UNIT I

UNIT I
TABLE OF CONTENTS

UNIT I: LIMITS AND CONTINUITY

Expectation:	The student will demonstrate an intuitive understanding of the limiting process and an understanding of continuity in terms of limits.

OVERVIEW:
The emphasis of this unit should be on an intuitive approach to limits and continuity through graphical investigation, numerical approximations and analytical methods where appropriate.

INDICATORS:

1. Demonstrate an intuitive understanding of the limiting process.

2. Determine the limit of a given function, including one-sided limits.

3. Estimate the limit of a function from a graph and table of data (numerical approach).

4. Determine the limit of a sum, product, and quotient of two or more functions as $x \to a$.

5. Determine the limit of a sum, product, and quotient of two or more functions as $x \to \infty$.

6. Demonstrate an understanding of the definition of continuity.

7. Determine the continuity of a function at a given point by using the definition of continuity.

8. Determine the continuity of a function over a given interval.

9. Demonstrate a geometric understanding of graphs of continuous functions including applications of the Intermediate Value Theorem and Extreme Value Theorem.

Indicators/ Objectives	Foerster: Calculus Key Curriculum 1998	Foerster: Calculus: Instructor's Resource Book Key Curriculum 1998	Finney, et al: Calculus S F A W 1999	Guide Pages
1-4	10; 25-31; 39-60	1.5	55-65	
5	60-66	2.5	65-73	
6-8	52-60	2.4	73-81	
9	67-70		79; 178	
Review				I-ReviewII

Indicators/ Objectives	Finney, et. al Calculus 1994	Guide Pages
1-5	105-116; 127- 144	
6-9	116-125	

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LIMITS REVIEW I

Evaluate each limit analytically.

1. $\lim_{x \to -3}(x^3 - 4x^2 + 2)$

-61

2. $\lim_{x \to 5} \frac{2x - 10}{x^2 - 4x - 5}$

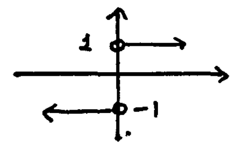
$\frac{1}{3}$

3. $\lim_{x \to 2} \frac{3x}{x^2 + 6x - 16}$

d.n.e.

4. $\lim_{x \to 0^+} \frac{|x|}{x}$

1



5. $\lim_{x \to 3} \sqrt[6]{81 - x^4}$

d.n.e.

$\left(\lim_{x \to 3^-} \sqrt[6]{81 - x^4} = 0\right.$, but $\lim_{x \to 3^+} \sqrt[6]{81 - x^4}$ d.n.e.)

6. $\lim_{x \to \infty} \frac{7x + 19}{4x - 3}$

$\frac{7}{4}$

7. $\lim_{x \to 0} e^x \cos x$

1

8. $\lim_{x \to 0} \frac{\sin 5x}{x}$

5

9. $\lim_{x \to \pi} \frac{\cos x}{x}$

$-\frac{1}{\pi}$

10. $\lim_{x \to \infty} \frac{3x - 9}{x}$

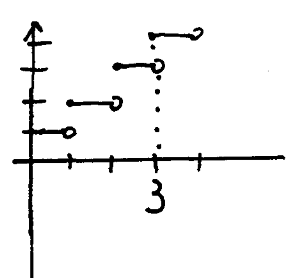
3

11. $\lim_{x \to \pi^-} \csc x$

∞
(d.n.e)

12. $\lim_{x \to 3} \lfloor x + 1 \rfloor$

d.n.e.



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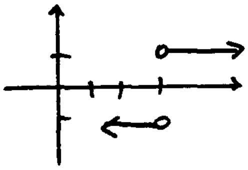
LIMITS REVIEW II

Evaluate each limit without using a calculator.

1. $\lim_{x\to 3}\frac{|x-3|}{x-3}$ d.n.e.



