UNIT IV

UNIT IV
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UNIT IV: THE INTEGRAL

Expectation:	The student will interpret the integral as an accumulation function and the limit of Reimann Sums as well as evaluate integrals.

OVERVIEW:

Students will explore different approaches to defining and calculating areas. The Fundamental Theorem of Calculus will be used to relate derivatives and antiderivatives.

INDICATORS:

1. Express the area under a curve as a limit of Trapezoidal Sums and Riemann Sums.
2. Compute Trapezoidal Sums and Riemann Sums using left, right and midpoint evaluation points.
3. Determine antiderivatives following directly from derivatives of basic functions.
4. Determine antiderivatives by substitution of variables ($\int f'(g(x))g'(x)\,dx$).
5. Apply the basic properties of integrals (for example, additivity and linearity).
6. Demonstrate an understanding of the Fundamental Theorem of Integral Calculus and use it to evaluate definite integrals.
7. Interpret and apply the integral as an accumulation function both analytically and geometrically.
8. Demonstrate an understanding of the definite integral of a rate of change of a quantity over an interval as the change of the quantity over the interval.
9. Apply geometric and numerical methods to compute approximate values of a given definite integral represented algebraically, geometrically, or by tables of values.

Unit IV: The Integral

Indicators/ Objectives	Foerster: Calculus Key Curriculum 1998	Foerster: Calculus: Instructor's Resource Book Key Curriculum 1998	Finney, et al: Calculus S F A W 1999	Guide Pages
1	195-196	1-4, 5-5	258-260, 289-290	
2	196-199; 18-20	1-4, 5-5	247-250, 290-291	IV 1-2, 6-8
3	119-121	3-9	190-191	
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6	215-217, 255	70-74	277-283	.
7	225-226	74	283-284	IV 9
8	225-226, 254-255		363-366	
9	196-199, 18-20	10-2, 5-5, 5-7	247-253, 290-291	

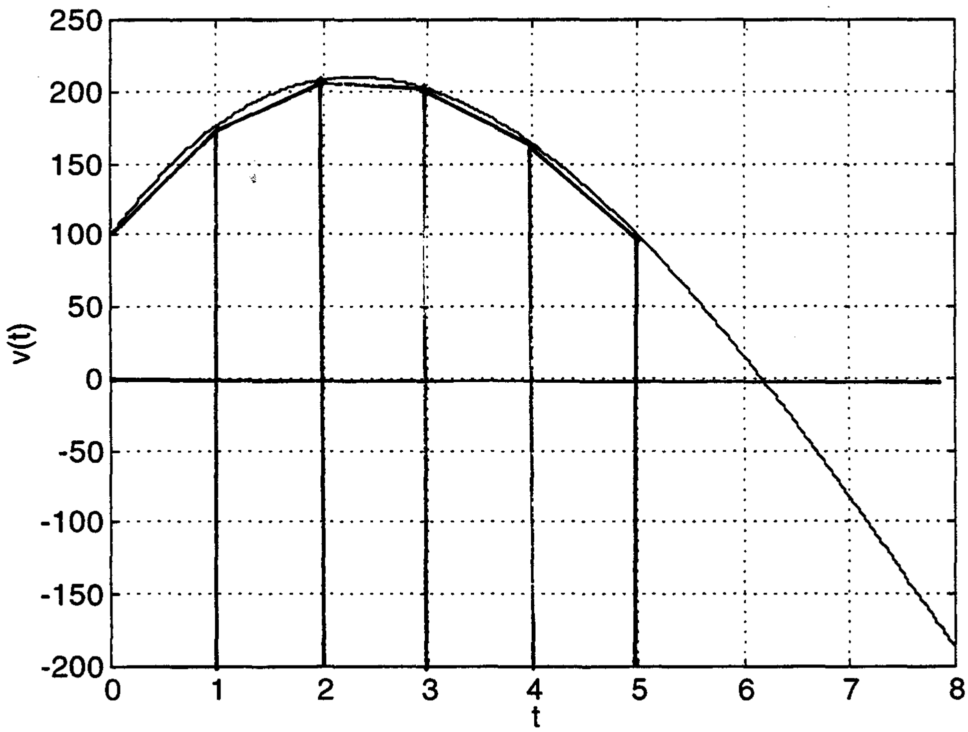
Indicators/ Objectives	Finney, et al: Calculus 1994	Guide Pages
1	361-336, 414-415	
2	362 - 365, 415 - 416	
3	394 - 412, 613 - 620	
4	405-410	
5	397 – 398	
6	380-386	
7	433	
8		
9	362-365, 415-416	

AP CALCULUS
UNIT IV - ACTIVITY 1

NAME Key
DATE

ESTIMATING AREA USING TRAPEZOIDAL RULE

Austin Powerless is traveling in his space ship on another "groovy" tour of duty. At $t=0$ (t being time in minutes), he fires his engines. His speed increases until he notices a ship from the planet Amazonia stalled up ahead. He slows down to help repair the ship. The graph of Austin's velocity, $v(t)$ miles per minute, is below.



1. How could you estimate the distance Austin traveled from $t = 0$ to $t = 5$ minutes using the graph? About how far did he travel in that time?

   Estimate the area under the curve.
   He traveled about 840 miles

2. Divide the region under the graph from $t = 0$ to $t = 5$ into five strips of equal width. Draw trapezoids that approximate the areas of these strips (the bases of the trapezoids should be parallel to the y-axis). To estimate Austin's distance traveled, find the sum of the areas of the trapezoids. Does your answer agree with your estimation in #1? See graph.

   distance $\approx (.5)(1)[(100+175)+(175+210)+(210+200)+(200+165)+(165+100)]=850$

   Yes. The answers are close.

3. If you were to use 10 trapezoids of equal width instead of five, how would this approximation compare to the area using five trapezoids? To the actual distance traveled? What would the width of each trapezoid be? Approximate Austin's distance traveled using 10 trapezoids.

