals in a variety of applications to model physical, social or economic situations.

OVERVIEW:
Use the integral of a rate of change to give the accumulated change and set up an approximating Riemann sum. Find the limit of these sums and evaluate them as a definite integral. Students should be able to adapt their knowledge and techniques to solve application problems in which a product has a factor that varies.

INDICATORS:

Specific applications should include construction of solutions for determining each of the following:

1. The area of a bounded region.
2. The position function from a given velocity or acceleration function.
3. The displacement and the distance traveled in a specified period of time from a given velocity or acceleration function.
4. The average value of a function, both analytically and geometrically.
5. The volume of a solid having a known cross section.
6. * The length of a curve (including a curve given in parametric form).
7. * Additional variable-factor products. (Examples: work, force, mass, moment)
8. Total change over a specific period of time given a rate of change.

* BC Calculus indicator only

Unit V: APPLICATIONS OF THE DEFINITE INTEGRAL

Indicators/ Objectives	Foerster: Calculus Key Curriculum 1998	Foerster: Calculus: Instructor's Resource Book Key Curriculum 1998	Finney, et al: Calculus S F A W 1999	Guide Pages
1	225-228		262-264;292; 374-379	V 1-4
2	225-227	1.3; 5.10	311; 313	
3	225-227; 194	3.9; 10.2	298; 366	V 6
4	549		270-271	
5	385-395	8.5 a, b	251; 383-389; 514-517	
6	403-409	8.7	395-399	V 5-9
7	584-490	11.2;11.3;11.4 a,b; 11.5	401-411	
8	584 - 585		363 - 370	V 8

Indicators/ Objectives	Finney, et al: Calculus 1994	Guide Pages
1	433 - 442	
2	332 - 335	
3	509 - 510	
4	374 - 375	
5	442 - 451,506 - 509	
6	466 - 471, 803 - 805	
7	480 - 502	
8		

AP CALCULUS
UNIT V

NAME Key
DATE ____________

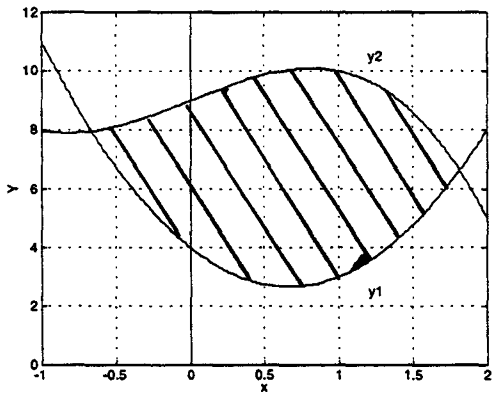
SETTING UP AND EVALUATING INTEGRAL EXPRESSIONS FOR AREA BETWEEN TWO CURVES

When finding the area between two curves, it is sometimes necessary to find the points of intersection by using a graphing calculator. The following example will illustrate the steps needed to do this and will be followed by several practice exercises.

Example: Set up an integral expression and find the area of the region between $Y_1 = 3x^2 - 4x + 4$ and $Y_2 = -x^3 + 2x + 9$ over [-1, 2].

View the graph of the equations using an appropriate window (showing points of intersection), and identify which function creates the top boundary for the region.

On the graph below, shade the region:



We will now find the points of intersection – these will serve as the limits of integration. In order to streamline the process, we will store the values for the limits using the calculator. Start by finding the left point of intersection. While the coordinates for the point of intersection are listed on the screen, return to the home screen and store the x value into the variable A.

X [STO] [ALPHA] A

Then repeat the process and find the right boundary and store it as the variable B.

X [STO] [ALPHA] B

Limits of integration: A = -.662 B = 1.812

Now, we can set up the integral expression $\int_{A}^{B} f(x)dx$

We have entered A and B as the values of the points of intersection. F(x) will be the expression of difference between the curves, in our case $Y_2 - Y_1$ (since Y_2 was the top boundary).

Therefore, the written integral expression would be

$$\int_{-.662391}^{1.81169141} ((-x^3 + 2x + 9) - (3x^2 - 4x + 4))dx$$

and to evaluate the expression using the calculator, the syntax would be

fnInt($Y_2 - Y_1$, X, A, B)

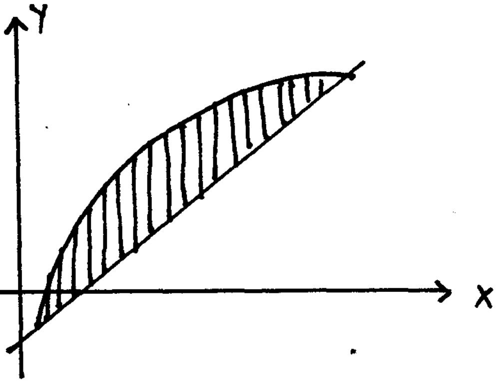
AP CALCULUS
UNIT V – ACTIVITY 1

NAME Key
DATE

SETTING UP AND EVALUATING INTEGRAL EXPRESSIONS FOR AREA BETWEEN TWO CURVES

For each system, sketch the graph and shade in the enclosed region. Then list the limits of integration, set up the integral, and evaluate the integral. Be sure that your final answer is correct to three decimal places.

1. $Y_1 = 4\sqrt{x} - 2$
   $Y_2 = 3x - \frac{3}{2}$



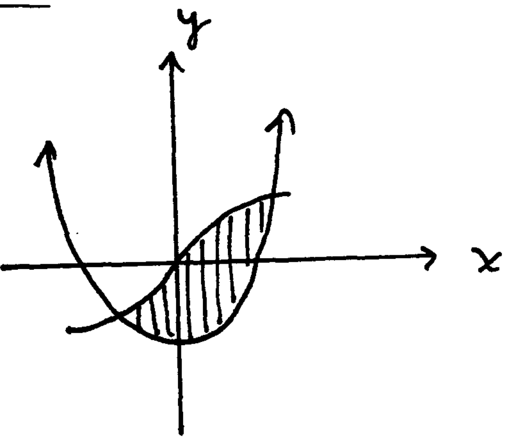
Remember to use stored values for limits of integration. (Do not truncate till the final answer is computed)

LIMITS OF INTEGRATION: A = .019... B = 1.425...

