UNIT III

UNIT III
TABLE OF CONTENTS

Expectation:	The student will use derivatives to analyze the behavior of a function and to solve real world problems.

OVERVIEW:

Although graphing calculators can give the student general information about the shape of a graph, the student will employ the derivative to identify key features of the graph. Derivatives will also be used to find solutions to applications dealing with optimization, related rates, and motion.

INDICATORS:

1. Construct an equation of a line tangent or normal to a given curve through a given point (including a parametrically defined equation *).
2. Apply the first and second derivative in the analysis of curves (including the notions of optimization, monotonicity and concavity).
3. Find the velocity, speed, and the acceleration from a distance function, graph, or table (including particle motion along a line).
4. Solve problems involving rates of change and related rates of change.
5. Solve optimization problems, both absolute (global) and relative (local) .
6. Demonstrate the meaning of the Mean Value Theorem geometrically and analytically.

* 7. Apply L'Hopital's rule to cases of the form 0/0 or ∞/∞.

8. Use the local linearity property of a differentiable function to approximate the change in the range of a function for a specified change in its domain.

* BC Calculus indicator only

Unit III: Applications of the Derivative

Indicators/ Objectives	Foerster: Calculus Key Curriculum 1998	Foerster: Calculus: Instructor's Resource Book Key Curriculum 1998	Finney, et al: Calculus S F A W 1999	Guide Pages
1	81, 84, 160, 183-184		144-145, 83-86	
2	385, 353-362	8-2a	188, 194-198	III 1 - 7
3	98-107	3-5	122-125, 198-199	III 8 - 10
4	517-518	10-4	232-236	
5	369-372, 523-524	8-3; 10-5	206-213	
6	202-207	5-6	186-189	
7	285-288	6-8	417-420	
8	183-187		225-227	III 11 - 14

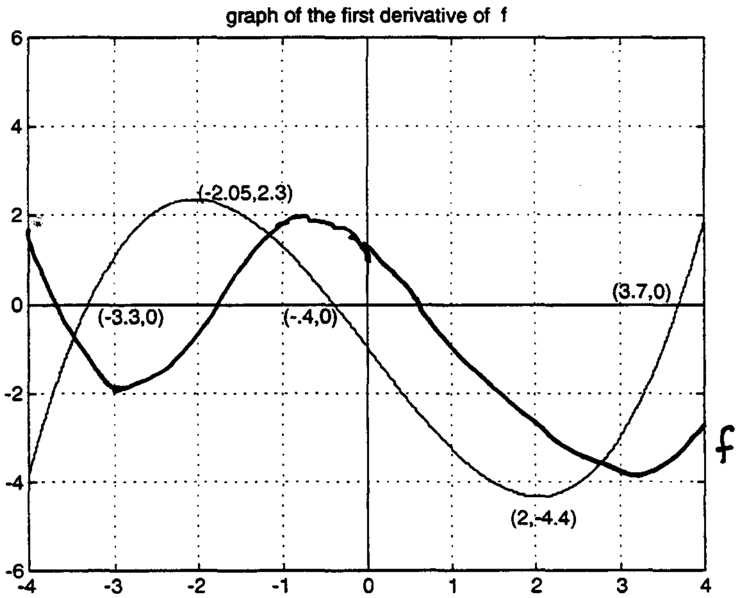
Indicators/ Objectives	Finney, et al: Calculus 1994	Guide Pages
1	189, 232 - 233	
2	259 - 262; 667 - 670	
3	201 - 205, 316 - 327	
4	205 - 212, 316 - 327	
5	290 - 305	
6	263 - 266	
7	566 - 576	
8	156, 239 - 248	

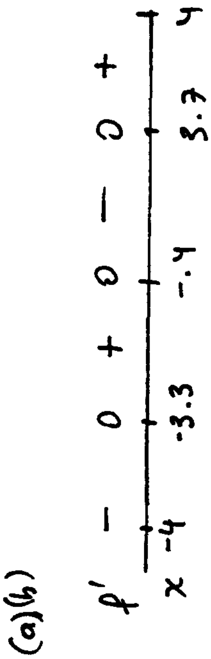
AP CALCULUS
UNIT III - ACTIVITY 1

NAME Key
DATE

GRAPHICAL ANALYSIS OF FUNCTIONS USING DERIVATIVES

1. A function, f, is defined on the closed interval [-4, 4]. The graph of f', the first derivative of f, is given below.





a. On which intervals is f increasing? Decreasing?

I: (-3.3, -.4) (3.7, 4] D: [-4, -3.3), (-.4, 3.7)

b. Where does f have a local maximum? Local minimum? Justify your answer.

Max. x: -.4, -4, 4 Min x: -3.3, 3.7

c. Where does f have points of inflection? Explain. f'' ______

At x = -2.05, 2 f'' changes sign

d. On which intervals is f concave up? Concave down?

u: [-4, -2.05), (2, 4]; D: (-2.05, 2)

e. Find the maximum and minimum values of f on the interval [-3.3, -.4]. Support your answer.

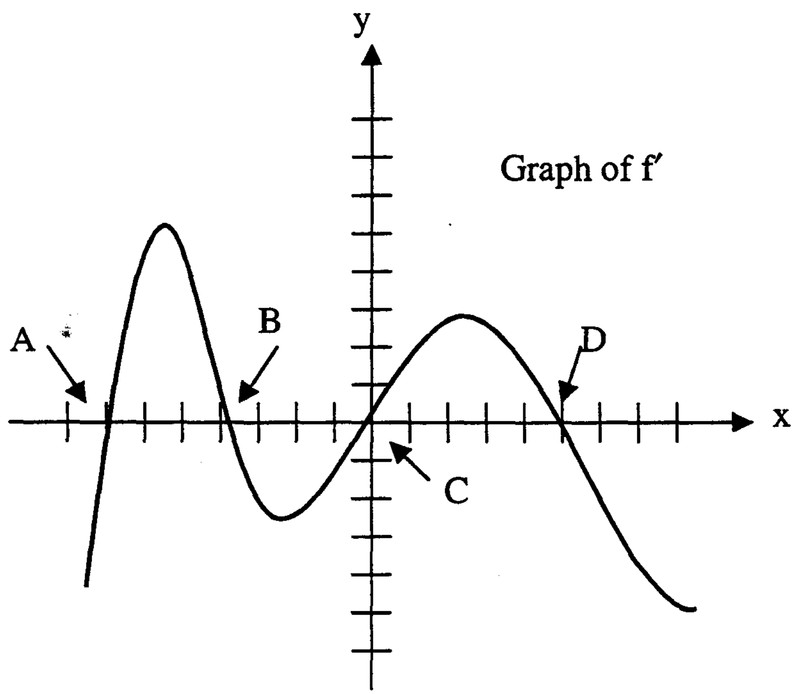
Max. at x = -.4, value ≈ 1.8 Min. at x = -3.3, value ≈ -1.9

f. Find the maximum and minimum values of f on the interval [2, 3.7]. Support your answer.

Max. at x = 2, value ≈ -2; Min. at x = 3.7, value ≈ 3.8

g. Given $f(0) = 1$, sketch the graph of f.

2. The graph of f', the derivative of a function f, is shown below. Study the graph carefully and answer the following questions.



a. Where does f have a local maximum? Local minimum? Justify your answers.

max. B, D   f' changes from + to −

min. A, C   "   − to +

b. Where is f concave up? Concave down? Explain your answers.

U: (-∞, -5.5) and (-2, 2)   answers will vary

D: (-5.5, -2) and (2, ∞)

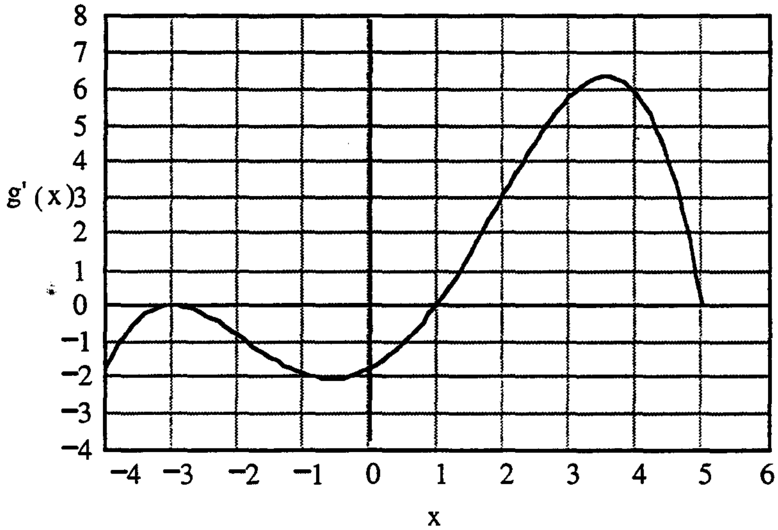
c. Where does f have an inflection point? Explain your answer.

at x = -5.5, -2, 2 ; f'' changes sign or the curve changes concavity.

AP CALCULUS
UNIT III – ACTIVITY 2

NAME Key
DATE

1. The graph of the *derivative* of g is shown in the figure.



a. Suppose that $g(4) = 4$. Find an equation of the line tangent to g at the point (4,4)

$g'(4) = 6$ $y - 4 = 6(x-4)$ or $y = 6x - 20$

b. Where does g have a local minimum? Support your answer.

at $x = 1$ g': − ∟ / +

c. Estimate $g''(3.5)$.

≈ 0 (from graph)

d. Where does g have a point of inflection? Support your answer.

at $x = -0.5,\ 3.5,\ -3$; g'' changes sign.