10 trapezoids should give a closer approximation. The width of each trapezoid should be 0.5 units.

distance ≈ (.5)(.5)[100 + 290 + 350 + 400 + 420 + 420 + 400 + 360 + 330 + 270 + 100] = 860

4. Write a definite integral that represents the distance Austin traveled in the first five seconds. Write an expression to approximate this definite integral by adding the areas of five trapezoids of equal width (follow #2 as a guide).

$$\text{distance} = \int_0^5 v(t)\,dt$$

$$\approx (.5)(1)\left[v(0) + 2v(1) + 2v(2) + 2v(3) + 2v(4) + v(5)\right]$$

5. A **general** equation for approximating a definite integral by adding areas of trapezoids of equal width using *v(t)* as the function defining the curve, *a* and *b* as starting and ending values of *t*, and *n* as the number of trapezoids is

$$\int_a^b v(t)\,dt \approx .5\Delta x[v(a) + 2v(a+\Delta x) + 2v(a+2\Delta x) + ... + 2v(a+(n-1)\Delta x) + v(a+n\Delta x)]$$

Define Δx in terms of *a*, *b*, and *n*.

$\Delta x = \dfrac{b-a}{n}$, n is the number of subdivisions of interval [a,b]; a and b are the endpoints of the interval.

Explain how to generate the terms in the brackets.

Read from graph.

6. The <u>exact</u> value of the definite integral is the <u>limit</u> of the area by trapezoids as the width of each trapezoid approaches zero. Austin's velocity is given by the function $v(t) = t^3 - 25t^2 + 100t + 100$. Evaluate the definite integral to find the actual distance Austin traveled from $t = 0$ to $t = 5$.

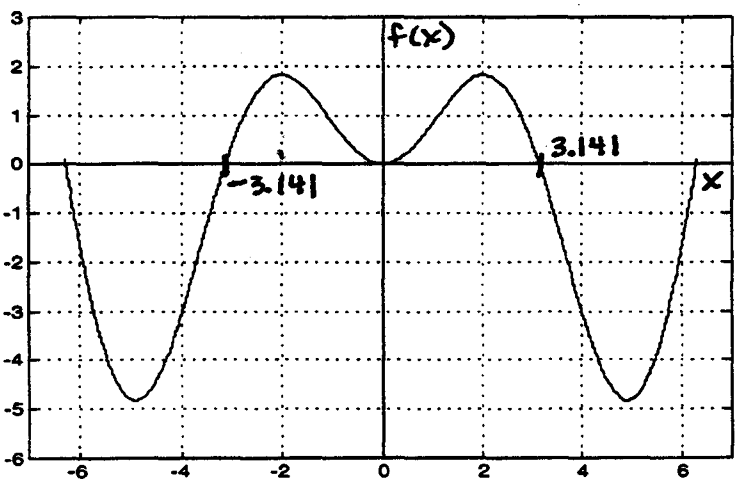
$$\int_0^5 (t^3 - 25t^2 + 100t + 100)\,dt \approx 864.583$$

AP CALCULUS
UNIT IV – ACTIVITY 2

NAME Key
DATE ______

INTRODUCTION TO THE PROPERTIES OF THE DEFINITE INTEGRAL

Given $\int_0^{3.141} f(x)\,dx = 3.245$, $\int_0^{2} f(x)\,dx = 1.745$ and the graph of a function, $f(x)$, below



Find

1) $\int_2^{3.141} f(x)dx$ 3.245 − 1.745 = 1.5

2) $\int_{-3.141}^{2} f(x)dx$ 3.245 + 1.745 = 4.99

3) $\int_2^{-2} f(x)dx$ 2(−1.745) = − 3.49

NAME Key
DATE

PROPERTIES OF THE DEFINITE INTEGRAL

1) Given that:

$$\int_{-1}^{6} f(x)dx = 7,\ \int_{-1}^{6} g(x)dx = 11,\ \text{and}\ \int_{-1}^{6} h(x)dx = -5$$

evaluate the following using the properties of the definite integral. BE WARNED! Some of these are not possible; briefly explain why.

a) $\int_{-1}^{6} (g(x)+h(x))dx$ $\quad 11 + (-5) = 6$

b) $\int_{-1}^{6} (2f(x)-g(x))dx$ $\quad 2(7) - 11 = 3$

c) $\int_{-1}^{6} g(x)f(x)dx$ $\quad$ cannot be determined

d) $\int_{-1}^{6} (f(x)+2)dx$ $\quad 7 + 14 = 21$

e) $\int_{6}^{-1} \frac{h(x)}{5}dx$ $\quad \frac{1}{5}\left(-(-5)\right) = 1$

f) $\int_{-1}^{6} \frac{g(x)}{f(x)}dx$ $\quad$ cannot be determined

2) Given that $\int_{a}^{b} h(x)dx = 2a-b$, find $\int_{a}^{b} (h(x)+3)dx$.

$$2a-b + \int_a^b 3\,dx = 2a-b + (3b-3a) = -a+2b$$

3) For an **odd** function g, suppose you know that $\int_0^3 g(x)dx = 4$ and $\int_0^5 g(x)dx = -2$.

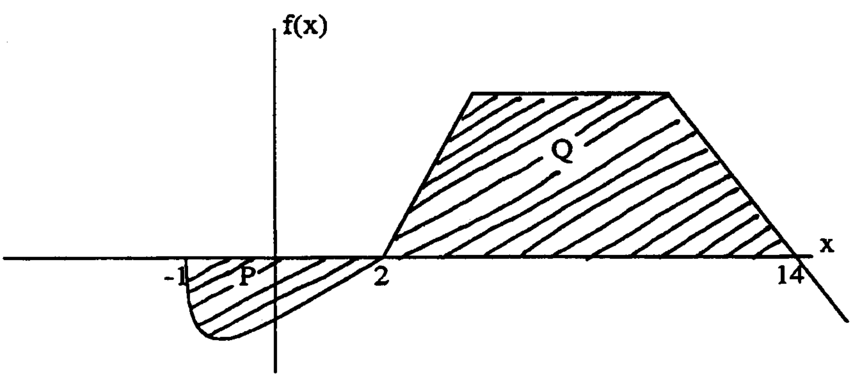
Find:

a) $\int_{-3}^{3} g(x)dx$ 0

b) $\int_{-5}^{-3} g(x)dx$ 6

c) $\int_{-3}^{0} (g(x)+2x)dx$ −13

4) For the graph of y = f(x) shown below, P and Q are positive integers that represent the area of the shaded regions as indicated.