WRITTEN INTEGRAL EXPRESSION: $\int_{.019...}^{1.425...} [(4\sqrt{x} - 2) - (3x - \frac{3}{2})]dx$

VALUE OF INTEGRAL: ≈ .781

2. $Y_1 = \sin(\frac{x}{3})$
   $Y_2 = x^2 - 3$

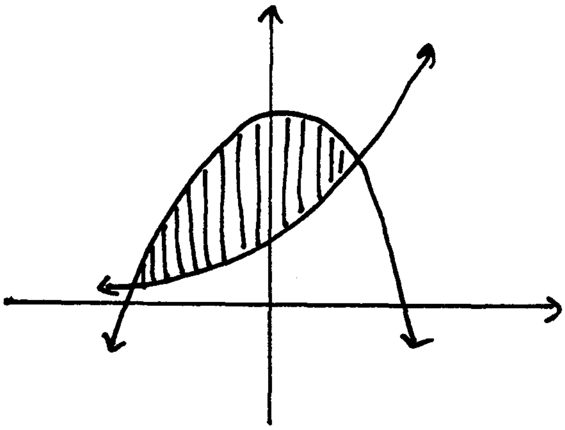


LIMITS OF INTEGRATION: A = -1.580... B = 1.895...

WRITTEN INTEGRAL EXPRESSION: $\int_{-1.580...}^{1.895...} (\sin\frac{x}{3} - (x^2 - 3))dx$

VALUE OF INTEGRAL: ≈ 7.014

3. $Y_1 = e^{3x}$
$Y_2 = -2x^2 + 4$

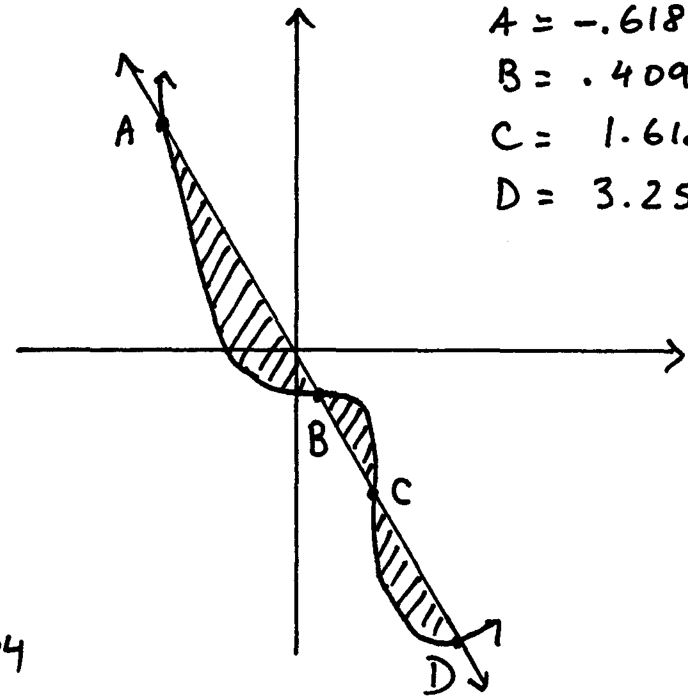


LIMITS OF INTEGRATION: A = $-1.411\ldots$ B = $.429\ldots$

WRITTEN INTEGRAL EXPRESSION: $\int_{-1.411}^{.429} (-2x^2+4-e^{3x})\,dx$

VALUE OF INTEGRAL: ≈ 4.232

4. CHALLENGE PROBLEM – FIND THE TOTAL AREA ENCLOSED BY THE CURVES.
$Y_1 = 3x^4 - 14x^3 + 12x^2 + x - 4$
$Y_2 = -6x$



$A \doteq -.618\ldots$
$B \doteq .409\ldots$
$C \doteq 1.618\ldots$
$D \doteq 3.257\ldots$

Total area:

$\int_A^B (Y_2 - Y_1)\,dx + \int_B^C (Y_1 - Y_2)\,dx + \int_C^D (Y_2 - Y_1)\,dx$

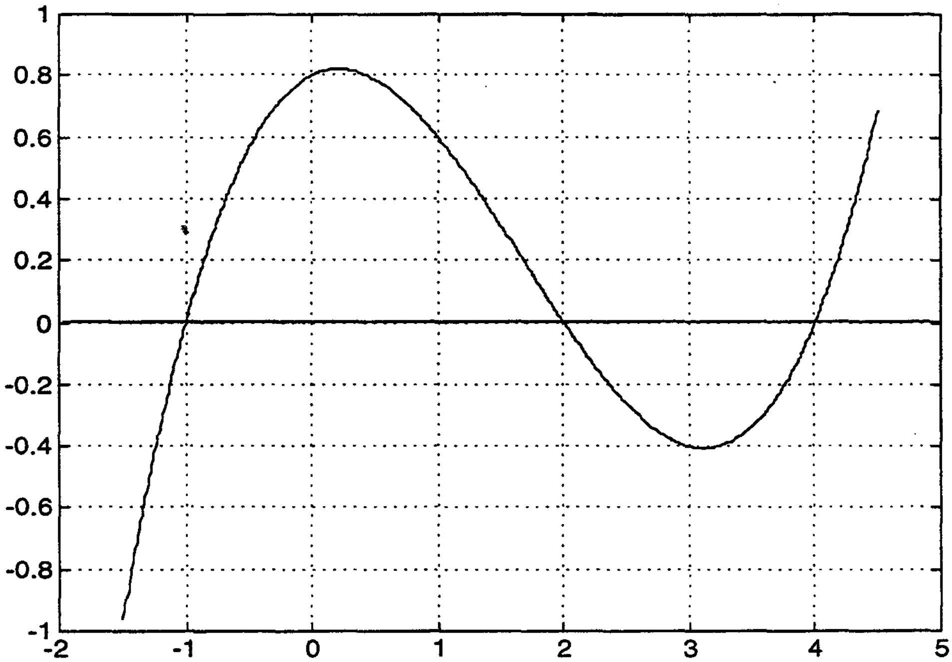
$\doteq 20.284$

AP CALCULUS
UNIT V – ACTIVITY 2

NAME Key
DATE

AVERAGE VALUE

The graph of f is pictured below.