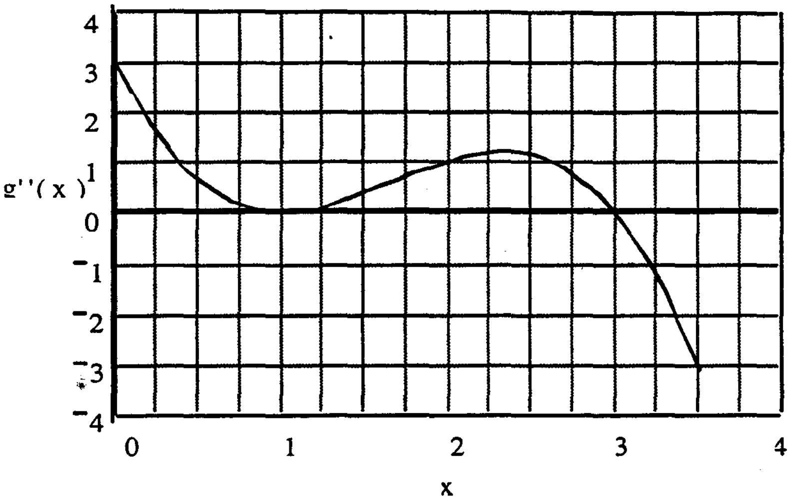
e. Where does g achieve its maximum on the interval [-3,0].

at $x = -3$, because f is decreasing on the interval.

f. Suppose that $g(1) = 2$. Find an equation for the line normal to g at (1, 2).

$g'(1) = 0$, ∴ g has a horizontal tangent : $y = 2$
and vertical normal: $x = 1$

2. The graph of g'', the second derivative of a function g, is shown below. Use this graph to answer the following questions.



a. Where is g concave down? $(3, 3.5)$

b. Where does g have points of inflection?

at $x = 3$; g'' changes sign here.

c. Rank the four numbers $g'(0)$, $g'(1)$, $g'(2)$, and $g'(3)$ in increasing order.

already in order. Since $g'' > 0$ on $(0, 3)$, g' is increasing

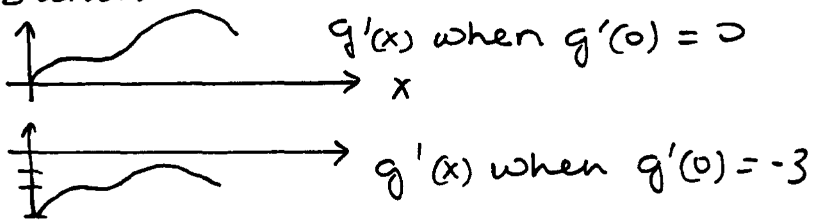
d. Suppose that $g'(0) = 0$. Is g increasing or decreasing at $x = 1.5$? Justify your answer.

Yes. $g'' > 0$ and $g' > 0$ at $x = 1.5$

Ans. will vary

e. Suppose that $g'(0) = -3$. Is g increasing or decreasing at $x = 1.5$? Justify your answer?

$g'(0)$ would vertically translate g' from part (d) 3 units below the x-axis.



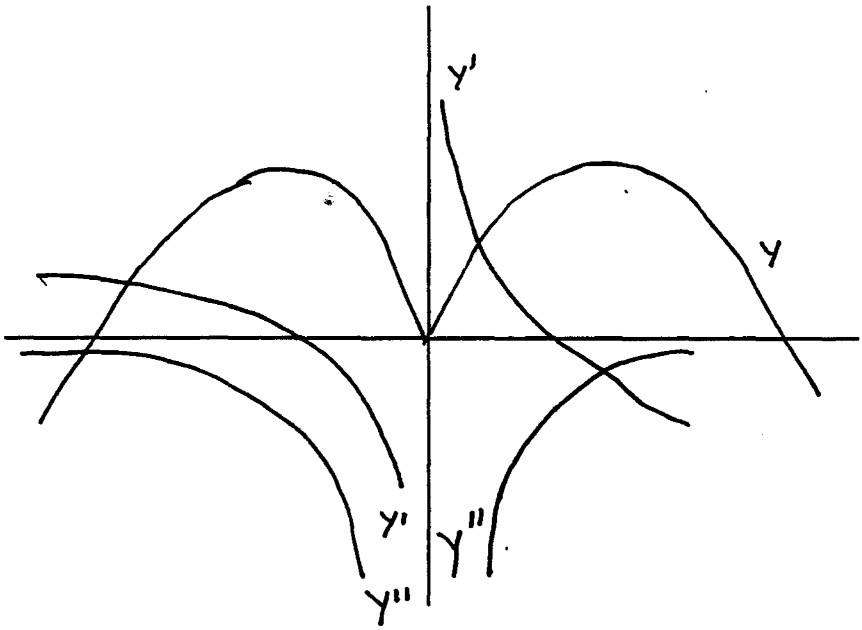
AP CALCULUS
UNIT III – ACTIVITY 3

NAME Key
DATE

RELATIONSHIPS BETWEEN DERIVATIVES

In this activity, you will observe the relationship between a function and its first and second derivatives.

1. On the axes below, sketch the graph of $y = 6x^{\frac{2}{5}} - x^{\frac{6}{5}}$ over [-10,10].



2. Find y'. Graph y' on your calculator and sketch it on the axes above.

$$y' = \frac{12}{5} x^{-3/5} - \frac{6}{5} x^{1/5}$$

3. For each given value(s) of y', give the corresponding value(s) of x and describe the graph.

y'	values of x	description of graph
=0	± 2.378	max. value of y
>0	[-10, -2.378) and (0, 2.378)	increasing
<0	x > 2.378 and -2.378 < x < 0	decreasing

4. Describe the behavior of y' as x approaches 0. What do you notice about the graph of y at x = 0?

$\lim_{x \to 0} y'$ d.n.e. f is not differentiable at $x = 0$; the graph of y has no tangent line at $x = 0$

5. Find y''.

$y'' = -\frac{36}{25} x^{-8/5} - \frac{6}{25} x^{-4/5}$

6. For each value(s) of y'', give the corresponding value(s) of x and describe the graph.

y''	values of x	description of graph
=0	N/A	N/A
>0	N/A	N/A
<0	all values of x	always concave down

7. Does y have any points of inflection? Explain.

NO. y" does not change signs

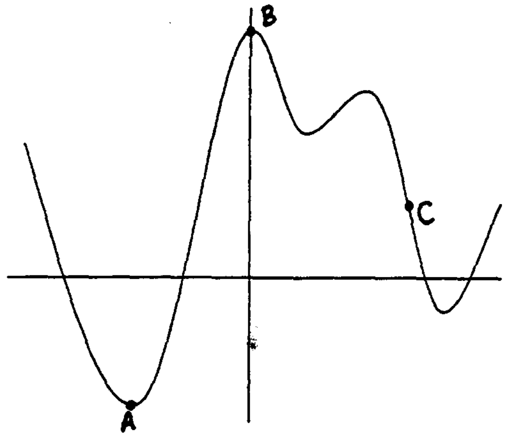
8. Write a paragraph summarizing the relationship between a function and its first and second derivatives.

The first derivative identifies the critical points and intervals on which the graph of f rises, falls, or is stationary.
The second derivatives identifies points of inflection and intervals of concavity.

Key

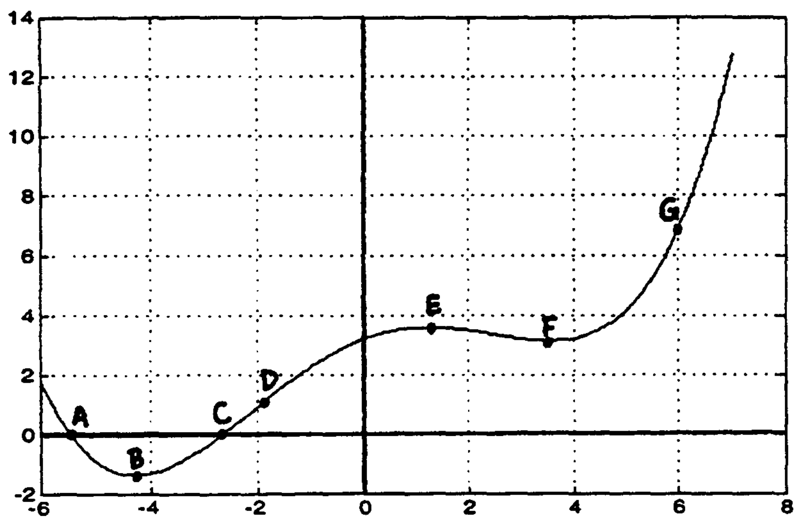
CURVE ANALYSIS REVIEW

1. The graph of f is given below. Determine whether f, f' and f'' are positive, negative, or zero at each point.



	A	B	C
f	−	+	+
f'	0	0	−
f''	+	−	0

2. The graph of g' is given below. Answer the following questions by choosing from points A - G.



g'

a) Where does g have a minimum? Explain.

C f' changes from decreasing to increasing

b) Where does g' have a minimum?

B f'' changes from − to +

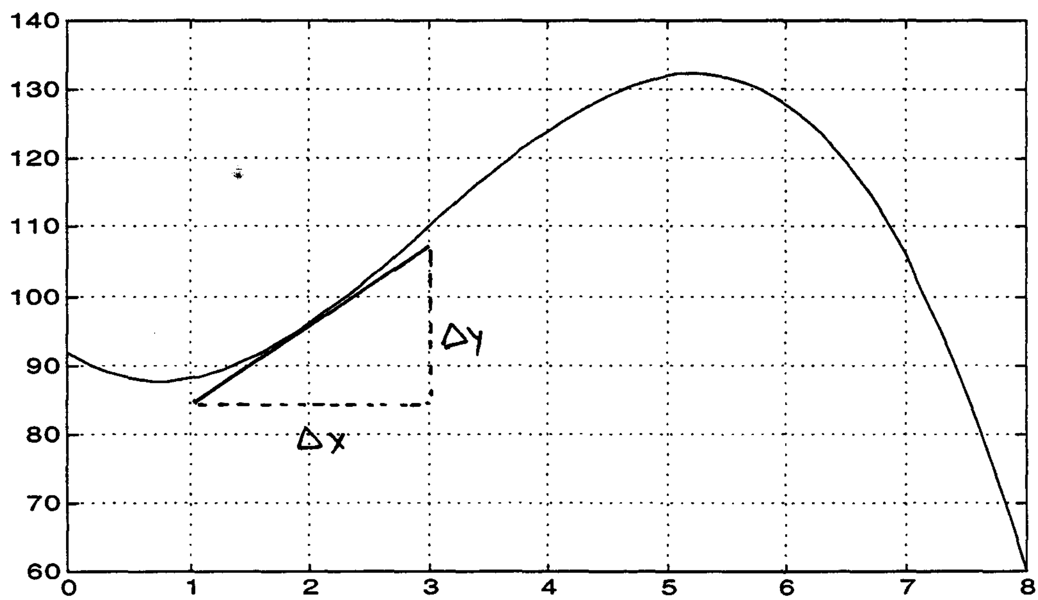
c) Where does g have points of inflection? Explain.