In terms of P and Q, find $3\int_2^{14} f(x)dx - \int_{-1}^{14} f(x)dx$.

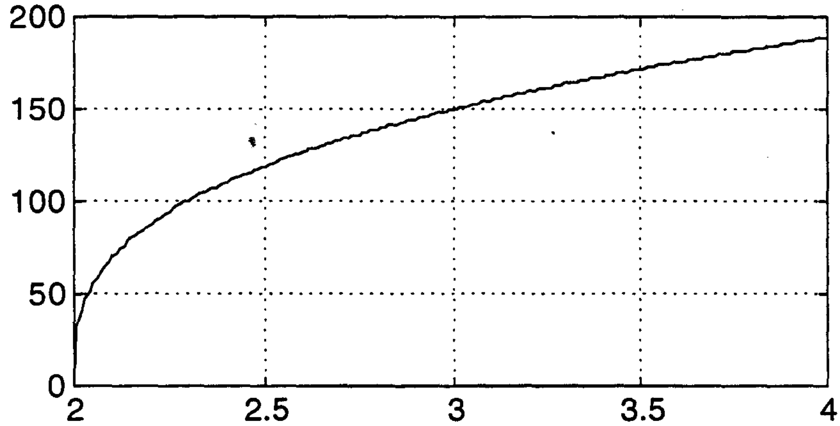
$3Q - (-P + Q) = 2Q + P$

AP CALCULUS
UNIT IV – ACTIVITY 4

NAME Key
DATE ______

APPROXIMATING DEFINITE INTEGRALS

1) Harry, Joe and Burley were all asked to give their best estimate for $\int_2^4 f(x)dx$ where f(x) is the function graphed below.



Here are their answers:

Harry:	400
Joe:	200
Burley:	-375.8

Clearly these guys are a bunch of stooges! Explain why each one is wrong in his estimate, AND give a better estimate. Show your work.

Harry: gave area of entire grid

Joe : gave area of half the grid

Burley: gave a negative area; the pictured area is not negative.

A better estimate is 275 (by counting the rectangles)

2) You are jet skiing on the Chesapeake Bay–smile on your face, the wind in your hair- when suddenly the engine stalls. Your best friend, who loves math, records your velocities as you glide to a stop:

Time since stall in seconds	0	2	4	6	8	10
Speed (ft/sec)	60	30	22	12	4	0

a) Use left- and right-hand rules to estimate how far the jet ski travels after the engine stalls. Show your reasoning.

$\Delta x = 2$; U: $2(60+30+22+12+4) = 256$; L: $2(30+22+12+4+0) = 136$

b) Unbeknownst to you, there is a huge rock just below the surface of the water directly in front of you 200 ft from where the engine stalled. Are you in any danger? EXPLAIN.

A better estimate: 196, using midpoint values.
You <u>are</u> in danger; it is a very close call!

3) The Air Force is testing its brand new, top secret heli-plane. The pilot takes off and heads due south. As planned, she radios her airspeed to the control tower every minute. After 5 minutes, the plane mysteriously disappears from the radar screen, and the radio transmissions stop. The pilot and the plane must be located quickly (to save the pilot) and quietly (to avoid a media "situation"). Assuming the plane crashed after the last contact, what is your <u>BEST</u> estimate of how far south of the point of takeoff the crash site is?

Minutes since take off	0	1	2	3	4	5
Speed (mph)	75	90	120	145	160	175

Using midpoint values, the plane crashed at about $10\frac{2}{3}$ mi after take off.

$\Delta x = 1$; $\dfrac{(82.5+105+132.5+152.5+167.5)}{60}$ mi.

AP CALCULUS
UNIT IV – ACTIVITY 5

NAME Key
DATE

APPLICATIONS OF THE DEFINITE INTEGRAL

1. The table below shows the velocity of a car, measured in km/s for 12 seconds.

Time (seconds), t	**Velocity** (km/s), V(t)
0	0
1	0.717
2	1.038
3	1.283
4	1.453
5	1.547
6	1.566
7	1.509
8	1.377
9	1.169
10	0.887
11	0.528
12	0.094

a. Use a midpoint Riemann Sum with four subdivisions of equal width to approximate $\int_0^{12} V(t)dt$. Using correct units, explain the meaning of your answer.

$\approx 3(1.038 + 1.547 + 1.377 + 0.528) = 13.47$ km

$s\left(\frac{km}{s}\right) = km$

In 12 seconds, the car traveled approximately 13.47 km.

b. Is there some time t, $0 < t < 12$, such that $V'(t) = 0$. State the significance of this.

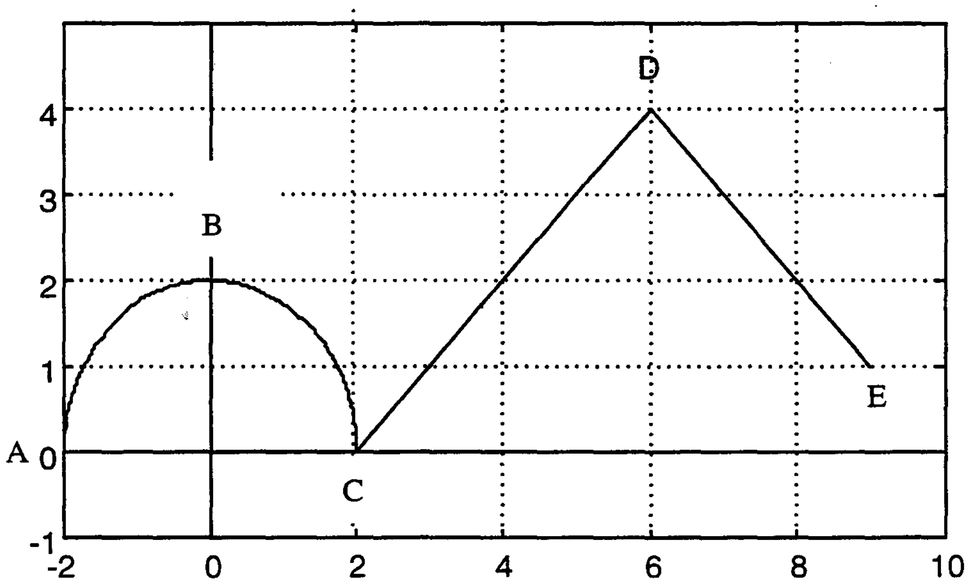
Yes, around $t = 6$; it is the maximum value of V.