List from least to greatest. d, c, b, a

a) $f'(-1)$ positive

b) $\int_{-1}^{4} f(x)dx$ negative

c) the average value of $f(x)$ over [-1, 4] $\dfrac{|\text{ans. to (b)}|}{4-(-1)} < |\text{Ans. to (b)}|$

(since (b) is negative, $\frac{b}{4} > b$)

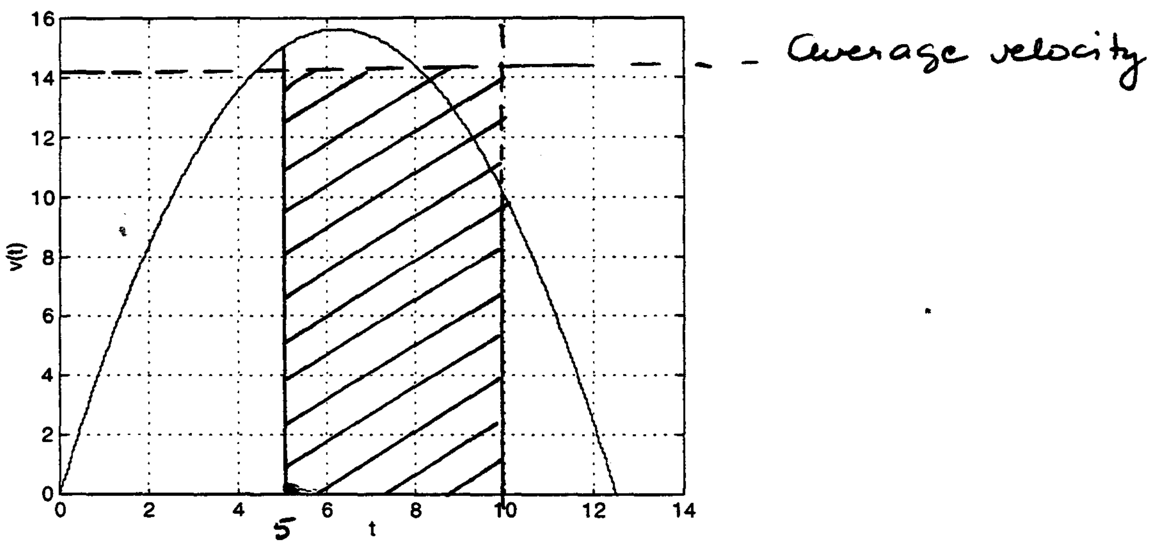
d) the average value of the rate of change of $f(x)$ over [-1, 4] $\dfrac{0}{4-(-1)} = 0$

AP CALCULUS
UNIT V – ACTIVITY 3

NAME Key
DATE

AVERAGE VELOCITY

It is the year 2025. You are navigating your space vehicle in rush hour air traffic. In moving from one "stationary" point to another, the velocity of your vehicle is given by the function $v(t) = -0.4t^2 + 5t$ (pictured below) which is measured in miles per minute.



1. Find your velocity at $t = 5$ and $t = 10$ minutes. Show that the graph agrees with these numbers.

$v(5) = 15$ and $v(10) = 10$

2. Write an expression for your displacement, $s(t)$, from $t = 5$ to $t = 10$ and evaluate it.

$s(t) = \int_5^{10} v(t)\,dt = 70.833 \text{ mi}$

3. a) If average velocity, $v_{av} = \frac{\text{change in position}}{\text{change in time}}$, write a **general equation** for v_{av} from $t = a$ to $t = b$.

$v(t)_{av} = \frac{\int_a^b v(t)\,dt}{b - a}$

b) Calculate your average velocity from $t = 5$ to $t = 10$.

14.167 mi/min

c) Represent your average velocity on the graph.

See graph.

4. If you were traveling at the average velocity from $t = 5$ to $t = 10$, represent the total distance traveled on the graph. What do you notice about this area and your answer from #2?

It is the shaded rectangle; it has an area of about 70.833 square units. Taking rounding into consideration, the area and the total distance are the same.

5. Does your average velocity equal the average of your initial and terminal velocities (from $t = 5$ to $t = 10$)? Justify.

No; $V_{av} = 14.167$

but $\frac{V_i + V_T}{2} = \frac{15 + 10}{2} = 12.5$

AP CALCULUS
UNIT V – ACTIVITY 4

NAME Key.
DATE

THE DEFINITE INTEGRAL
WHAT DOES IT MEAN?

1) If $f(t)$ is measured in miles per hour, and t is measured in hours, what are the units of $\int_a^b f(t)dt$?

$t \cdot f(t) \Rightarrow hr \cdot \frac{mi}{hr} = mi$

2) If $C(x)$ is measured in apples per bushel, and x is measured in bushels, what are the units of $\int_a^b C(x)dx$?

apples

3) A swimming pool is being filled at a rate of $R(t)$ gallons per minute, where t is in minutes. Write an integral expression to represent the total amount of water in the pool after being filled for half an hour.

$\int_0^{30} R(t)\,dt$

4) Ima Driver is a traveling calculator salesperson. With all that quiet time to think, Ima figures out a function, $C(t)$, to represent the number of calculators per day she sells. If t is measured in days, and $t = 0$ corresponds to September 1, 1999, interpret AND compare $\int_0^{30} C(t)dt$ and $\frac{1}{30-0}\int_0^{30} C(t)dt$. Be specific.

$\int_0^{30} C(t)dt$ represents the no. of calculators sold from Sept 1. to Sept. 30; $\frac{1}{30}\int_0^{30} C(t)dt$ is the average no. of calculators she sold per day during the same time period.

5) The value of a Star Wars lunch box, worth \$3 in 1977, increases at 12% per year. Its value in dollars, V, where t is time in years since 1977 is given by $V(t) = 3(1.12)^t$.

a) Find the value of the lunch box in 1999.

$V(22) = \$36.30$

b) Set up and evaluate an integral expression to find the average value of the lunch box over the period 1977 to 1999.

$\frac{1}{22}\int_0^{22} v(t)\,dt \approx 13.36$ dollars.

6) Stranded on a volcanic desert island with nothing better to do, you find that after an eruption, lava cools from a temperature of 2000°C to air temperature of 20°C, according to the equation $L(t) = 20+1880e^{-.01t}$, t is time in minutes since the eruption.

a) Find the temperature of the lava after 2 hours.

$$L(120) = 20 + 1880e^{-.01(120)} \doteq 586.245°C$$

b) Set up and evaluate an integral expression to find the average temperature of the lava during the second hour after the eruption.

$$\frac{1}{120-60}\int_{60}^{120} L(t)\,dt \doteq 795.868°C$$

7) $R(x)$ is the growth rate in thousands of bacteria per hour. Given the graph of R below, estimate

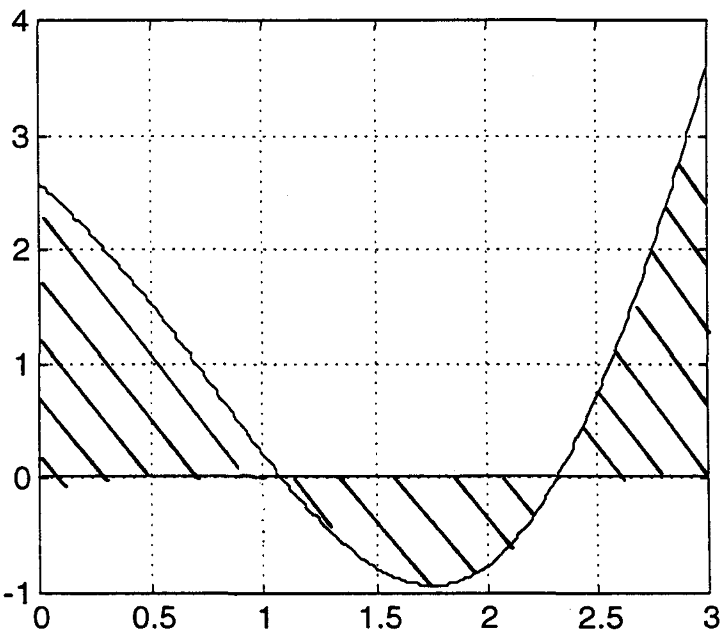
a) the average value of the growth rate of this bacteria colony over the first three hours. EXPLAIN/SHOW your reasoning.