2. $\lim_{x\to 25}\frac{\sqrt{x}-5}{x-25}$

$\frac{(\sqrt{x}-5)}{(\sqrt{x}-5)(\sqrt{x}+5)} \to \frac{1}{10}$

3. $\lim_{x\to\infty}\sqrt{x^2+2}-x$ 0

$\sqrt{x^2+2}-x = \frac{(x^2+2)-x^2}{\sqrt{x^2+2}+x}$

4. $\lim_{x\to 0}\frac{(3+x)^3-27}{x}$ 27

Factor and simplify, or use L'Hôpital's Rule

5. $\lim_{x\to\infty}\frac{\sqrt{x^2+1}}{x+2}$ 1

$\frac{x\sqrt{1+\frac{1}{x^2}}}{x\left(1+\frac{2}{x}\right)} \to \frac{\sqrt{1+\frac{1}{x^2}}}{1+\frac{2}{x}} \to 1$ (with $\frac{1}{x^2}\to 0$, $\frac{2}{x}\to 0$)

6. $\lim_{x\to\infty}\frac{2x^2+3x+5}{3x^2+4x-7}$ $\frac{2}{3}$

7. $\lim_{x\to 3}\frac{x^2-9}{x^2-x-6}$

$\frac{6}{5}$

8. $\lim_{x\to 1^+}\begin{cases}4-x^2, & x\le 1\\ 3x-1, & x>1\end{cases}$ 2

Note: $\lim_{x\to 1} f$ d.n.e. because $\lim_{x\to 1^-} f = 3$, $\lim_{x\to 1^+} f = 2$, $3 \neq 2$.

9. $\lim_{x\to\infty}e^{-x}\sin x$

0

10. $\lim_{x\to-\infty} Arc\tan x$

$-\frac{\pi}{2}$

11. $\frac{x\left(\frac{3}{x}+1\right)}{|x|\sqrt{\frac{5}{x^2}+6}}$ (Since $x\to-\infty$, $\frac{x}{|x|}=-1$; $\frac{3}{x}\to 0^-$ etc.)

11. $\lim_{x\to-\infty}\frac{3+x}{\sqrt{5+6x^2}}$ $-\frac{1}{\sqrt{6}}$

12. $\lim_{x\to 0}\frac{\frac{1}{4+x}-\frac{1}{4}}{x}$ $-\frac{1}{16}$

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LIMITS REVIEW III

Evaluate each limit, if it exists. If a limit does not exist, explain why.

1. $\lim\limits_{x\to 2}\frac{x^3-8}{x^2-4}$ $\to \frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}$

2. $\lim\limits_{h\to 0}\frac{(1+h)^6-1}{h}$ 6 → Def. of the derivative: $f(x)=x^6$, $f'(1)=6$

3. $\lim\limits_{x\to\infty}\frac{1}{3^{-x}}$ $\to 3^x$

4. $\lim\limits_{x\to -\frac{1}{2}}[3x]$ -2

5. $\lim\limits_{x\to\infty}\sin^2 x$ d.n.e.

6. $\lim\limits_{x\to\infty}x^2\sin\left(\frac{1}{x}\right)$ Let $u=\frac{1}{x}$: $\lim\limits_{u\to 0}\frac{1}{u}\cdot\frac{\sin u}{u}$

7. $\lim\limits_{x\to a^+}\frac{|x-a|}{x^2-a^2}$ $\frac{1}{2a}$

Here $x>a$, so $x-a>0$ and $|x-a|=x-a$

8. $\lim\limits_{x\to a^-}\frac{|x-a|}{x^2-a^2}$ $-\frac{1}{2a}$ Here $x<a$, so $x-a<0$, so $|x-a|=-(x-a)$

9. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+3x}-x\right)$ $=\frac{3}{2}$

$$\frac{(\sqrt{x^2+3x}-x)(\sqrt{x^2+3x}+x)}{\sqrt{x^2+3x}+x}=\frac{x^2+3x-x^2}{\sqrt{x^2+3x}+x}=\frac{x(3)}{x[\sqrt{1+\frac{3}{x}}+1]}\to\frac{3}{2}$$

10. $\lim\limits_{x\to\infty}\left(2x-\sqrt{4x^2-5x}\right)$ $=\frac{5}{4}$ *

11. $\lim\limits_{x\to 3^+}\frac{x-[x]}{x}$ 0

12. $\lim\limits_{x\to\infty}\frac{3x+4}{\sqrt{2x^2-5}}$ $\frac{x(3+\frac{4}{x})}{x\sqrt{2-5/x^2}}\to\frac{3}{\sqrt{2}}$

13. $\lim\limits_{x\to\infty}\frac{\sqrt{1+x}}{\sqrt{9x-1}}$ $\to\sqrt{\frac{x+1}{9x-1}}\to\frac{1}{3}$.