B, E, F f'' changes sign.

NAME Key
DATE

RELATING GRAPHS OF POSITION, VELOCITY, AND ACCELERATION

1. The graph below shows the height h(t) of a bird soaring in the sky after t seconds of observation.



a. Fill in the values of $h'(t)$ in the chart using either a +, –, or 0.

t	.8	2	3	4	5.2	6	7
$h'(t)$	0	+	+	+	0	–	–

b. Fill in the values of $h''(t)$ in the chart using either a +, –, or 0.

t	.8	2	3	4	5.2	6	7
$h''(t)$	+	+	0	–	–	–	–

c. Estimate the value of $h'(2)$. What does this mean in terms of the situation?

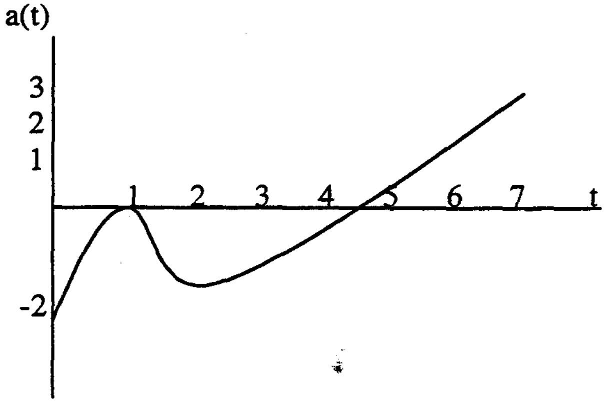
$h'(2) \approx 11$ speed of the bird is about 11 ft/s.

d. Over what interval is the bird's velocity positive? How can you tell from the graph?

on (0.8, 5.2) The slope of the curve is positive

e. Over what interval is the bird's acceleration positive? How can you tell from the graph? when $h'' > 0$, on (0, 3)

2. The graph below shows a(t), the acceleration of an object in $\frac{m}{s^2}$ at time t seconds.



When is the object's velocity

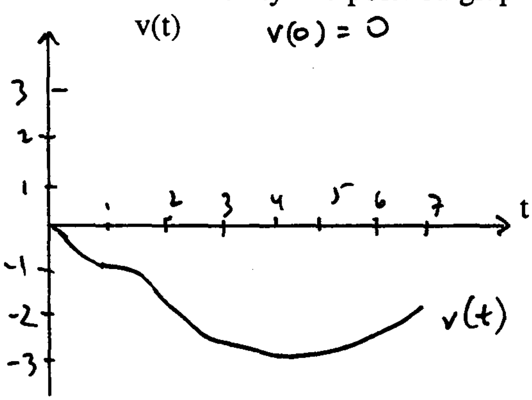
a.	at a maximum	$t = 0, 7$
b.	at a minimum	$t = 5$
c.	increasing	$5 < t < 7$
d.	decreasing	$0 < t < 5$

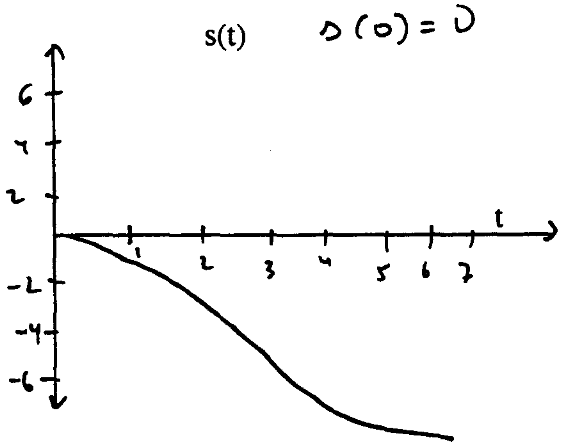
answers will vary

When does the object's position have

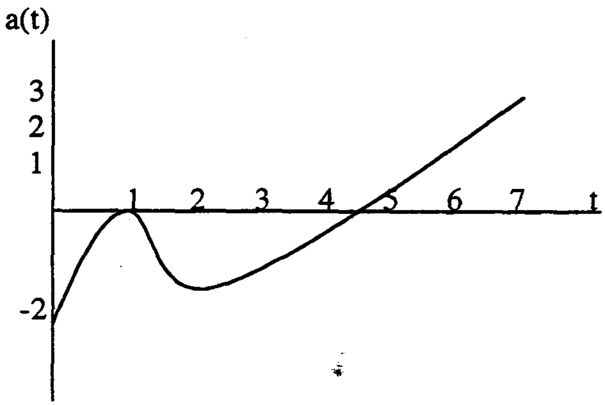
a.	points of inflection	$t = 5$
b	upward concavity	$(5, 8]$
c.	downward concavity	$(0, 5)$

Sketch the velocity and position graphs.





2. The graph below shows a(t), the acceleration of an object in $\frac{m}{s^2}$ at time *t* seconds.



When is the object's velocity

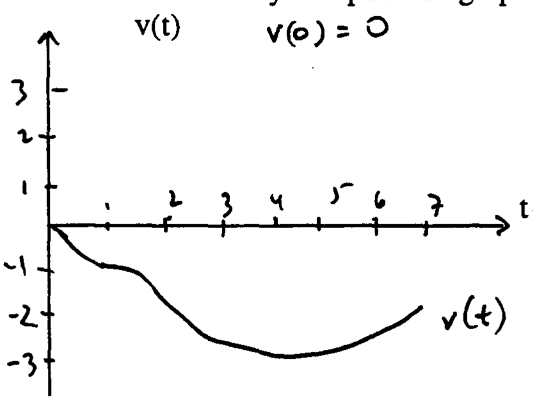
a.	at a maximum	t = 0, 7
b.	at a minimum	t = 5
c.	increasing	5 < t < 7
d.	decreasing	0 < t < 5

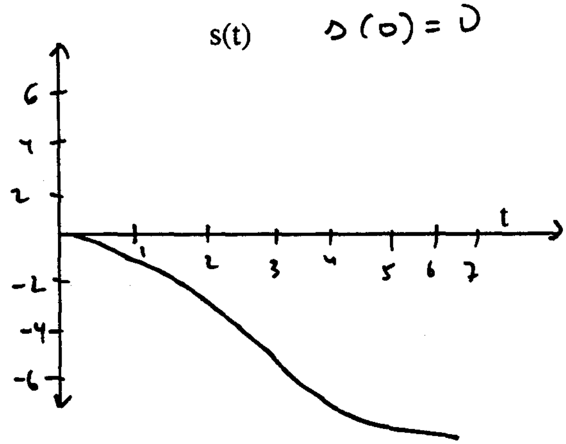
answers will vary

When does the object's position have

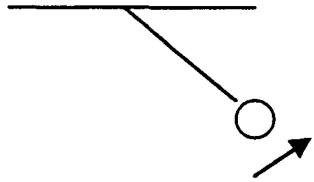
a.	points of inflection	t = 5
b	upward concavity	(5, 8]
c.	downward concavity	(0, 5)

Sketch the velocity and position graphs.

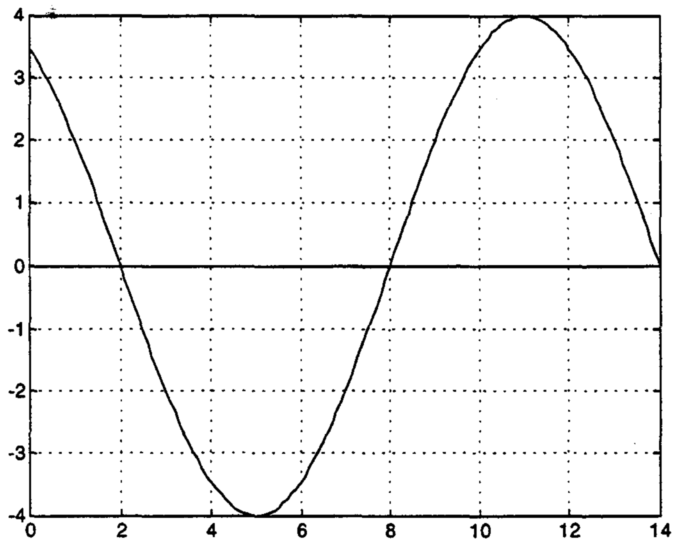




3. A student displaced a pendulum as shown in the diagram



At $t = 0$ she pushed it and it began to swing back and forth. Below is the graph of the velocity v(t), for the first 14 seconds of motion. (Let the motion to the right be positive, to the left be negative.)



a. When is the pendulum's position farthest from its final resting position? Explain.

$t = 2, 8, 14$. From $t = 0$ to $t = 2$, the pendulum moves to the right. At $t = 0$, $v = 0$, the pendulum changes direction.

b. Give the intervals where the pendulum accelerates; decelerates.

accel. when $v' > 0$: $(5, 11)$

decel. when $v' < 0$: $(0, 5)$ and $(11, 14)$

c. After how many seconds is the pendulum accelerating at its maximum rate? Explain.

$t = 8$ The pendulum accelerates at its max. rate. v' is the greatest

d. Over what intervals is the pendulum's distance from its final resting position increasing and decreasing?

I : $(0, 2)$, $(5, 8)$, $(11, 14)$

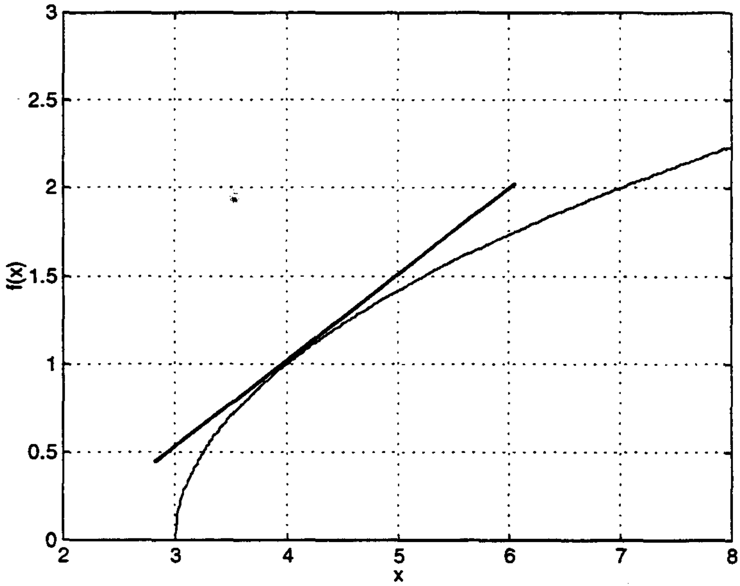
D : $(2, 5)$, $(8, 11)$

AP CALCULUS
UNIT III – ACTIVITY 5

NAME Key
DATE ______

LOCAL LINEARITY

1. Use your calculator to graph $f(x) = \sqrt{x-3}$ in the window shown below.



a) Find an equation of the line tangent to f at $x = 4$, and sketch it on the grid above and on your calculator.