c. $V(t)$, the rate of travel, can be approximated by $U(t) = \frac{1}{53}(-2t^2 + 23t + 17)$. Evaluate $\int_0^{12} U(t)dt$. Indicate units of measure. Compare your answer to #1a and explain its significance.

$\int_0^{12} u(t)\,dt \approx 13.358$ km

The answer is very close to #1a, which is an estimation

2. Given the graph of f, answer the following questions. Note that the graph is NOT drawn to scale; it consists of a semicircle and two line segments.



a. Let $F(x) = \int_2^x f(t)dt$. Compute $F(-2)$, $F(2)$ and $F(6)$.

$F(-2) = -\frac{1}{2}(\pi \cdot 2^2) = -2\pi$

$F(2) = 0$

$F(6) = 8$

b. Find the instantaneous rate of change of F, with respect to x, at $x = 4$.

$F'(4) = 2 \qquad f(x) = F'(x)$

c. Find the absolute maximum value of F on the closed interval [-2, 9]. Justify your answer.

The maximum is at $x=9$. Although F is decreasing from $x=2$ to $x=-2$, it is increasing from $x=2$ to $x=9$

d. At what value(s) of x does F have a point (or points) of inflection? Justify your answer.

$f'(x) = F''(x)$ where $f'(x)$ changes from decreasing to increasing, or from increasing to decreasing, there is a point of inflection. This happens at $x=0,2,6$.

UNIT IV

STUDENT WORKSHEETS

AP CALCULUS
UNIT IV - ACTIVITY 1

NAME ____________________
DATE ____________________

ESTIMATING AREA USING TRAPEZOIDAL RULE

Austin Powerless is traveling in his space ship on another "groovy" tour of duty. At $t=0$ (t being time in minutes), he fires his engines. His speed increases until he notices a ship from the planet Amazonia stalled up ahead. He slows down to help repair the ship. The graph of Austin's velocity, $v(t)$ miles per minute, is below.

1. How could you estimate the distance Austin traveled from $t = 0$ to $t = 5$ minutes using the graph? About how far did he travel in that time?

2. Divide the region under the graph from $t = 0$ to $t = 5$ into five strips of equal width. Draw trapezoids that approximate the areas of these strips (the bases of the trapezoids should be parallel to the y-axis). To estimate Austin's distance traveled, find the sum of the areas of the trapezoids. Does your answer agree with your estimation in #1?

3. If you were to use 10 trapezoids of equal width instead of five, how would this approximation compare to the area using five trapezoids? To the actual distance traveled? What would the width of each trapezoid be? Approximate Austin's distance traveled using 10 trapezoids.

4. Write a definite integral that represents the distance Austin traveled in the first five seconds. Write an expression to approximate this definite integral by adding the areas of five trapezoids of equal width (follow #2 as a guide).

5. A **general** equation for approximating a definite integral by adding areas of trapezoids of equal width using $v(t)$ as the function defining the curve, a and b as starting and ending values of t, and n as the number of trapezoids is

$$\int_a^b v(t)\,dt \approx .5\Delta x[v(a)+2v(a+\Delta x)+2v(a+2\Delta x)+...+2v(a+(n-1)\Delta x)+v(a+n\Delta x)]$$

Define Δx in terms of a, b, and n.

Explain how to generate the terms in the brackets.

6. The <u>exact</u> value of the definite integral is the <u>limit</u> of the area by trapezoids as the width of each trapezoid approaches zero. Austin's velocity is given by the function $v(t)=t^3-25t^2+100t+100$. Evaluate the definite integral to find the actual distance Austin traveled from $t = 0$ to $t = 5$.

NAME ____________________
DATE ____________________

INTRODUCTION TO THE PROPERTIES OF THE DEFINITE INTEGRAL

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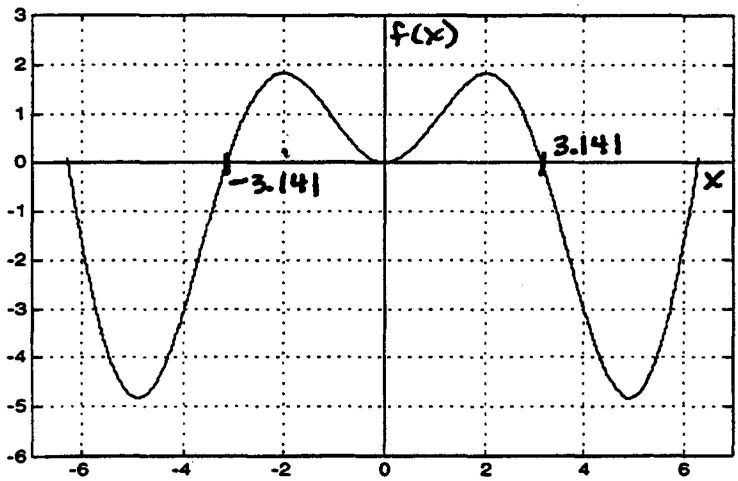

Find

1) $\int_2^{3.141} f(x)dx$

2) $\int_{-3.141}^{2} f(x)dx$

3) $\int_2^{-2} f(x)dx$

NAME ______________________
DATE ______________________

PROPERTIES OF THE DEFINITE INTEGRAL

1) Given that:

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evaluate the following using the properties of the definite integral. BE WARNED! Some of these are not possible; briefly explain why.

a) $\int_{-1}^{6} (g(x) + h(x))dx$

b) $\int_{-1}^{6} (2f(x) - g(x))dx$

c) $\int_{-1}^{6} g(x)f(x)dx$

d) $\int_{-1}^{6} (f(x) + 2)dx$

e) $\int_{6}^{-1} \frac{h(x)}{5}dx$

f) $\int_{-1}^{6} \frac{g(x)}{f(x)}dx$

2) Given that $\int_{a}^{b} h(x)dx = 2a - b$, find $\int_{a}^{b} (h(x) + 3)dx$.