*10: $$\frac{(2x-\sqrt{4x^2-5x})(2x+\sqrt{4x^2-5x})}{2x+\sqrt{4x^2-5x}}$$

$$=\frac{5x}{2x+\sqrt{4x^2-5x}}=\frac{x(5)}{x(2+\sqrt{4-5/x})}\to\frac{5}{4}$$

UNIT I
STUDENT WORKSHEETS

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LIMITS REVIEW I

Evaluate each limit analytically.

1. $\lim\limits_{x\to -3}(x^3 - 4x^2 + 2)$

2. $\lim\limits_{x\to 5}\dfrac{2x-10}{x^2-4x-5}$

3. $\lim\limits_{x\to 2}\dfrac{3x}{x^2+6x-16}$

4. $\lim\limits_{x\to 0^+}\dfrac{|x|}{x}$

5. $\lim\limits_{x\to 3}\sqrt[6]{81-x^4}$

6. $\lim\limits_{x\to\infty}\dfrac{7x+19}{4x-3}$

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8. $\lim\limits_{x\to 0}\dfrac{\sin 5x}{x}$

9. $\lim\limits_{x\to\pi}\dfrac{\cos x}{x}$

10. $\lim\limits_{x\to\infty}\dfrac{3x-9}{x}$

11. $\lim\limits_{x\to\pi^-}\csc x$

12. $\lim\limits_{x\to 3}\lfloor x+1\rfloor$

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LIMITS REVIEW II

Evaluate each limit without using a calculator.

1. $\lim_{x \to 3} \frac{|x-3|}{x-3}$

2. $\lim_{x \to 25} \frac{\sqrt{x}-5}{x-25}$

3. $\lim_{x \to \infty} \sqrt{x^2+2} - x$

4. $\lim_{x \to 0} \frac{(3+x)^3 - 27}{x}$

5. $\lim_{x \to \infty} \frac{\sqrt{x^2+1}}{x+2}$

6. $\lim_{x \to \infty} \frac{2x^2+3x+5}{3x^2+4x-7}$

7. $\lim_{x \to 3} \frac{x^2-9}{x^2-x-6}$

8. $\lim_{x \to 1^+} \begin{cases} 4-x^2, & x \le 1 \\ 3x-1, & x > 1 \end{cases}$

9. $\lim_{x \to \infty} e^{-x} \sin x$

10. $\lim_{x \to -\infty} Arc\tan x$

11. $\lim_{x \to -\infty} \frac{3+x}{\sqrt{5+6x^2}}$

12. $\lim_{x \to 0} \frac{\frac{1}{4+x} - \frac{1}{4}}{x}$

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LIMITS REVIEW III

Evaluate each limit, if it exists. If a limit does not exist, explain why.

1. $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2-4}$

2. $\lim\limits_{h\to 0}\dfrac{(1+h)^6-1}{h}$

3. $\lim\limits_{x\to\infty}\dfrac{1}{3^{-x}}$

4. $\lim\limits_{x\to -\frac{1}{2}}[3x]$

5. $\lim\limits_{x\to\infty}\sin^2 x$

6. $\lim\limits_{x\to\infty}x^2\sin\left(\dfrac{1}{x}\right)$

7. $\lim\limits_{x\to a^+}\dfrac{|x-a|}{x^2-a^2}$

8. $\lim\limits_{x\to a^-}\dfrac{|x-a|}{x^2-a^2}$

9. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+3x}-x\right)$

10. $\lim\limits_{x\to\infty}\left(2x-\sqrt{4x^2-5x}\right)$

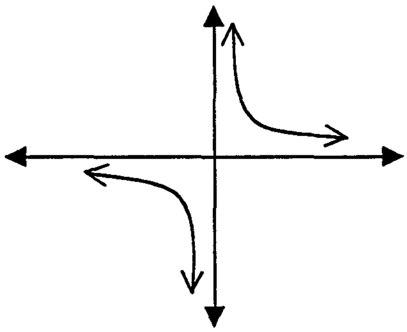
11. $\lim\limits_{x\to 3^+}\dfrac{x-[x]}{x}$