$y - 1 = \frac{1}{2}(x - 4)$

b) Describe the proximity of the line to the curve near $x = 4$.

The line is close to the curve when x is close to 4.

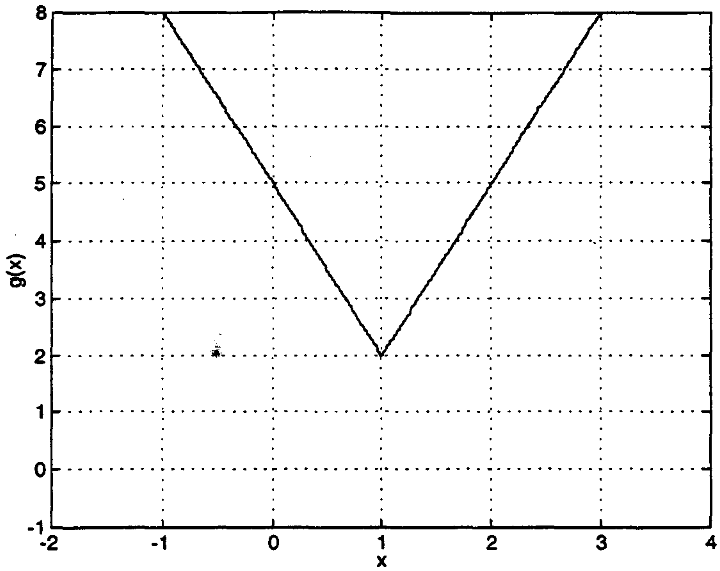
c) Zoom in at the point (4, *f(4)*) three times. How are the line and the curve related now?

Almost indistinguishable.

d) We say that f has "local linearity" at $x = 4$. Based on your investigation, explain what this term means.

When you zoom in, the curve appears to be linear in a small local area. The tangent line will give a good approximation in that area.

2. Pictured below is $g(x) = 3|x - 1| + 2$.



a) Is g locally linear at (1, 2)? Justify your answer.

No. No matter how much you zoom in, the graph will never appear linear. The function is not differentiable at $x = 1$. No tangent line exists there.

b) Explain why g does not have a derivative at $x = 1$.

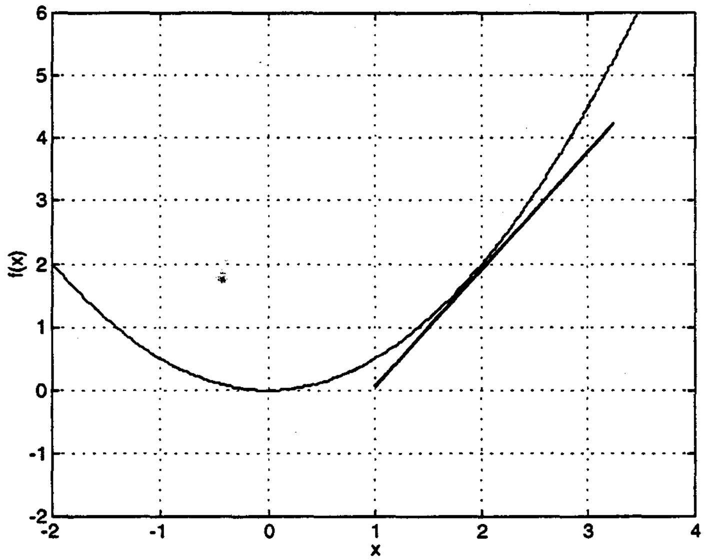
The slope of the tangent for $x < 1$ is different from the slope for $x > 1$. There is no unique tangent at $x = 1$

AP CALCULUS
UNIT III – ACTIVITY 6

NAME Key
DATE

LINEAR APPROXIMATIONS

1. The graph of function $f(x) = 0.5x^2$ is shown below.



a) Find an equation y, for the linearization of f(x), that best fits $f(x)$ at $x = 2$ (the tangent at $x = 2$). Graph y on the axes above.

$f'(x) = x\Big|_{x=2} = 2$ $y - 2 = 2(x-2)$

or $y = 2x - 2$

b) Approximate $f(1.7)$ using y. Is your estimate reasonable? Why or why not?

$f(1.7) = 0.5(1.7)^2 = 1.445$, $y(1.7) = 2(1.7) - 2$
$= 1.4$

Error $= |1.445 - 1.4| = 0.045$

c) What is the error in using y to approximate $f(1.7)$, where error = |expected value – approximate value|?

d) Complete the table.

x	$f(x)$	Y	error
1.96	1.9208	1:92	0.0008
1.97	1.9405	1.94	0.0005
1.98	1.9602	1.96	0.0002
1.99	1.9801	1.98	0.0001
2	2	2	0
2.01	2.0201	2.02	0.0001
2.02	2.0402	2.04	0.0002
2.03	2.0605	2.06	0.0005

e) What do you notice about the error as x approaches 2?

As x approaches 2, the error approaches zero.

f) Is the approximation too large or too small? Why?

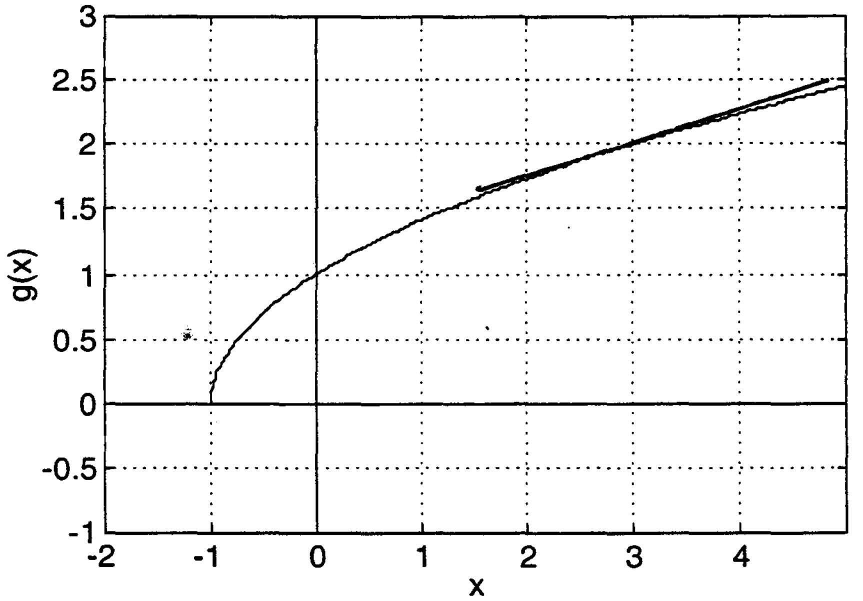
Too small; the graph is concave up.

g) How close should x be to 2 so that the error would be <u>less than</u> .0003?

$$|0.5x^2 - (2x-2)| < .0003$$

$$|x-2| < .0245$$

2. The graph of $g(x) = \sqrt{x+1}$ is pictured below.



a) Find the linearization of g at $x = 3$ and sketch it on the graph.

$g'(x) = \frac{1}{2\sqrt{x+1}}$ $\quad g'(3) = \frac{1}{4}$

$y = \frac{1}{4}(x-3) + 2$ or $y = \frac{1}{4}x + \frac{5}{4}$

b) Compute $\sqrt{3.98}$ with your calculator and compare this answer with the linear approximation of $g(x)$ from above.

$\sqrt{x+1} = \sqrt{3.98}$ so $x = 2.98$

$\sqrt{3.98} \doteq 1.99499$ $\quad y(2.98) = 1.995$

c) Calculate the error in using the linear approximation to estimate g(2.98).

$E = |\sqrt{3.98} - g(2.98)| = 10^{-5}$

d) Is the approximation too large or too small? Why?

Too large. The curve is concave down (The tangent lies above the curve)

e) For what values of x is your approximation accurate to within .002 units?

$\left|\sqrt{x+1} - \left(\frac{1}{4}x + \frac{5}{4}\right)\right| < .002$

AP CALCULUS
UNIT III – ACTIVITY 7

NAME Key
DATE

LIMITS AND THE DEFINTION OF DERIVATIVE

1. If h is a function such that $\lim_{x \to -1} \frac{h(x)-h(-1)}{x+1} = -2$, which of the following must be true?

   I. h is continuous at x = -1
   II. h is differentiable at x = -1
   III. $h(-1) = -2$
   IV. $h'(-1) = -2$

   a) I only
   b) II only
   c) II and IV
   d) I, II, and III
   e) I, II and IV

   (E)

2. If g is a differentiable function, then $g'(c) =$

   a) $\frac{g(c)}{c}$
   b) $\frac{g(x)-g(c)}{x-c}$
   c) $\lim_{x \to c} \frac{g(x)-g(c)}{x-c}$
   d) $\lim_{x \to c} \frac{g(x+c)-g(x)}{x-c}$
   e) $\lim_{h \to 0} \frac{g(x+h)-g(h)}{h}$

   (C)

3. $\lim_{t \to 0} \frac{e^{2(x+t)} - e^{2x}}{t}$ at $x = 0$

   a) 0
   b) 1
   c) 2
   d) e
   e) does not exist

   (C)

4. $\lim_{h \to 0} \frac{\cos 2(x+h) - \cos 2x}{h} =$

   a) $2\cos 2x$
   b) $-2\sin 2x$
   c) $-2\sin x \cos x$
   d) $-4\sin x \cos x$
   e) $4\cos^2 x - 1$

   (B)