3) For an **odd** function g, suppose you know that $\int_0^3 g(x)dx = 4$ and $\int_0^5 g(x)dx = -2$.

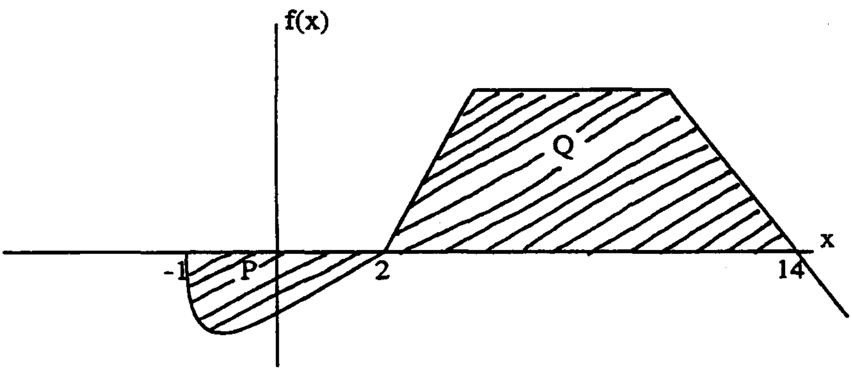
Find:

a) $\int_{-3}^{3} g(x)dx$

b) $\int_{-5}^{-3} g(x)dx$

c) $\int_{-3}^{0} (g(x)+2x)dx$

4) For the graph of y = f(x) shown below, P and Q are positive integers that represent the area of the shaded regions as indicated.



In terms of P and Q, find $3\int_2^{14} f(x)dx - \int_{-1}^{14} f(x)dx$.

NAME ____________________
DATE ____________________

APPROXIMATING DEFINITE INTEGRALS

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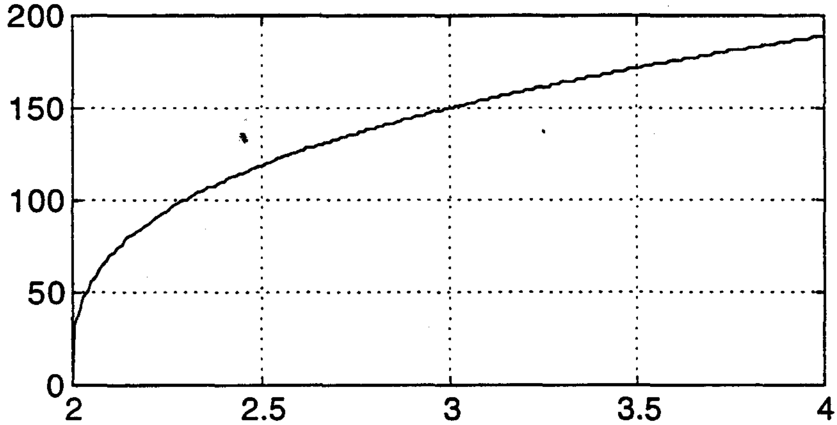

Here are their answers:

Harry:	400
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Clearly these guys are a bunch of stooges! Explain why each one is wrong in his estimate, AND give a better estimate. Show your work.

2) You are jet skiing on the Chesapeake Bay–smile on your face, the wind in your hair- when suddenly the engine stalls. Your best friend, who loves math, records your velocities as you glide to a stop:

Time since stall in seconds	0	2	4	6	8	10
Speed (ft/sec)	60	30	22	12	4	0

a) Use left- and right-hand rules to estimate how far the jet ski travels after the engine stalls. Show your reasoning.

b) Unbeknownst to you, there is a huge rock just below the surface of the water directly in front of you 200 ft from where the engine stalled. Are you in any danger? EXPLAIN.

3) The Air Force is testing its brand new, top secret heli-plane. The pilot takes off and heads due south. As planned, she radios her airspeed to the control tower every minute. After 5 minutes, the plane mysteriously disappears from the radar screen, and the radio transmissions stop. The pilot and the plane must be located quickly (to save the pilot) and quietly (to avoid a media "situation"). Assuming the plane crashed after the last contact, what is your <u>BEST</u> estimate of how far south of the point of takeoff the crash site is?

Minutes since take off	0	1	2	3	4	5
Speed (mph)	75	90	120	145	160	175

NAME________________
DATE ________________

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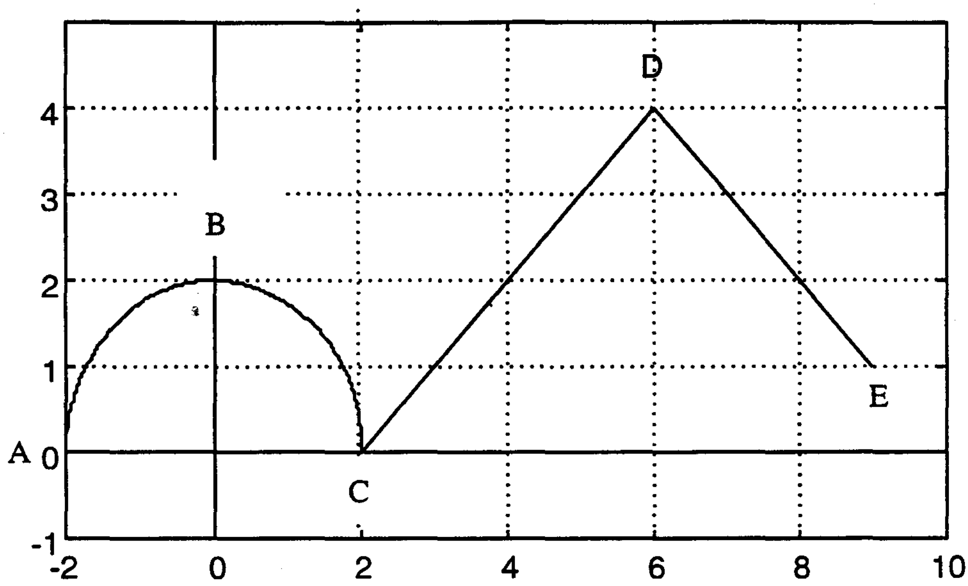
a. Use a midpoint Riemann Sum with four subdivisions of equal width to approximate $\int_0^{12} V(t)dt$. Using correct units, explain the meaning of your answer.