12. $\lim\limits_{x\to\infty}\dfrac{3x+4}{\sqrt{2x^2-5}}$

13. $\lim\limits_{x\to\infty}\dfrac{\sqrt{1+x}}{\sqrt{9x-1}}$

1. Graph the function: $g(x) = \ln x$ and its inverse on the same coordinate system.

2. Write the equation for the function graphed below:



3. Find f(3) if:

$$f(x) = \begin{cases} 2x+3, & x<3 \\ \dfrac{2}{x^2}, & x \geq 3 \end{cases}$$

4. Evaluate: $\tan\dfrac{\pi}{3}$

5. Factor completely: $x^3 - 6x^2 + 8x$

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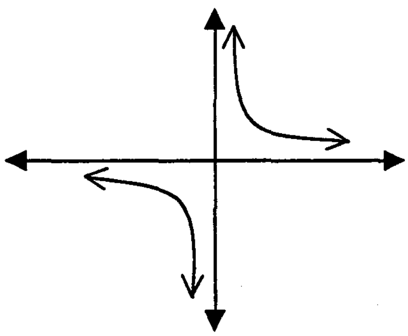

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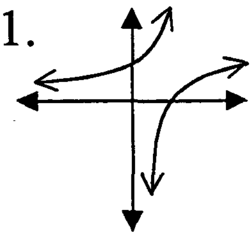
ANSWERS:

1.



2. $f(x)=\dfrac{1}{x}$

3. $\dfrac{2}{9}$

4. $\sqrt{3}$

5. x(x-4)(x-2)

1. Sketch the graph of a function with a removable discontinuity.

2. Graph the function:

$$f(x) = \frac{|x-4|}{x-4}$$

3. Simplify: $\dfrac{x^2-4}{x^2+x-6}$

4. Find $\lim\limits_{x \to -5} (x+4)^{2001}$

5. Is f(x) = cos x increasing or decreasing at x = 2?

1. Sketch the graph of a function with a removable discontinuity.

2. Graph the function:

$$f(x) = \frac{|x-4|}{x-4}$$

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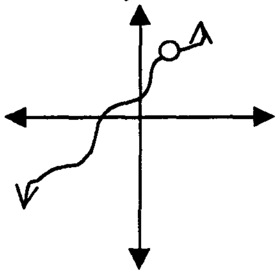
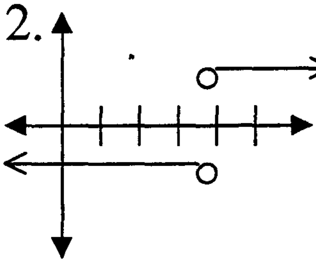
ANSWERS:

1. Answers vary



2.



3. $\dfrac{x+2}{x-3}$

4. -1

5. decreasing

1. Sketch the graph of a function with a removable discontinuity.

2. Graph the function:

$$f(x) = \frac{|x-4|}{x-4}$$

3. Simplify: $\dfrac{x^2-4}{x^2+x-6}$

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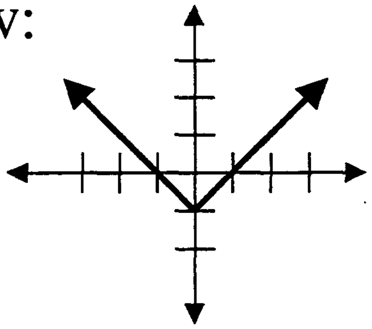


2. What is the graphical meaning of the average rate of change between two points, A and B, on a graph.

3. Given: $f(x) = \dfrac{x^2 + 2x - 15}{x + 5}$

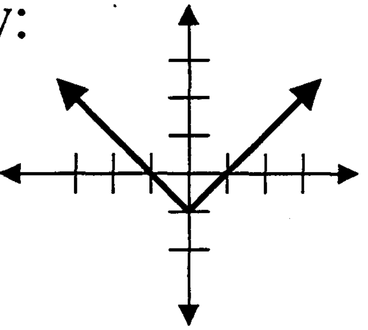
What value should be assigned to f(-5) to make the extended function continuous?