5. If $\lim_{h \to 0} \frac{\ln\left[(x+h)^2\right] - \ln(x^2)}{h}$ exists, then

   I. $f(x) = 2\ln|x|$
   II. $x \neq 0$
   III. $f'(2) = 1$
   IV. $f'(0)$ does not exist

   a) I only
   b) IV only
   c) I and II only
   d) I and III only
   e) I, II, III and IV

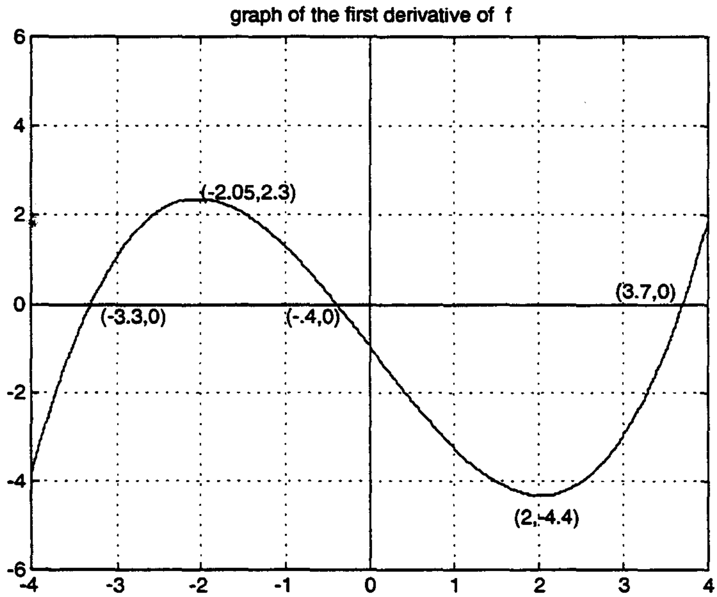
   (E)

UNIT III
STUDENT WORKSHEETS

NAME________________
DATE________________

GRAPHICAL ANALYSIS OF FUNCTIONS USING DERIVATIVES

1. A function, f, is defined on the closed interval [-4, 4]. The graph of f', the first derivative of f, is given below.



a. On which intervals is f increasing? Decreasing?

b. Where does f have a local maximum. Local minimum? Justify your answer.

c. Where does f have points of inflection? Explain.

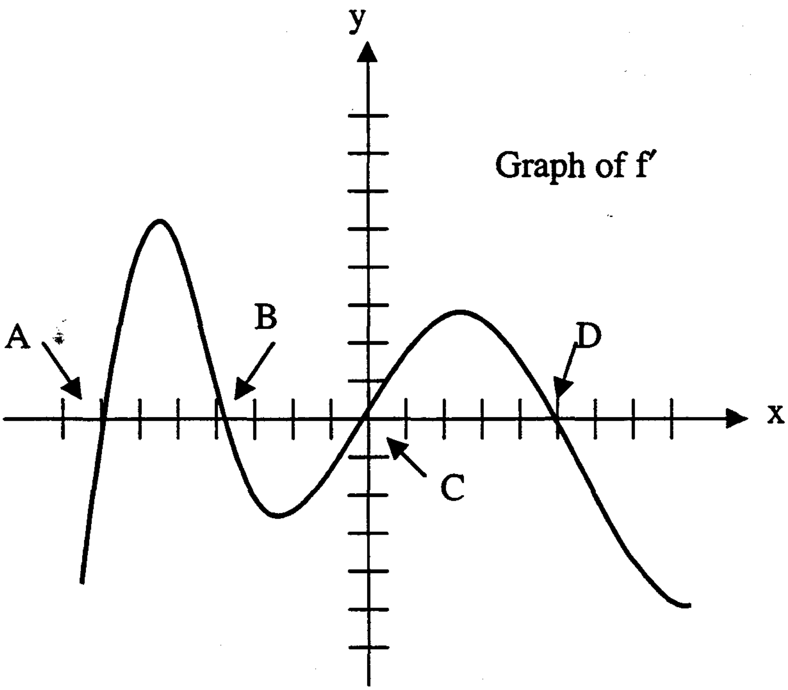
d. On which intervals is f concave up? Concave down?

e. Find the maximum and minimum values of f on the interval [-3.3, -.4]. Support your answer.

f. Find the maximum and minimum values of f on the interval [2, 3.7]. Support your answer.

g. Given $f(0) = 1$, sketch the graph of f.

2. The graph of f', the derivative of a function f, is shown below. Study the graph carefully and answer the following questions.



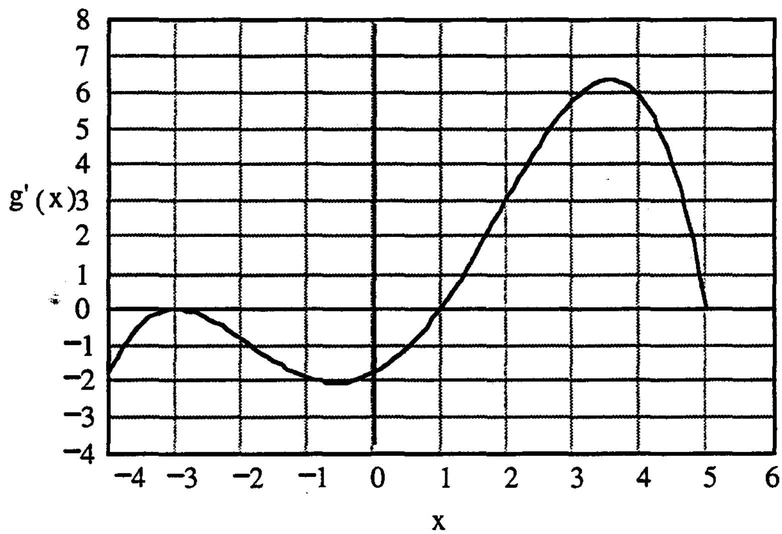
a. Where does f have a local maximum? Local minimum? Justify your answers.

b. Where is f concave up? Concave down? Explain your answers.

c. Where does f have an inflection point? Explain your answer.

NAME________________
DATE________________

1. The graph of the *derivative* of g is shown in the figure.



a. Suppose that $g(4) = 4$. Find an equation of the line tangent to g at the point (4,4)

b. Where does g have a local minimum? Support your answer.

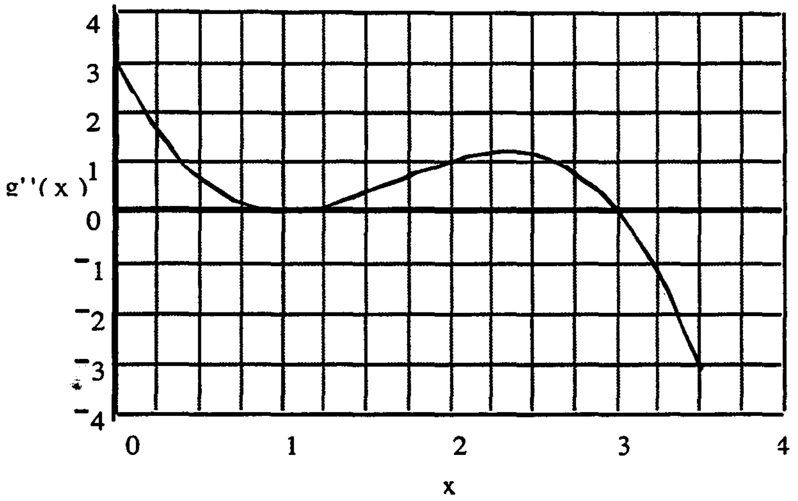
c. Estimate $g''(3.5)$.

d. Where does g have a point of inflection? Support your answer.

e. Where does g achieve its maximum on the interval [-3,0].

f. Suppose that $g(1) = 2$. Find an equation for the line normal to g at (1, 2).

2. The graph of g'', the second derivative of a function g, is shown below. Use this graph to answer the following questions.



a. Where is g concave down?

b. Where does g have points of inflection?

c. Rank the four numbers $g'(0)$, $g'(1)$, $g'(2)$, and $g'(3)$ in increasing order.

d. Suppose that $g'(0) = 0$. Is g increasing or decreasing at $x = 1.5$? Justify your answer.

e. Suppose that $g'(0) = -3$. Is g increasing or decreasing at $x = 1.5$? Justify your answer?

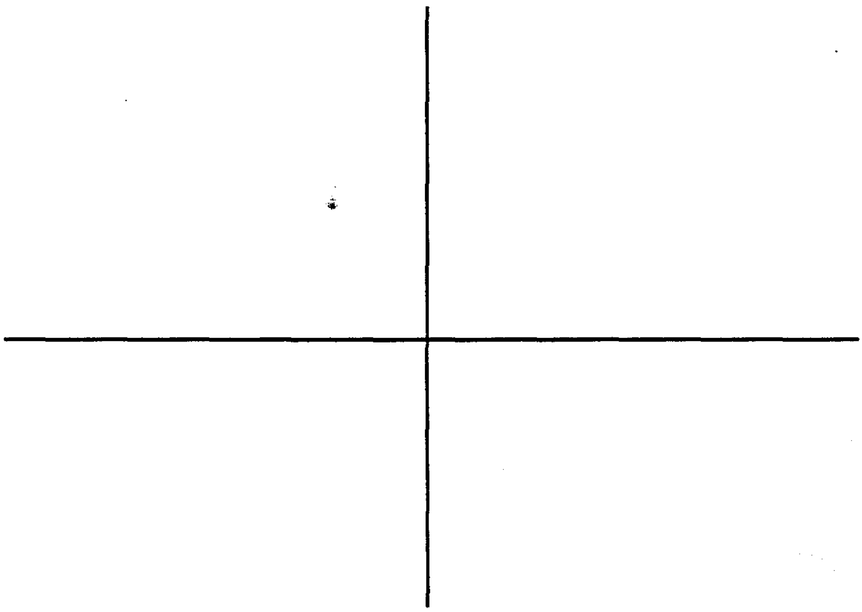
AP CALCULUS
UNIT III – ACTIVITY 3

NAME__________________
DATE __________________

RELATIONSHIPS BETWEEN DERIVATIVES

In this activity, you will observe the relationship between a function and its first and second derivatives.