↙ counting squares

$$\frac{1}{3-0}\int_0^3 R(x)\,dx \approx \frac{1}{3}(1.5-.75+1.2) = .65 \text{ thousand}$$

b) the total change in the number of bacteria in this colony after 3 hours.

$$\int_0^3 R(x)\,dx = 1.95 \text{ thous.}$$



R(x) is the growth rate of bacteria per hour - in thousands.

UNIT V

STUDENT WORKSHEETS

UNIT V

STUDENT WORKSHEETS

NAME ____________________
DATE ____________________

SETTING UP AND EVALUATING INTEGRAL EXPRESSIONS FOR AREA BETWEEN TWO CURVES

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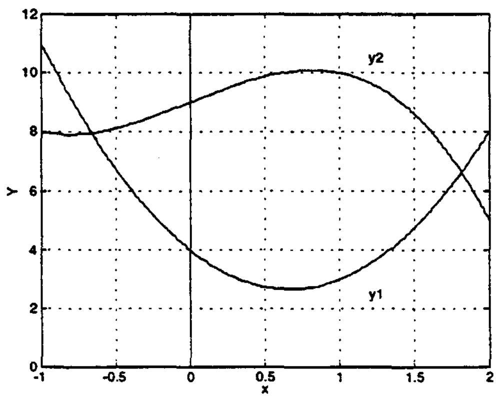

We will now find the points of intersection – these will serve as the limits of integration. In order to streamline the process, we will store the values for the limits using the calculator. Start by finding the left point of intersection. While the coordinates for the point of intersection are listed on the screen, return to the home screen and store the x value into the variable A.

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NAME ____________________
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For each system, sketch the graph and shade in the enclosed region. Then list the limits of integration, set up the integral, and evaluate the integral. Be sure that your final answer is correct to three decimal places.

1. $Y_1 = 4\sqrt{x} - 2$
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LIMITS OF INTEGRATION: A = ______________ B = ______________

WRITTEN INTEGRAL EXPRESSION: __________________________

VALUE OF INTEGRAL: __________________

2. $Y_1 = \sin(\frac{x}{3})$
   $Y_2 = x^2 - 3$

LIMITS OF INTEGRATION: A = ______________ B = ______________

WRITTEN INTEGRAL EXPRESSION: __________________________

VALUE OF INTEGRAL: __________________

3. $Y_1 = e^{3x}$

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NAME ____________________
DATE ____________________

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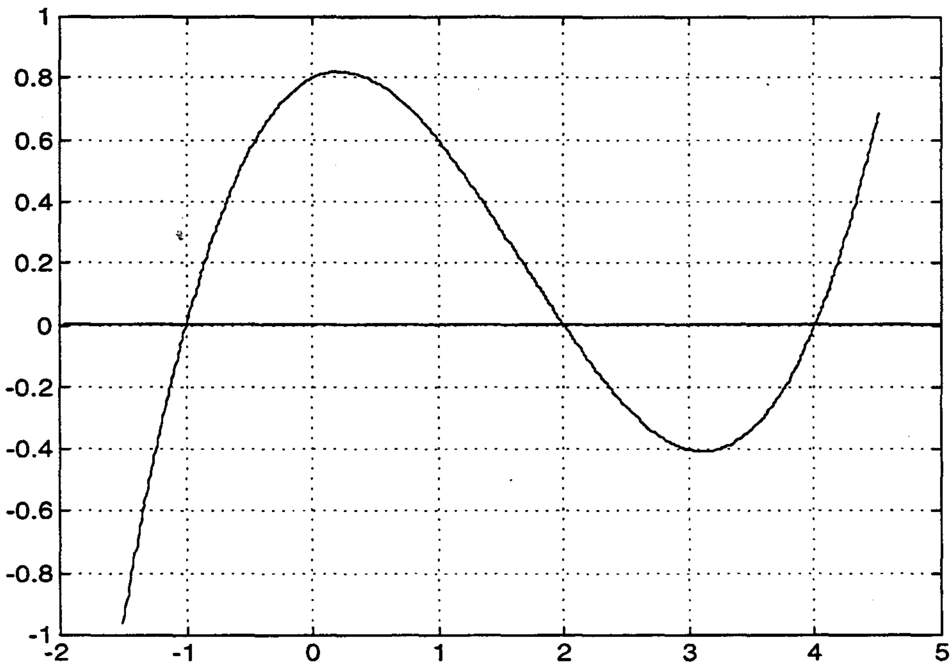

List from least to greatest.

a) $f'(-1)$

b) $\int_{-1}^{4} f(x)dx$

c) the average value of $f(x)$ over [-1, 4]

d) the average value of the rate of change of $f(x)$ over [-1, 4]

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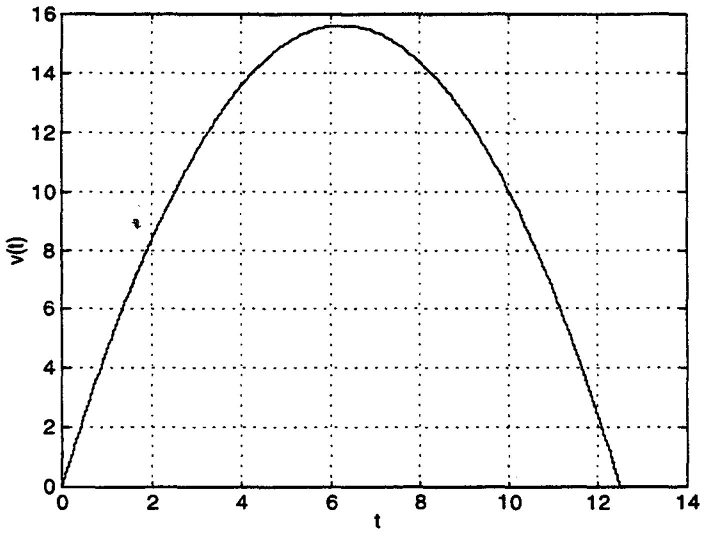

1. Find your velocity at $t = 5$ and $t = 10$ minutes. Show that the graph agrees with these numbers.

2. Write an expression for your displacement, $s(t)$, from $t = 5$ to $t = 10$ and evaluate it.

3. a) If average velocity, $v_{av} = \dfrac{\text{change in position}}{\text{change in time}}$, write a **general equation** for v_{av} from $t = a$ to $t = b$.