4. Find $\lim\limits_{x \to 2^-} \lfloor x + 1 \rfloor$.

5. State the domain of f(x) = ln(x+3).

ANSWERS:

1. Write the equation for the function graphed below:



Answer 1. $f(x) = |x| - 1$

2. What is the graphical meaning of the average rate of change between two points, A and B, on a graph.

Answer 2. Slope of line AB

3. Given: $f(x) = \dfrac{x^2 + 2x - 15}{x + 5}$

What value should be assigned to f(-5) to make the extended function continuous?

Answer 3. –8

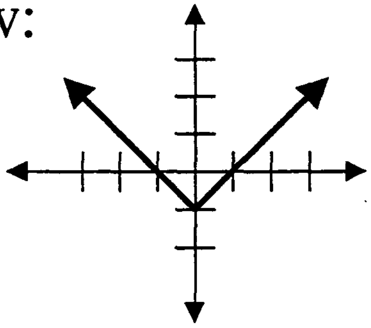
4. Find $\lim_{x \to 2^-} \lfloor x + 1 \rfloor$.

Answer 4. 2

5. State the domain of f(x) = ln(x+3).

Answer 5. $x > -3$

1. Write the equation for the function graphed below:



2. What is the graphical meaning of the average rate of change between two points, A and B, on a graph.

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5. State the domain of f(x) = ln(x+3).

1. Graph: $f(x)=\begin{cases}3x-7, & x>2\\ 1-x^2, & x\le 2\end{cases}$

2. For the function above, find each limit or explain why it does not exist:

   $a)\ \lim_{x\to 2^-}$ $\qquad b)\ \lim_{x\to 2^+}$ $\qquad c)\ \lim_{x\to 2}$

3. Simplify: $\frac{1}{2}+\frac{2}{3}+\frac{1}{4}$

4. Given: $f(x)=x^2+2$

   Evaluate $\frac{f(3)-f(1)}{3-1}$

5. Write the point-slope form of the equation of the line that passes through points A(3, 11) and B(1, 3).

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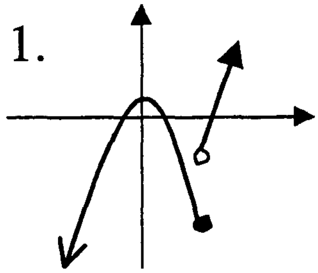
ANSWERS:

1.



2. a) –3
   b) –1
   c) DNE
   since $-3 \neq -1$

3. $\frac{17}{12}$

4. 4

5. y=4(x-1)+3
   or
   y=4(x-3)+11

1. $\lim_{x\to\infty} \frac{1}{x} = \lim_{x\to 0^+}$ ______ = ______

2. $\lim_{x\to\infty} \cos\frac{1}{x} = \lim_{x\to 0^+}$ ______ = ______

3. Given: f(x) = tan x
Graph f(x), $f^{-1}(x)$, and y = x on one coordinate system.

4. Find: $\lim_{x\to 0} \frac{\sin x}{x}$

5. Name at least 4 types of functions that are continuous for all real numbers.

1. $\lim\limits_{x\to\infty} \frac{1}{x} = \lim\limits_{x\to 0^+}$ ______ = ______

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4. Find: $\lim\limits_{x\to 0} \frac{\sin x}{x}$

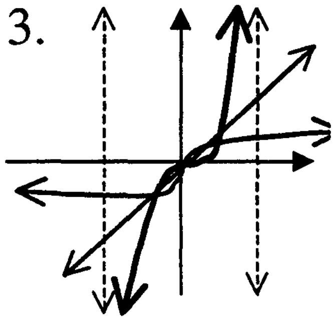
5. Name at least 4 types of functions that are continuous for all real numbers.

ANSWERS:

1. x, 0

2. cos x, 1

3.



4. 1

5.
polynomials
exponential
sine
cosine
abs value

UNIT I INTERNET RESOURCES

http://www.math.odu.edu/cbii/calcanim/ and http://www.math.psu.edu/dna/graphics.html

Both sites have animated demonstrations of the limiting process.

http://www.netsrq.com/~hahn/calculus.html, http://www.barzilai.org/archive,
http://archives.math.utk.edu.visual.calculus/ and
http://www.hofstra.edu/~matscw/RealWorld/index.html

Sites feature some tutorials, but mostly have good drill and quiz resources for either in-class practice or at-home practice.