1. On the axes below, sketch the graph of $y = 6x^{\frac{2}{5}} - x^{\frac{6}{5}}$ over [-10,10].



2. Find y'. Graph y' on your calculator and sketch it on the axes above.

3. For each given value(s) of y', give the corresponding value(s) of x and describe the graph.

y'	values of x	description of graph
$=0$		
>0		
<0		

4. Describe the behavior of y' as x approaches 0. What do you notice about the graph of y at $x = 0$?

5. Find y''.

6. For each value(s) of y'', give the corresponding value(s) of x and describe the graph.

y''	values of x	description of graph
$=0$		
>0		
<0		

7. Does y have any points of inflection? Explain.

8. Write a paragraph summarizing the relationship between a function and its first and second derivatives.

4. Describe the behavior of y' as x approaches 0. What do you notice about the graph of y at x = 0?

5. Find y''.

6. For each value(s) of y'', give the corresponding value(s) of x and describe the graph.

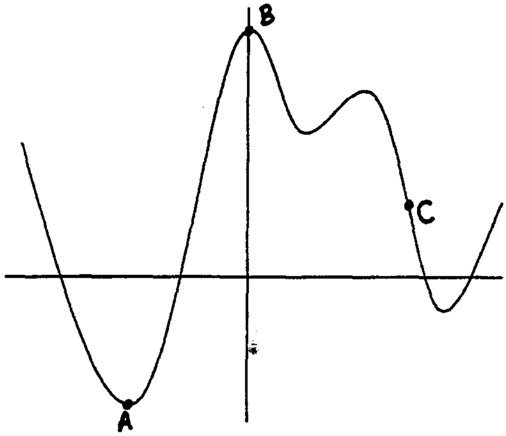
y''	values of x	description of graph
$=0$		
>0		
<0		

7. Does y have any points of inflection? Explain.

8. Write a paragraph summarizing the relationship between a function and its first and second derivatives.

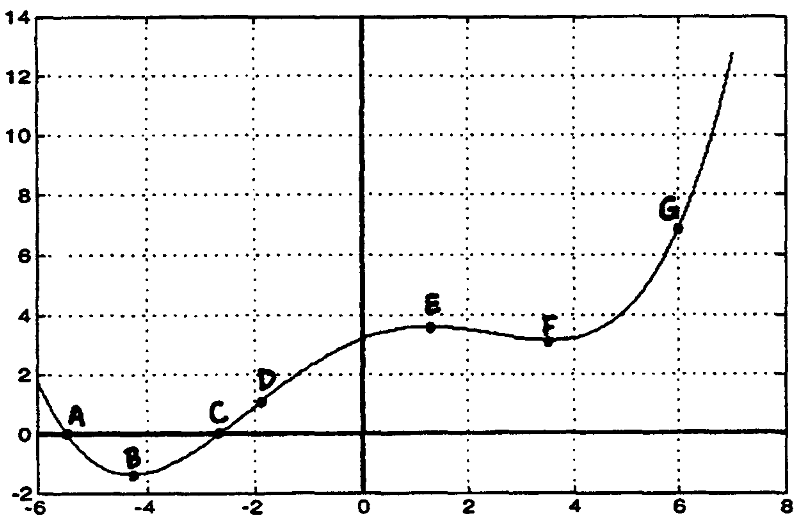
CURVE ANALYSIS REVIEW

1. The graph of f is given below. Determine whether f, f' and f'' are positive, negative, or zero at each point.



	A	B	C
f			
f'			
f''			

2. The graph of g' is given below. Answer the following questions by choosing from points A - G.



g'

a) Where does g have a minimum? Explain.

b) Where does g' have a minimum?

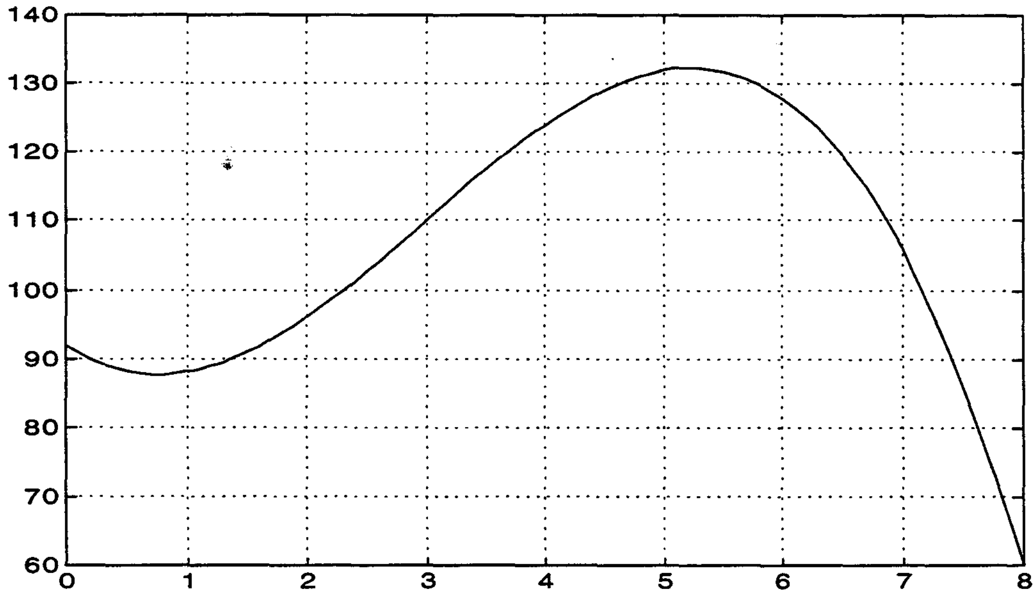
c) Where does g have points of inflection? Explain.

AP CALCULUS
UNIT III-ACTIVITY 4

NAME ____________________
DATE ____________________

RELATING GRAPHS OF POSITION, VELOCITY, AND ACCELERATION

1. The graph below shows the height h(t) of a bird soaring in the sky after t seconds of observation.



a. Fill in the values of $h'(t)$ in the chart using either a +, −, or 0.

t	.8	2	3	4	5.2	6	7
$h'(t)$							

b. Fill in the values of $h''(t)$ in the chart using either a +, −, or 0.

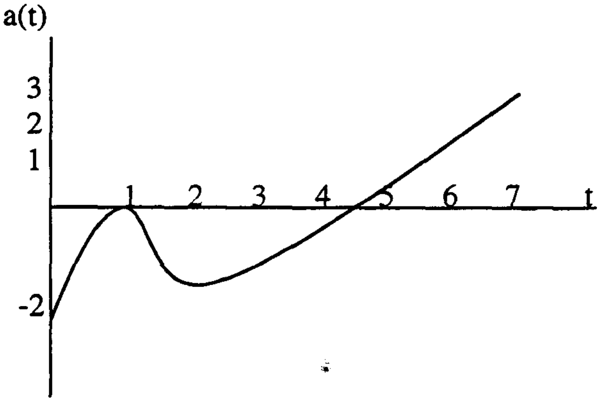
t	.8	2	3	4	5.2	6	7
$h''(t)$							

c. Estimate the value of $h'(2)$. What does this mean in terms of the situation?

d. Over what interval is the bird's velocity positive? How can you tell from the graph?

e. Over what interval is the bird's acceleration positive? How can you tell from the graph?

2. The graph below shows a(t), the acceleration of an object in $\frac{m}{s^2}$ at time t seconds.



When is the object's velocity

a. at a maximum ___________

b. at a minimum ___________

c. increasing ___________

d. decreasing ___________

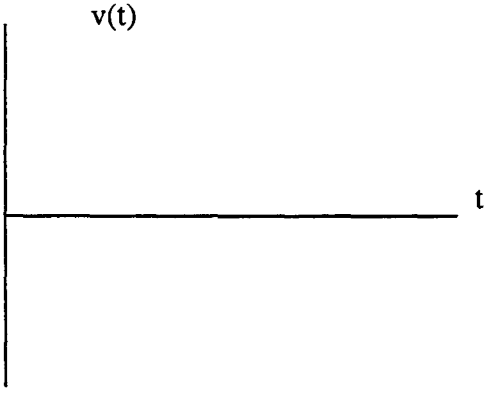
When does the object's position have

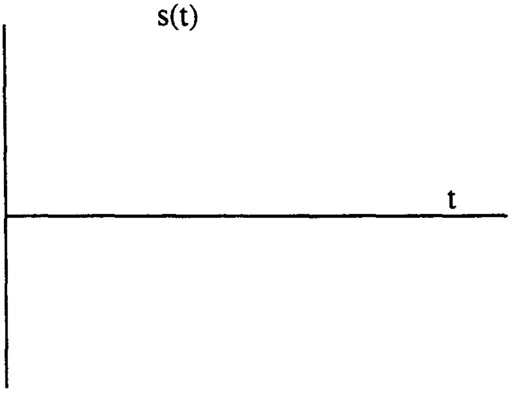
a. points of inflection ___________

b upward concavity ___________

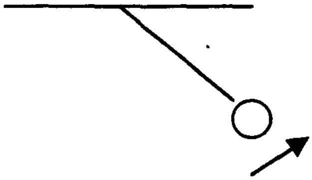
c. downward concavity ___________

Sketch the velocity and position graphs.

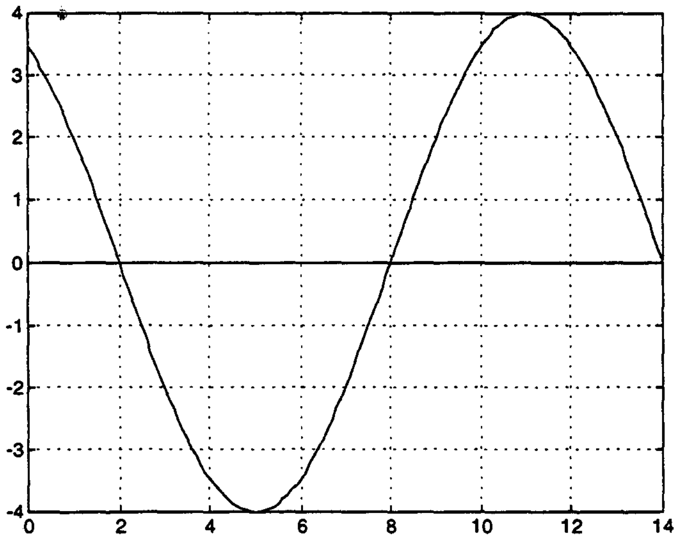




3. A student displaced a pendulum as shown in the diagram



At $t = 0$ she pushed it and it began to swing back and forth. Below is the graph of the velocity v(t), for the first 14 seconds of motion. (Let the motion to the right be positive, to the left be negative.)



a. When is the pendulum's position farthest from its final resting position? Explain.

b. Give the intervals where the pendulum accelerates; decelerates.