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NAME ______________________
DATE ______________________

THE DEFINITE INTEGRAL
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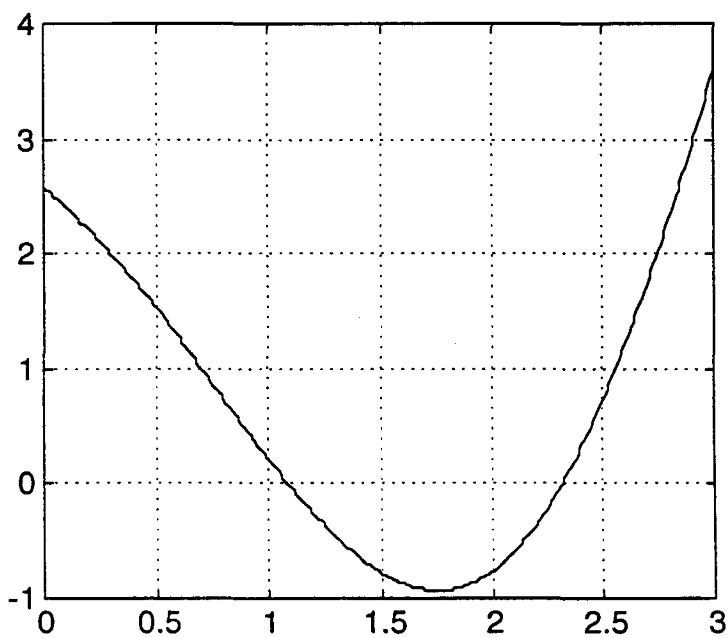


NAME ____________________
DATE ____________________

MULTIPLE CHOICE PRACTICE

A CALCULATOR MAY NOT BE USED ON THE FOLLOWING QUESTIONS

1. The area of the region enclosed by the graph of $y = x^2 + 2$ and the line $y = 6$ is

$A)\ 0 \qquad B)\ \frac{8}{3} \qquad C)\ \frac{16}{3} \qquad D)\ \frac{32}{3} \qquad E)\ \frac{48}{3}$

2. The average value of *sin* x on the interval [-2, 6] is

$A)\ \frac{\cos 2 + \cos 6}{8} \qquad B)\ \frac{\cos 6 - \cos 2}{8} \qquad C)\ \frac{-\cos 6 - \cos 2}{8}$

$D)\ \frac{-\cos 6 - \cos 2}{8} \qquad E)\ \frac{\cos 2 - \cos 6}{8}$

3. If the region enclosed by the y-axis, the line $y = 1$ and the curve $y = 2\sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

$A)\ \frac{\pi}{160}\, units^3 \qquad B)\ \frac{\pi}{80}\, units^3 \qquad C)\ \frac{\pi}{40}\, units^3 \qquad D)\ \frac{\pi}{8}\, units^3 \qquad E)\ 2\pi\, units^3$

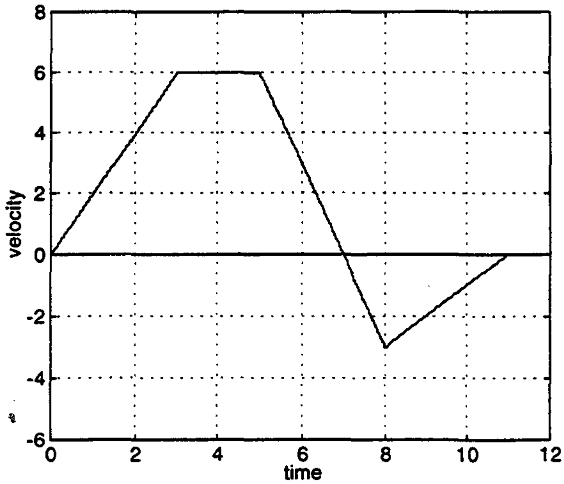
4. A particle moves along the x-axis so that its acceleration at any time t is $a(t) = 2t - 3$. If the initial velocity of the particle is 2, at what time t during the interval $0 \le t \le 2$ is the particle farthest to the right?

$A)\ 0 \qquad B)\ .5 \qquad C)\ 1 \qquad D)\ 1.5 \qquad E)\ 2$

5. The length of the curve described by the parametric equations $x = \cos^2 t$ and $y = \sin^3 t$ for $0 \le t \le \frac{\pi}{3}$ is given by

$A)\ \int_0^{\frac{\pi}{3}} \cos t \sin t \sqrt{4 + 9\sin^2 t}\, dt \qquad B)\ \int_0^{\frac{\pi}{3}} \cos t \sin t \sqrt{1 + \sin^2 t}\, dt \qquad C)\ \int_0^{\frac{\pi}{3}} \cos t \sin t \sqrt{4\cos^2 t + 9}\, dt$

$D)\ \int_0^{\frac{\pi}{3}} \cos t \sin t \sqrt{\cos^2 t + 1}\, dt \qquad E)\ 5\int_0^{\frac{\pi}{3}} \cos t \sin t\, dt$



A bird tries to hop across a branch at time t = 0. The velocity v of the bird at time t, $0 \le t \le 11$ is given by the function whose graph is shown above.

6. At what time does the bird change direction?

$A)\,3$ $B)\,5$ $C)\,7$ $D)\,8$ $E)\,11$

7. Which of the following is the best estimate of the total distance the bird travels from t = 0 to t = 11?

$A)\,9$ $B)\,16$ $C)\,23$ $D)\,27$ $E)\,33$

A GRAPHING CALCULATOR MAY BE REQUIRED ON SOME PARTS OF THE FOLLOWING QUESTIONS.

8. What is the area of the region in the first quadrant enclosed by the graphs of $y = 4\sin x$ and y = x?

$A)\,3.065$ $B)\,3.726$ $C)\,4.081$ $D)\,4.197$ $E)\,8$

9. The base of a solid S is the region in the first quadrant enclosed by the graph of $y = \sqrt{e^x}$, the line $x = \ln 2$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

$A)\,0.693$ $B)\,1$ $C)\,2$ $D)\,7.389$ $E)\,10.660$

10. At time $t \geq 0$, the acceleration of a particle moving on the x-axis is $a(t) = .25t + 34\cos(.7t)$. At t = 0, the velocity of the particle is 4. For what value of t will the velocity of the particle first be zero?