b. Is there some time t, $0 < t < 12$, such that $V'(t) = 0$. State the significance of this.

c. $V(t)$, the rate of travel, can be approximated by $U(t) = \frac{1}{53}(-2t^2 + 23t + 17)$. Evaluate $\int_0^{12} U(t)dt$. Indicate units of measure. Compare your answer to #1a and explain its significance.

2. Given the graph of f, answer the following questions. Note that the graph is NOT drawn to scale; it consists of a semicircle and two line segments.



a. Let $F(x) = \int_2^x f(t)dt$. Compute $F(-2)$, $F(2)$ and $F(6)$.

b. Find the instantaneous rate of change of F, with respect to x, at $x = 4$.

c. Find the absolute maximum value of F on the closed interval [-2, 9]. Justify your answer.

d. At what value(s) of x does F have a point (or points) of inflection? Justify your answer.

AP CALCULUS
UNIT IV

NAME ____________________
DATE ____________________

MULTIPLE CHOICE PRACTICE

1. What is the error in approximating $\int_1^4 \ln(x^2)dx$ by using three trapezoids of equal width?

   a) .121
   b) .171
   c) .694
   d) 1.792
   e) 5.090

2. The expression $\frac{\pi}{16}\left(\tan\left(\frac{\pi}{16}\right)+\tan\left(\frac{\pi}{8}\right)+\tan\left(\frac{3\pi}{16}\right)+\ldots+\tan\left(\frac{\pi}{2}\right)\right)$ is a Riemann sum approximation for

   a) $\int_0^{\frac{\pi}{16}} \tan x\,dx$
   b) $\frac{\pi}{16}\int_0^{\frac{\pi}{16}} \tan x\,dx$
   c) $\int_0^{\frac{\pi}{2}} \tan x\,dx$
   d) $\frac{\pi}{2}\int_0^{\frac{\pi}{2}} \tan x\,dx$
   e) $\frac{\pi}{16}\int_0^{\frac{\pi}{2}} \tan x\,dx$

3. If the interval [a, b] is divided into n equal subintervals of width Δx by the numbers $x_0, x_1, x_2, \ldots, x_n$, where $a = x_0 < x_1 < x_2 < \ldots < x_n = b$, what is $\lim_{n\to\infty}\sum_{k=1}^{n}\sqrt[3]{x_k}\,\Delta x$?

   a) $b^{\frac{1}{3}} - a^{\frac{1}{3}}$
   b) $\frac{3}{4}(b-a)$
   c) $\frac{3}{4}(b-a)^{\frac{4}{3}}$
   d) $\frac{3}{4}\left(b^{\frac{4}{3}} - a^{\frac{4}{3}}\right)$
   e) $\frac{1}{3}\left(b^{\frac{4}{3}} - a^{\frac{4}{3}}\right)$

*4. $\frac{d}{dx}\int_x^{x^3} 3\tan(t^2)dt =$

   a) $3(\tan(x^6) - \tan(x^2))$
   b) $3(\tan(x^3) - \tan x)$
   c) $3(x^2\tan(x^6) - \tan(x^2))$
   d) $3(3x^2\tan(x^6) - \tan(x^2))$
   e) $3(3x^2\tan(x^3) - \tan x)$

*5. $\lim_{t\to 1}\frac{\int_1^t e^{x^3}dx}{t^3 - 1} =$

   a) 0
   b) $\frac{e}{3}$
   c) 1
   d) $\frac{e}{2}$
   e) $e - 2$

* Questions 4 and 5 are BC level questions.

AP CALCULUS
UNIT IV

NAME Key
DATE

MULTIPLE CHOICE PRACTICE

1. What is the error in approximating $\int_1^4 \ln(x^2)dx$ by using three trapezoids of equal width?

Ⓐ
a) .121
b) .171
c) .694
d) 1.792
e) 5.090

2. The expression $\frac{\pi}{16}\left(\tan\left(\frac{\pi}{16}\right)+\tan\left(\frac{\pi}{8}\right)+\tan\left(\frac{3\pi}{16}\right)+...+\tan\left(\frac{\pi}{2}\right)\right)$ is a Riemann sum approximation for

a) $\int_0^{\frac{\pi}{16}} \tan x\, dx$
b) $\frac{\pi}{16}\int_0^{\frac{\pi}{16}} \tan x\, dx$
Ⓒ c) $\int_0^{\frac{\pi}{2}} \tan x\, dx$
d) $\frac{\pi}{2}\int_0^{\frac{\pi}{2}} \tan x\, dx$
e) $\frac{\pi}{16}\int_0^{\frac{\pi}{2}} \tan x\, dx$

3. If the interval [a, b] is divided into n equal subintervals of width Δx by the numbers $x_0, x_1, x_2, ..., x_n$, where $a = x_0 < x_1 < x_2 < ... < x_n = b$, what is $\lim_{n\to\infty}\sum_{k=1}^{n}\sqrt[3]{x_k}\,\Delta x$?