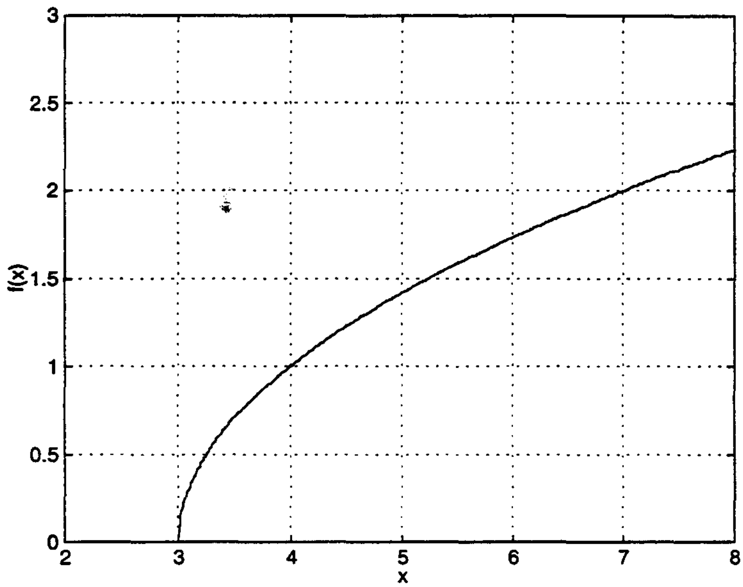
c. After how many seconds is the pendulum accelerating at its maximum rate? Explain.

d. Over what intervals is the pendulum's distance from its final resting position increasing and decreasing?

NAME ______________
DATE ______________

LOCAL LINEARITY

1. Use your calculator to graph $f(x) = \sqrt{x-3}$ in the window shown below.



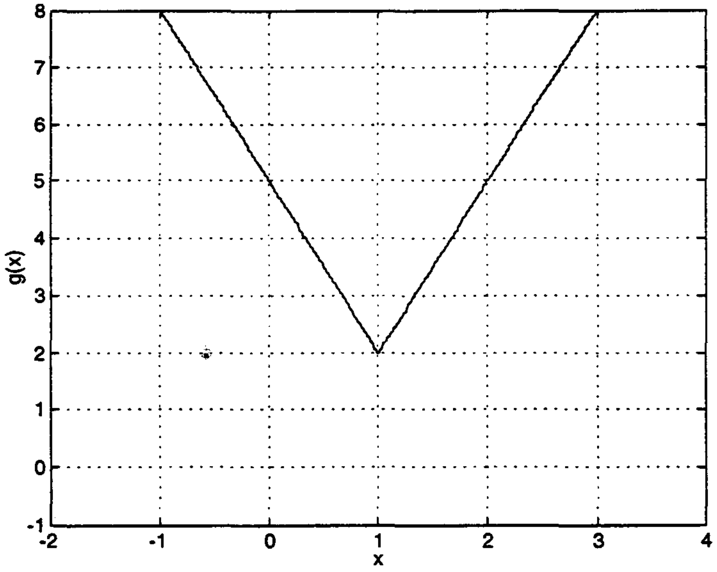
a) Find an equation of the line tangent to f at $x = 4$, and sketch it on the grid above and on your calculator.

b) Describe the proximity of the line to the curve near $x = 4$.

c) Zoom in at the point (4, *f(4)*) three times. How are the line and the curve related now?

d) We say that f has "local linearity" at $x = 4$. Based on your investigation, explain what this term means.

2. Pictured below is $g(x) = 3|x-1| + 2$.



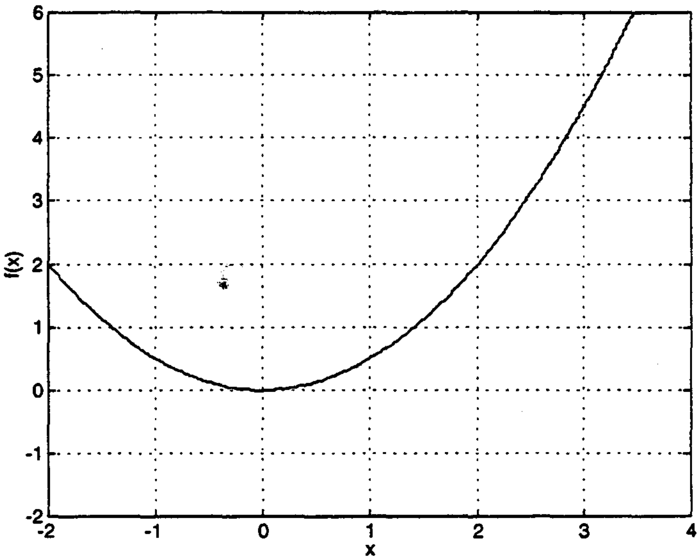
a) Is g locally linear at $(1, 2)$? Justify your answer.

b) Explain why g does not have a derivative at $x = 1$.

NAME ____________________
DATE ____________________

LINEAR APPROXIMATIONS

1. The graph of function $f(x) = 0.5x^2$ is shown below.



a) Find an equation y, for the linearization of f(x), that best fits $f(x)$ at $x = 2$ (the tangent at $x = 2$). Graph y on the axes above.

b) Approximate $f(1.7)$ using y. Is your estimate reasonable? Why or why not?

c) What is the error in using y to approximate $f(1.7)$, where error = $|$expected value – approximate value$|$?

d) Complete the table.

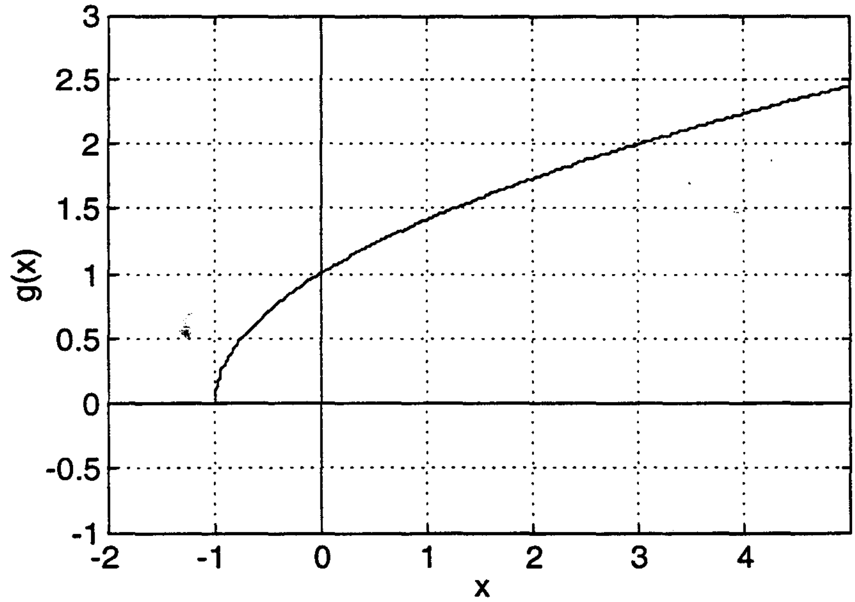
x	$f(x)$	Y	error
1.96			
1.97			
1.98			
1.99			
2			
2.01			
2.02			
2.03			

e) What do you notice about the error as x approaches 2?

f) Is the approximation too large or too small? Why?

g) How close should x be to 2 so that the error would be <u>less than</u> .0003?

2. The graph of $g(x) = \sqrt{x+1}$ is pictured below.



a) Find the linearization of g at $x = 3$ and sketch it on the graph.

b) Compute $\sqrt{3.98}$ with your calculator and compare this answer with the linear approximation of $g(x)$ from above.

c) Calculate the error in using the linear approximation to estimate g(2.98).

d) Is the approximation too large or too small? Why?

e) For what values of x is your approximation accurate to within .002 units?

NAME________________
DATE ________________

LIMITS AND THE DEFINTION OF DERIVATIVE

1. If h is a function such that $\lim_{x\to -1} \frac{h(x)-h(-1)}{x+1} = -2$, which of the following must be true?

   I. h is continuous at x = -1
   II. h is differentiable at x = -1
   III. $h(-1) = -2$
   IV. $h'(-1) = -2$

   a) I only
   b) II only
   c) II and IV
   d) I, II, and III
   e) I, II and IV

2. If g is a differentiable function, then $g'(c) =$

   a) $\frac{g(c)}{c}$
   b) $\frac{g(x)-g(c)}{x-c}$
   c) $\lim_{x\to c} \frac{g(x)-g(c)}{x-c}$
   d) $\lim_{x\to c} \frac{g(x+c)-g(x)}{x-c}$
   e) $\lim_{h\to 0} \frac{g(x+h)-g(h)}{h}$

3. $\lim_{t\to 0} \frac{e^{2(x+t)}-e^{2x}}{t}$ at $x = 0$

   a) 0
   b) 1
   c) 2
   d) e
   e) does not exist

4. $\lim_{h\to 0} \frac{\cos 2(x+h)-\cos 2x}{h} =$

   a) $2\cos 2x$
   b) $-2\sin 2x$
   c) $-2\sin x\cos x$
   d) $-4\sin x\cos x$
   e) $4\cos^2 x - 1$

5. If $\lim_{h\to 0} \frac{\ln\left[(x+h)^2\right]-\ln(x^2)}{h}$ exists, then

   I. $f(x) = 2\ln|x|$
   II. $x \neq 0$
   III. $f'(2) = 1$
   IV. $f'(0)$ does not exist

   a) I only
   b) IV only
   c) I and II only
   d) I and III only
   e) I, II, III and IV

1. Draw a tangent line to the graph of $y = \sqrt{4 - x^2}$ at x = 1.

2. Graph $f(x) = \frac{1}{2}x^2$ and $f'(x)$ on the same coordinate system.

3. For what value(s) of x does $49x^2 - 28x + 4$ equal zero?

4. Find $\lim_{h \to 0} \frac{\cos(\frac{\pi}{3} + h) - \frac{1}{2}}{h}$

5. If $f(x) = \ln(\frac{\pi}{2})$, find $f'(x)$.

1. Draw a tangent line to the graph of $y = \sqrt{4 - x^2}$ at x = 1.

2. Graph $f(x) = \frac{1}{2}x^2$ and $f'(x)$ on the same coordinate system.

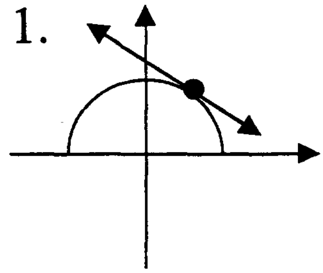
3. For what value(s) of x does $49x^2 - 28x + 4$ equal zero?