A) .169 B) 1.554 C) 4.687 D) 4.775 E) 4.778

11. The vertical position of a person riding an exciting amusement park ride is given by $y(t) = \frac{1}{3}\cos(6t) - \frac{1}{5}\sin(6t)$, where t is time in minutes. In the first three minutes of the ride, how many times is the vertical component of the velocity of the person equal to zero?

A) 4 B) 5 C) 6 D) 7 E) 8

12. When the region enclosed by the graphs of $y = \frac{1}{3}x$ and $y = 5x - x^2$ is revolved about the x-axis, the volume of the solid generated is given by

A) $\pi \int_0^{4.667} \left[(5x - x^2)^2 - (\frac{1}{3}x)^2 \right] dx$ B) $\pi \int_0^{4.667} \left[(5x - x^2 - \frac{1}{3}x)^2 \right] dx$ C) $-\pi \int_0^5 \left[(-5x + x^2 + \frac{1}{3}x)^2 \right] dx$

D) $\pi \int_0^5 \left[(5x - x^2)^2 - (\frac{1}{3}x)^2 \right] dx$ E) $\pi \int_0^5 \left[(5x - x^2 - \frac{1}{3}x)^2 \right] dx$

AP CALCULUS
UNIT V

NAME Key
DATE

MULTIPLE CHOICE PRACTICE

A CALCULATOR MAY NOT BE USED ON THE FOLLOWING QUESTIONS

1. The area of the region enclosed by the graph of $y = x^2 + 2$ and the line $y = 6$ is

(D)

$A)\ 0 \quad B)\ \frac{8}{3} \quad C)\ \frac{16}{3} \quad D)\ \frac{32}{3} \quad E)\ \frac{48}{3}$

2. The average value of *sin* x on the interval [-2, 6] is

(E)

$A)\ \frac{\cos 2 + \cos 6}{8} \quad B)\ \frac{\cos 6 - \cos 2}{8} \quad C)\ \frac{-\cos 6 - \cos 2}{8}$

$D)\ \frac{-\cos 6 - \cos 2}{8} \quad E)\ \frac{\cos 2 - \cos 6}{8}$

3. If the region enclosed by the y-axis, the line $y = 1$ and the curve $y = 2\sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

(B)

$A)\ \frac{\pi}{160} units^3 \quad B)\ \frac{\pi}{80} units^3 \quad C)\ \frac{\pi}{40} units^3 \quad D)\ \frac{\pi}{8} units^3 \quad E)\ 2\pi\, units^3$

4. A particle moves along the x-axis so that its acceleration at any time t is $a(t) = 2t - 3$. If the initial velocity of the particle is 2, at what time t during the interval $0 \le t \le 2$ is the particle farthest to the right?

(C)

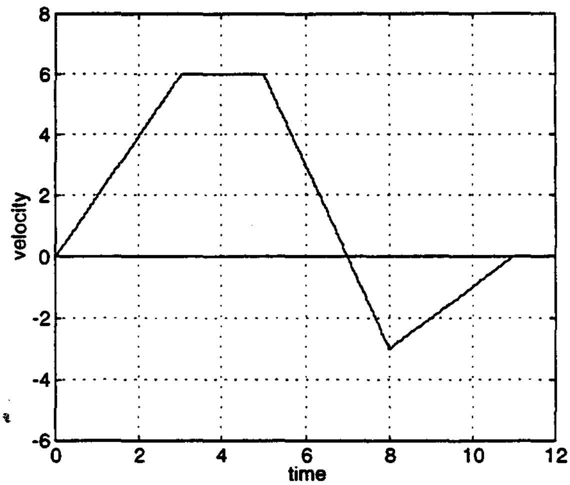
$A)\ 0 \quad B)\ .5 \quad C)\ 1 \quad D)\ 1.5 \quad E)\ 2$

5. The length of the curve described by the parametric equations $x = \cos^2 t$ and $y = \sin^3 t$ for $0 \le t \le \frac{\pi}{3}$ is given by

(A)

$A)\ \int_0^{\frac{\pi}{3}} \cos t \sin t \sqrt{4 + 9\sin^2 t}\, dt \quad B)\ \int_0^{\frac{\pi}{3}} \cos t \sin t \sqrt{1 + \sin^2 t}\, dt \quad C)\ \int_0^{\frac{\pi}{3}} \cos t \sin t \sqrt{4\cos^2 t + 9}\, dt$

$D)\ \int_0^{\frac{\pi}{3}} \cos t \sin t \sqrt{\cos^2 t + 1}\, dt \quad E)\ 5\int_0^{\frac{\pi}{3}} \cos t \sin t\, dt$



A bird tries to hop across a branch at time t = 0. The velocity v of the bird at time t, $0 \le t \le 11$ is given by the function whose graph is shown above.

6. At what time does the bird change direction?

A) 3 *B*) 5 *C*) 7 *D*) 8 *E*) 11

7. Which of the following is the best estimate of the total distance the bird travels from t = 0 to t = 11?

A) 9 *B*) 16 *C*) 23 *D*) 27 *E*) 33

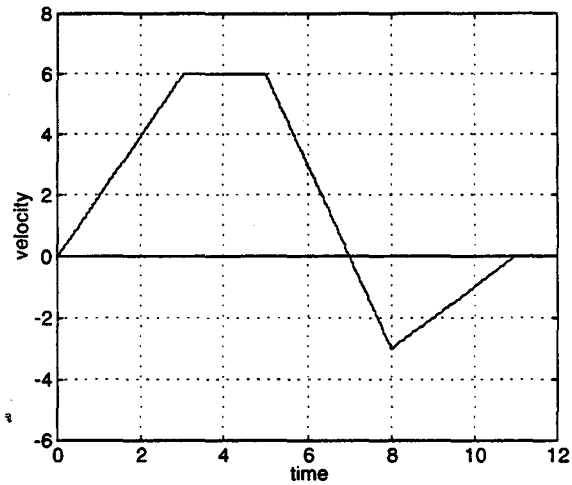
A GRAPHING CALCULATOR MAY BE REQUIRED ON SOME PARTS OF THE FOLLOWING QUESTIONS.