a) $b^{\frac{1}{3}} - a^{\frac{1}{3}}$
b) $\frac{3}{4}(b-a)$
c) $\frac{3}{4}(b-a)^{\frac{4}{3}}$
Ⓓ d) $\frac{3}{4}\left(b^{\frac{4}{3}} - a^{\frac{4}{3}}\right)$
e) $\frac{1}{3}\left(b^{\frac{4}{3}} - a^{\frac{4}{3}}\right)$

*4. $\frac{d}{dx}\int_x^{x^3} 3\tan(t^2)dt =$

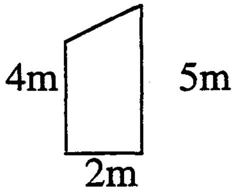
a) $3(\tan(x^6) - \tan(x^2))$
b) $3(\tan(x^3) - \tan x)$
c) $3(x^2\tan(x^6) - \tan(x^2))$
Ⓓ d) $3(3x^2\tan(x^6) - \tan(x^2))$
e) $3(3x^2\tan(x^3) - \tan x)$

*5. $\lim_{t\to 1}\frac{\int_1^t e^{x^3}dx}{t^3 - 1} =$

a) 0
Ⓑ b) $\frac{e}{3}$
c) 1
d) $\frac{e}{2}$
e) $e - 2$

* Questions 4 and 5 are BC level questions.

1. Find the area of the following trapezoid:



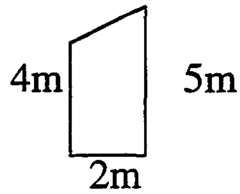
2. Given: $y = \arctan\sqrt{x}$
Find: y'.

3. If $h = \frac{1}{4}t^2$, where h is in meters and t is in seconds, how fast is h changing when t is 8 seconds?

4. $y = (6^{12})(4^3)$
$y' =$ ____________

5. Solve the following inequality by factoring and using a sign chart:
$x^3 - 9x < 0$

1. Find the area of the following trapezoid:



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4. $y = (6^{12})(4^3)$
$y' =$ ____________

5. Solve the following inequality by factoring and using a sign chart: $x^3 - 9x < 0$

ANSWERS:

1. $9m^2$

2. $\frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}}$

3. $\frac{dh}{dt} = \frac{1}{2}t$

$h'(8) = 4 m/s$

4. 0

5. $x(x-3)(x+3)$

x		-3		0		3	
f(x)	-		+		-		+

$(-\infty, -3)(0, 3)$

1. Evaluate the sum: $\sum_{n=1}^{3} n^2$

2. $f(x) = x^3 + 3x - 5$
   $f'(x) =$ ________________

3. The linearization of f(x) at x = 2 is ____________________.

4. Calculate the error that occurs if the linear approximation in #3 is used to approximate f(3)?

5. A balloon is expanding at a rate of $5\text{cm}^3/\text{sec}$. How fast is its radius increasing when the radius is 3 cm?

	ANSWERS:
1. Evaluate the sum: $\sum_{n=1}^{3} n^2$	1. $1^2 + 2^2 + 3^2$ $= 14$
2. $f(x) = x^3 + 3x - 5$ $f'(x) =$ _______________	2. $f'(x) = 3x^2 + 3$ $f'(2) = 15$
3. The linearization of f(x) at x = 2 is ____________________.	3. $L(x) = f(2) + f'(2)(x-2)$ $L(x) = 14 + 15(x-2)$
4. Calculate the error that occurs if the linear approximation in #3 is used to approximate f(3)?	4. $f(3) = 31$ $L(3) = 29$ Error = 2
5. A balloon is expanding at a rate of $5 cm^3/sec$. How fast is its radius increasing when the radius is 3 cm?	5. $V = \frac{4}{3}\pi r^3$ $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{5}{36\pi} cm/sec$

ANSWERS:

1. Evaluate the sum: $\sum_{n=1}^{3} n^2$

1. $1^2 + 2^2 + 3^2 = 14$

2. $f(x) = x^3 + 3x - 5$
   $f'(x) =$ ________________

2. $f'(x) = 3x^2 + 3$
   $f'(2) = 15$

3. The linearization of f(x) at x = 2 is ____________________.

3. $L(x) = f(2) + f'(2)(x-2)$

   $L(x) = 14 + 15(x-2)$

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   $L(3) = 29$

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   $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

   $\frac{dr}{dt} = \frac{5}{36\pi} cm/se$

1. Evaluate: $\frac{d}{dx}\int_{2}^{x} \ln(3t)\, dt$

2. Water flows over a dam at a rate of $200\text{m}^3/\text{sec}$. How many cubic meters of water pass over the dam in an hour?

3. Evaluate: $\int \sec^2 x\, dx$

4. If $\int_{3}^{5} f(x)dx = 7$,

   then $\int_{5}^{3} f(x)dx = ?$

5. Given: $\ln(xy) = 5$
   Find: y'

AP CALCULUS
UNIT IV – WARMUP SET 3

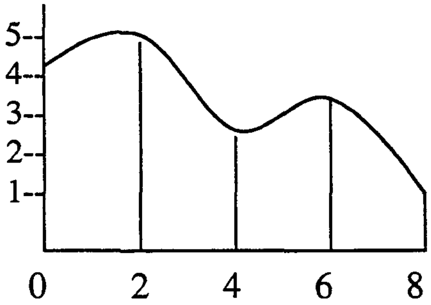
	ANSWERS:
1. Evaluate: $\frac{d}{dx}\int_2^x \ln(3t)\,dt$	1. ln 3x
2. Water flows over a dam at a rate of 200m^3/sec. How many cubic meters of water pass over the dam in an hour?	2. $\frac{m^3}{\text{sec}} \cdot \frac{\text{sec}}{hour} = \frac{m^3}{hour}$ $\frac{200}{1} \cdot \frac{3600}{1}$ $= 720{,}000\ m^3/hr$
3. Evaluate: $\int \sec^2 x\,dx$	3. tan x + c
4. If $\int_3^5 f(x)dx = 7$, then $\int_5^3 f(x)dx = ?$	4. $-\int_3^5 f(x)dx$ $= -7$
5. Given: ln(xy) = 5 Find: y′	5. lnx + ln y = 5 $\frac{1}{x} + \frac{1}{y} \cdot y' = 0$ $y' = -\frac{y}{x}$