4. Find $\lim_{h \to 0} \frac{\cos(\frac{\pi}{3} + h) - \frac{1}{2}}{h}$

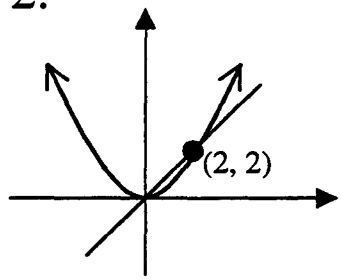
5. If $f(x) = \ln(\frac{\pi}{2})$, find $f'(x)$.

ANSWERS:

1.



2.



3. $(7x-2)^2=0$

$x = \frac{2}{7}$

4. Since

$\frac{1}{2} = \cos\frac{\pi}{3}$

$f'(\cos\frac{\pi}{3}) = -\sin\frac{\pi}{3}$

$= -\frac{\sqrt{3}}{2}$

5. 0

1. If $g'(x) = 3x^2 \cos x^3$,
   Then $g(x) =$ ______________.

2. Given: $\ln(xy) = 3$, where $x, y \neq 0$
   Find: y'

3. If $y = \dfrac{e^{2x}}{\tan x}$, find y'

4.
$$f(x) = \begin{cases} x^2 + 3x, & x > 2 \\ 7x - 5, & x \leq 2 \end{cases}$$

$$f'(x) = \begin{cases} \rule{4cm}{0.4pt} \\ \rule{4cm}{0.4pt} \end{cases}$$

5. Is $f'(x)$ in problem #4 a continuous function? Give a reason for your answer.

1. If $g'(x) = 3x^2 \cos x^3$,
Then $g(x) =$ ______________.

2. Given: $\ln(xy) = 3$, where $x, y \neq 0$
Find: y'

3. If $y = \dfrac{e^{2x}}{\tan x}$, find y'

4.

$$f(x) = \begin{cases} x^2 + 3x, & x > 2 \\ 7x - 5, & x \le 2 \end{cases}$$

$$f'(x) = \begin{cases} \_\_\_\_\_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_\_\_\_\_ \end{cases}$$

5. Is $f'(x)$ in problem #4 a continuous function? Give a reason for your answer.

ANSWERS:

1. $\sin x^3$

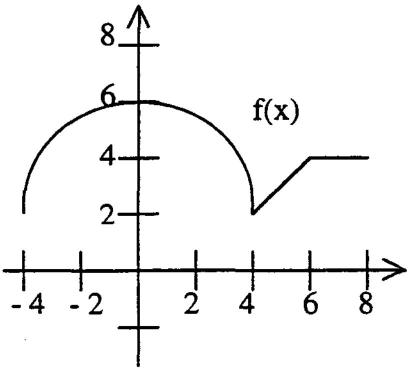
2.
$$\frac{1}{xy}(xy' + y \cdot 1) = 0$$
$$\frac{y'}{y} + \frac{1}{x} = 0$$
$$y' = -\frac{1}{x} \cdot y = -\frac{y}{x}$$

3.
$$\frac{\tan x \cdot e^{2x} \cdot 2 - e^{2x} \cdot \sec^2 x}{\tan^2 x}$$

4.
$$f'(x) = \begin{cases} 2x + 3, & x > 2 \\ 7, & x \le 2 \end{cases}$$

5. Yes

- $\lim_{x \to 2^+} (2x + 3) = \lim_{x \to 2-} 7$
  7   7

1. 

Use the figure on the left to evaluate:

a) $f'(0) =$ ______

b) $f'(4) =$ ______

c) $f'(5) =$ ______

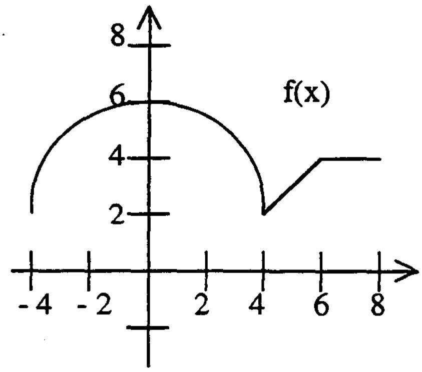
2. Sketch the derivative of the function above.

3. If $h(x) = \ln x + 3x^2$, find $h'(x)$.

4. Use your answer in #3 to find $h'(1)$.

5. Use #3, to find the equation of the line tangent to $h(x)$ at $x = 1$.

ANSWERS:

1. 

Use the figure on the left to evaluate:

a) $f'(0) =$ ______

b) $f'(4) =$ ______

c) $f'(5) =$ ______

2. Sketch the derivative of the function above.

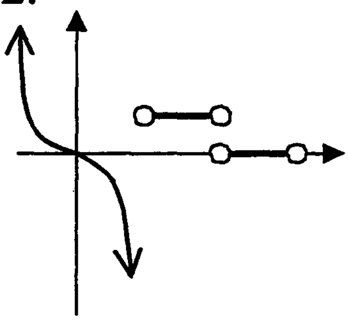
3. If $h(x) = \ln x + 3x^2$, find $h'(x)$.

4. Use your answer in #3 to find $h'(1)$.

5. Use #3, to find the equation of the line tangent to $h(x)$ at $x = 1$.

ANSWERS:

1.
a) 0
b) undefined
c) 1

2.


3.
$$h'(x) = \frac{1}{x} + 6$$

4.
$h'(1) = 7$

5.
$y = 3 + 7(x - 1)$

1. Given: $y = e^x \cos^{-1} x$
   Find: y'

2. Find the volume of a right circular cylinder with radius 3cm and height 7cm.

3. If $f(x) = x^{\frac{1}{5}}$, then $f'(x) =$ __________

4. State the domain of f and f′ in #3.

5. Evaluate:

$$\lim_{x \to \infty} \frac{400}{1 + 20e^{-.7x}}$$

ANSWERS:

1. Given: $y = e^x \cos^{-1}x$
Find: y'

1. $\frac{-e^x}{\sqrt{1-x^2}} + e^x \cos^{-1} x$

2. Find the volume of a right circular cylinder with radius 3cm and height 7cm.

2. $V=\pi r^2 h$
$V=63\pi\ cm^3$

3. If $f(x) = x^{\frac{1}{5}}$, then $f'(x) =$ ______

3. $\frac{1}{5}x^{-\frac{4}{5}}$

4. State the domain of f and f′ in #3.

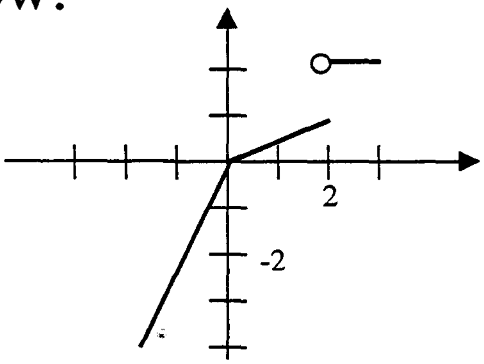
4. $D_f = \Re$
$D_{f'} = \Re,\ x \neq 0$

5. Evaluate:

$$\lim_{x\to\infty} \frac{400}{1+20e^{-.7x}}$$

5. 400

1. Sketch the derivative of the graph below:



2. The instantaneous rate of change of a function is also known as the ________________ of the function.

3. Name the critical values of

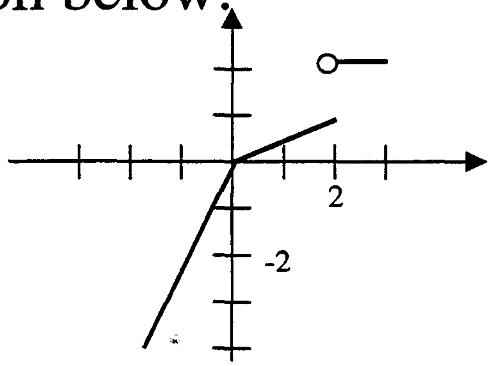
$$g(x) = \frac{x^3}{3} + \frac{x^2}{2} - 12x$$

4. If $p(t) = \sin^{-1}x$, represents the position of a particle, find the velocity function.

5. If $f(x) = x\,e^{2x}$, find $f'(x)$.

AP CALCULUS
UNIT III – WARMUP SET 5

1. Sketch the derivative of the graph below:



2. The instantaneous rate of change of a function is also known as the ______________ of the function.

3. Name the critical values of

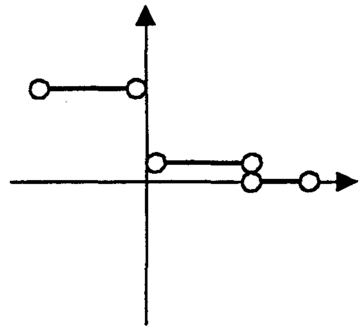
$$g(x) = \frac{x^3}{3} + \frac{x^2}{2} - 12x$$

4. If p(t) = $\sin^{-1}x$, represents the position of a particle, find the velocity function.

5. If f(x) = x e^{2x}, find f′(x).

ANSWERS:

1.



2. derivative

3. x = - 4, 3

4.
$$v(t) = \frac{1}{\sqrt{1-x^2}}$$

5. $2xe^{2x} + e^{2x}$
or $e^{2x}(2x + 1)$

UNIT III INTERNET RESOURCES

http://www.math.odu.edu/cbii/calcanim/ and http://www.math.psu.edu/dna/graphics.html

Both sites have animated demonstrations of the derivative by way of local linearity.

http://www.netsrq.com/~hahn/calculus.html, http://www.barzilai.org/archive, http://archives.math.utk.edu.visual.calculus/ and http://www.hofstra.edu/~matscw/RealWorld/index.html

Sites feature some tutorials, but mostly have good drill and quiz resources for either in-class practice or at-home practice.