8. What is the area of the region in the first quadrant enclosed by the graphs of $y = 4\sin x$ and y = x?

A) 3.065 *B*) 3.726 *C*) 4.081 *D*) 4.197 *E*) 8

9. The base of a solid S is the region in the first quadrant enclosed by the graph of $y = \sqrt{e^x}$, the line $x = \ln 2$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

A) 0.693 *B*) 1 *C*) 2 *D*) 7.389 *E*) 10.660



A bird tries to hop across a branch at time t = 0. The velocity v of the bird at time t, $0 \le t \le 11$ is given by the function whose graph is shown above.

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(C)

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(E)

A GRAPHING CALCULATOR MAY BE REQUIRED ON SOME PARTS OF THE FOLLOWING QUESTIONS.

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9. The base of a solid S is the region in the first quadrant enclosed by the graph of $y = \sqrt{e^x}$, the line $x = \ln 2$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

(B)

$A)\ 0.693 \qquad B)\ 1 \qquad C)\ 2 \qquad D)\ 7.389 \qquad E)\ 10.660$

10. At time $t \geq 0$, the acceleration of a particle moving on the x-axis is $a(t) = .25t + 34\cos(.7t)$. At t = 0, the velocity of the particle is 4. For what value of t will the velocity of the particle first be zero?

(C)

A) .169 B) 1.554 C) 4.687 D) 4.775 E) 4.778

11. The vertical position of a person riding an exciting amusement park ride is given by $y(t) = \frac{1}{3}\cos(6t) - \frac{1}{5}\sin(6t)$, where t is time in minutes. In the first three minutes of the ride, how many times is the vertical component of the velocity of the person equal to zero?

(B)

A) 4 B) 5 C) 6 D) 7 E) 8

12. When the region enclosed by the graphs of $y = \frac{1}{3}x$ and $y = 5x - x^2$ is revolved about the x-axis, the volume of the solid generated is given by

(A)

A) $\pi \int_0^{4.667} \left[(5x - x^2)^2 - (\frac{1}{3}x)^2 \right] dx$ B) $\pi \int_0^{4.667} \left[(5x - x^2 - \frac{1}{3}x)^2 \right] dx$ C) $-\pi \int_0^{5} \left[(-5x + x^2 + \frac{1}{3}x)^2 \right] dx$

D) $\pi \int_0^{5} \left[(5x - x^2)^2 - (\frac{1}{3}x)^2 \right] dx$ E) $\pi \int_0^{5} \left[(5x - x^2 - \frac{1}{3}x)^2 \right] dx$

1.

x	1	4	6	9
f(x)	4	8	6	2

If f(x) is continuous on [1, 9], approximate $\int_1^9 f(x)dx$ using 3 trapezoids?

2. Evaluate: $\lim_{h \to 0} \frac{\cot(5x+h)-\cot(5x)}{h}$

3. Evaluate: $\int \frac{x^2+3}{x}dx$

4. Given: $xy - e^{2y} = 7$
   Find: y'

5. Evaluate: $\lim_{x \to 2} \frac{\int_2^x e^t dt}{x^2-3}$

1.

x	1	4	6	9
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ANSWERS:

1.
$3\cdot\frac{4+8}{2}+2\cdot\frac{8+6}{2}+3\cdot\frac{6+2}{2}$

$18+14+12$

$=44$

2.
$f(x)=\cot 5x$

$f'(x)=-5\csc^2 5x$

3.
$\int (x+\frac{3}{x})dx$

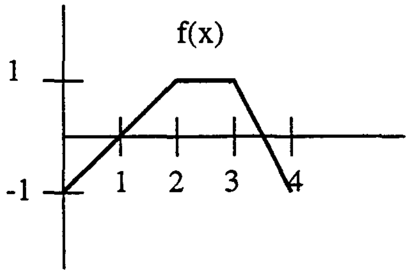
$=\frac{x^2}{2}+3\ln x + C$

4.
$xy'+y(1)-2\frac{dy}{dx}e^{2y}=0$

$y'=\frac{dy}{dx}=\frac{-y}{x-2e^{2y}}$

5. Use L's Rule
$\lim_{x\to 2}\frac{e^x}{2x}=\frac{e^2}{4}$

1. Given:



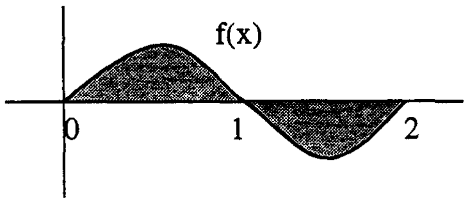
Find: $\int_0^3 f(x)dx =$ _____

2. How fast is a 10-foot ladder sliding down a wall if its base is moving at a rate of 5 ft/sec when it is 8 feet away from the wall?

3. Differentiate: $f(x) = \sqrt{e^x + 2}$

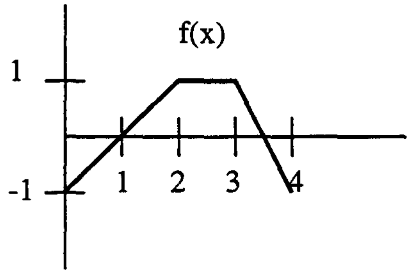
4. Write the definition of the derivative of f(x) in terms of a limit.

5. Set up an integral to express the shaded region below.



AP CALCULUS
UNIT V – WARMUP SET 2

1. Given:



Find: $\int_0^3 f(x)dx =$ ____

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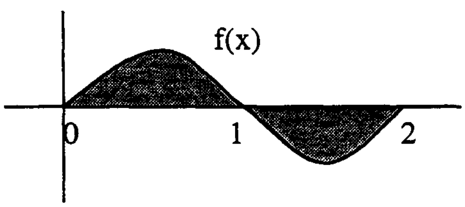
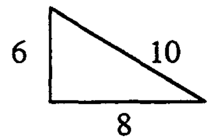


ANSWERS:

1.
$$-\frac{1}{2}+\frac{3}{2}=1$$

2.



$x^2 + y^2 = 10^2$

$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$

$2(6)\frac{dx}{dt} + 2(8)5 = 0$

$\frac{dx}{dt} = -\frac{20}{3}$ ft/sec

3. $f(x) = (e^x + 2)^{\frac{1}{2}}$

$$f'(x) = \frac{e^x}{2\sqrt{e^x+2}}$$

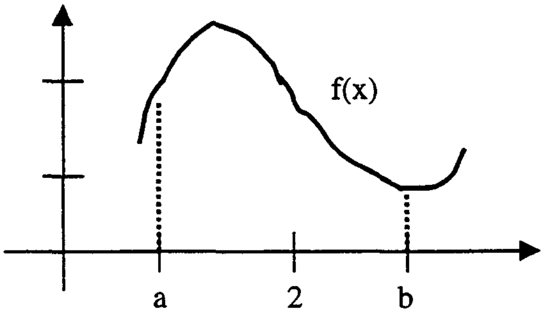
4.
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

5.
$$\int_0^1 f(x)dx - \int_1^2 f(x)dx$$

1. $\int \sin^2 x \cos x \, dx$

2. If $F(x) = \int_5^{\arctan x} \sqrt{1+e^{2t}}\, dt$,
then $F'(x) =$ ____________

3.