1. Simplify: $\int_1^4 f(x)dx - \int_3^4 f(x)dx$

2. If $y = e^u$ and $u = 5x + 3$, find $\dfrac{dy}{dx}$.

3. Evaluate: $\lim_{h\to 0}\dfrac{\ln(2+h)-\ln 2}{h}$

4. Approximate the area of the region below by a Riemann Sum using right rectangles.



5. $y = \log_5(3x)$
$y' =$ ________________

ANSWERS:

1. Simplify: $\int_1^4 f(x)dx - \int_3^4 f(x)dx$

1. $\int_1^4 f(x)dx + \int_4^3 f(x)dx$
$= \int_1^3 f(x)dx$

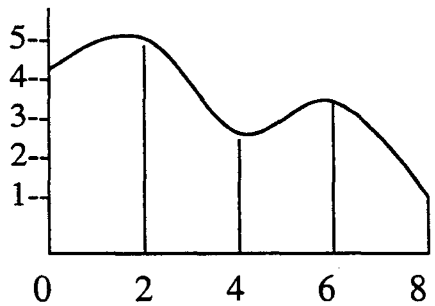
2. If y = e^u and u = 5x + 3, find $\frac{dy}{dx}$.

2. $5e^{5x+3}$

3. Evaluate: $\lim_{h \to 0} \frac{\ln(2+h) - \ln 2}{h}$

3. f(x) = ln x
$f'(x) = \frac{1}{x}$
$f'(2) = \frac{1}{2}$

4. Approximate the area of the region below by a Riemann Sum using right rectangles.



4.

2(5+3+3+2)
= 26 units^2

5. $y = \log_5(3x)$
$y' =$ ____________________

5. $y = \frac{\ln 3x}{\ln 5}$
$y' = \frac{1}{3x \cdot \ln 5} \cdot 3$
$y' = \frac{1}{x \cdot \ln 5}$

1. Simplify: $\int_1^4 f(x)dx - \int_3^4 f(x)dx$

2. If $y = e^u$ and $u = 5x + 3$, find $\frac{dy}{dx}$.

3. Evaluate: $\lim_{h \to 0} \frac{\ln(2+h) - \ln 2}{h}$

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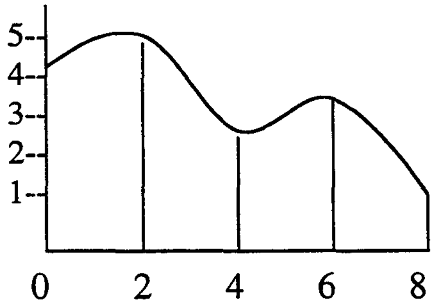

5. $y = \log_5(3x)$
$y' =$ ______________

1. Sketch a graph illustrating the meaning of the definite integral $\int_a^b f(x)dx$

2. Is $f(x) = e^{2x}$ concave up or concave down on [-4, 4]. Justify your answer.

3. Given: $y = \text{arcsec } x$
   Find: y'

4. Does the derivative of $\sin(x^2)$ equal to the derivative of $\sin^2 x$. Justify your answer.

5. Evaluate:

$$\frac{d}{dx}\int_2^{3x} e^{t^2}\,dt$$

1. Sketch a graph illustrating the meaning of the definite integral $\int_a^b f(x)dx$

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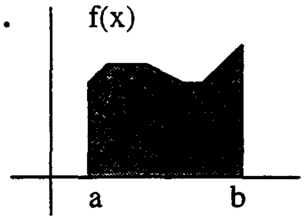
5. Evaluate:

$$\frac{d}{dx}\int_2^{3x} e^{t^2}\,dt$$

ANSWERS:

1. 

2. concave up since $y''=4e^{2x} > 0$

3. $y' = \dfrac{1}{|x|\sqrt{x^2-1}}$

4. No,
$\frac{d}{dx}(\sin x^2) = \cos x^2(2x)$
$\frac{d}{dx}(\sin^2 x) = 2\sin x\cos x$

5. $e^{(3x)^2}(3)$
$= 3e^{9x^2}$

Without attempting to evaluate the integrals, decide which of the following statements are true and which are false. Justify your answers.

1. $\int_1^5 \sqrt{x^6+1}\,dx \geq \int_1^5 \sqrt{x^2+1}\,dx$

2. $\int_{-1}^{3} f(x)\,dx = \int_{3}^{-1} f(-x)dx$

3. $\textit{If } a < b, \textit{ then} \int_a^b (\cos^2 x + 1)\,dx \geq 0$

4. $\int_0^{\pi} \sin x\,dx = \int_0^{\pi} \sin(-x)\,dx$

Without attempting to evaluate the integrals, decide which of the following statements are true and which are false. Justify your answers.

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2. $\int_{-1}^{3} f(x)\,dx = \int_{3}^{-1} f(-x)\,dx$

3. *If* $a < b$, *then* $\int_a^b (\cos^2 x + 1)\,dx \geq 0$

4. $\int_0^{\pi} \sin x\,dx = \int_0^{\pi} \sin(-x)\,dx$

ANSWERS:

1. True, $f(x) \geq g(x)$ $\therefore \int f(x) \geq \int g(x)$

2. True, if f(x) is even
   False, if f(x) is odd

3. True because $f(x) \geq 0$

4. False because f(x) is odd

UNIT IV INTERNET RESOURCES

http://www.netsrq.com/~hahn/calculus.html, http://www.barzilai.org/archive, http://archives.math.utk.edu.visual.calculus/ and http://www.hofstra.edu/~matscw/RealWorld/index.html

Sites feature some tutorials, but mostly have good drill and quiz resources for either in-class practice or at-home practice.

http://www.integrals.com

Site features an integral calculator (indefinite integrals) and has a history of integration.

http://www.math.odu.edu/~bogacki/citat/

Site has an integration test. If you use this site, please email the professor to let him know that you are using his material. He is interested in your feedback.