Given the graph above, draw and shade a rectangle with base (b-a) to approximate the area of
$\int_a^b f(x)dx$

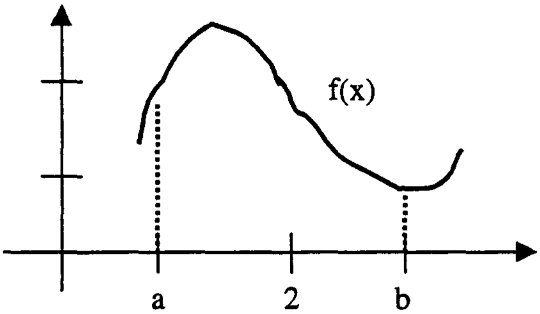
4. Evaluate: $\lim_{x\to 1} \dfrac{x^2 - 4x + 3}{\ln x}$

5. Sketch a graph showing: $\int_0^4 \sqrt{x}\, dx$

1. $\int \sin^2 x \cos x \, dx$

2. If $F(x) = \int_5^{\arctan x} \sqrt{1+e^{2t}}\, dt$,
then $F'(x) =$ ____________

3.



Given the graph above, draw and shade a rectangle with base (b-a) to approximate the area of $\int_a^b f(x)dx$

4. Evaluate: $\lim_{x \to 1} \frac{x^2 - 4x + 3}{\ln x}$

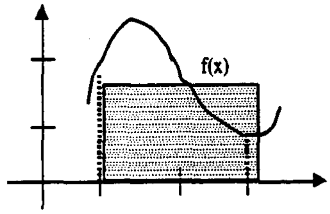
5. Sketch a graph showing: $\int_0^4 \sqrt{x}\, dx$

ANSWERS:

1. u = sin x
du = cosx dx
$\therefore \int u^2 du$
$= \frac{\sin^3 x}{3} + C$

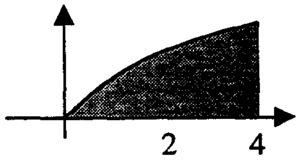
2.
$\frac{\sqrt{1+e^{2\arctan x}}}{1+x^2}$

3.



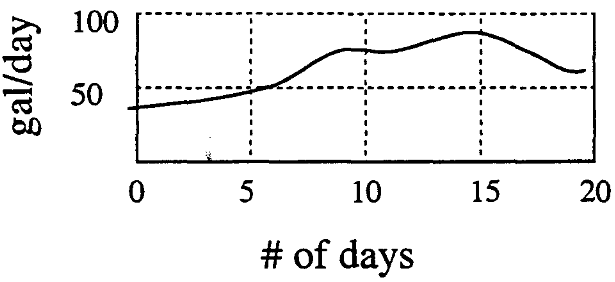
4.Use L's Rule
$\lim_{x \to 1} \frac{2x-4}{\frac{1}{x}}$
$= -2$

5.



1. The following graph represents the flow of maple syrup during a 20 day interval:

Maple Syrup Flow



Approximate the total number of gallons flowing during the 20 days shown.

2. $\int (\cos^2 x + \sin^2 x)dx =$

3. Evaluate: $\lim_{x \to \frac{\pi}{6}} \frac{\sin x - \frac{1}{2}}{x - \frac{\pi}{6}}$

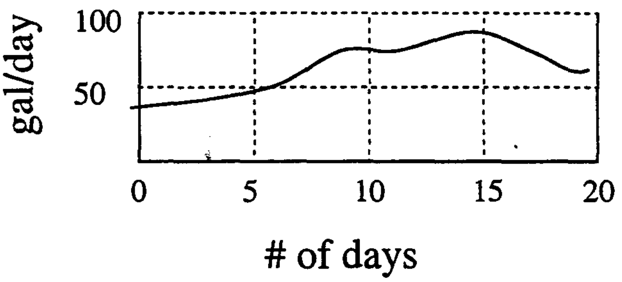
4. Σ g(x) Δx is called a(n) ________

5. $\int g(x)\,dx$ is called a(n) ____________

ANSWERS:

1. The following graph represents the flow of maple syrup during a 20 day interval:

Maple Syrup Flow



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4. Σ g(x) Δx is called a(n) ________

5. $\int g(x)\,dx$ is called a(n) ________

ANSWERS:

1. $5(5 \cdot 50)$
$= 1250$ gal

2. $\int 1\,dx$
$= x + C$

3. $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

4. Riemann Sum

5. indefinite integral (antiderivative)

1. Set up an integral to find the length of curve f(x) = csc x on [1, 3]

2. $\int 96x^{1.4}dx$

3. What is the domain of
$y = 5 - |x + 2|$

4. $F(x) = \int_1^{x^2} \cos^{-1} t \, dt$
$F'(x) =$ ________

5. Given: $f(x) = e^{\sin x}$
Set up an integral to represent the average function value from x = 2 to x = 5

1. Set up an integral to find the length of curve f(x) = csc x on [1, 3]

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3. What is the domain of $y = 5 - |x+2|$

4. $F(x) = \int_1^{x^2} \cos^{-1} t \, dt$
   $F'(x) =$ ________

5. Given: $f(x) = e^{\sin x}$
   Set up an integral to represent the average function value from x = 2 to x = 5

ANSWERS:

1. $\int_1^3 \sqrt{1+\csc^2 x \cdot \cot^2 x}dx$

2. $40x^{2.4} + C$

3. $\Re$, x ≠ -2

4. $\cos^{-1}(x^2)(2x)$
   $= 2x \cdot \cos^{-1}(x^2)$

5. $f_{av} = \frac{\int_2^5 (e^{\sin x})dx}{3}$

UNIT V INTERNET RESOURCES

http://www.netsrq.com/~hahn/calculus.html, http://www.barzilai.org/archive, http://archives.math.utk.edu.visual.calculus/ and http://www.hofstra.edu/~matscw/RealWorld/index.html

Sites feature some tutorials, but mostly have good drill and quiz resources for either in-class practice or at-home practice.

http://www.integrals.com

Site features an integral calculator (indefinite integrals) and has a history of integration.

http://www.library.thinkquest.org/3616/Calc/S3/TDM.html

Animated demonstration of the disc method for calculating volume

http://www.math.odu.edu/cbii/calcanim/

Site has a nice animation depicting the calculation of arc length by using line segments.