BC CALCULUS
SEMESTER B REVIEW

NAME *Key*
DATE ______

how all work on a separate sheet

1. Let R be the region in the first quadrant enclosed by the graphs of $y = x^2 - 2x + 2$ and $y = 1 + 2\sin x$.

(a) $\int_{.270}^{2.248} \left(\sqrt{1+(2\cos x)^2} + \sqrt{1+(2x-2)^2}\right) dx$

   a) Write an expression involving one or more integrals to find the length of the boundary of R.

(b) $\int_{.270}^{2.248} \left[(1+2\sin x)^2 - (x^2-2x+2)^2\right] dx$

   b) Write an expression involving one or more integrals to find the volume of the solid generated when R is revolved about the x-axis.
   c) The base of a solid is the region R. Each cross section of the solid perpendicular to the x-axis is a semicircle. Write an expression involving one or more integrals that gives the volume of the solid.

(c) $\frac{\pi}{2}\int_{.270}^{2.248} \left[\frac{(1+2\sin x) - (x^2-2x+2)}{2}\right]^2 dx$

2. The sales of a small company are expected to grow at a rate given by $\frac{ds}{dt} = 100(3^{.2t})$ where t is measured in days and sales in hundreds of dollars.

(a) ≈ 364.096 hundreds of \$ / day

   a) Determine the average rate of growth over the first 10 days.
   b) Show the geometric meaning of the average rate of growth on a graph of the rate function and explain its significance.
   c) Find the accumulated sales of the company through the first 20 days.

(b) 

It is the constant rate of sales needed to achieve the same total sales using the variable rate of sales over the 10 day period

(c) \$ 36,409.60 (Rounded)

Evaluate:

a) $\int x\cos x\,dx$ b) $\int e^x \sin x\,dx$ c) $\int \frac{dx}{x^2+x}$ d) $\int \frac{2\,dx}{x^2-3x-4}$

$x\sin x + \cos x + C$ $\qquad \frac{e^x(\sin x - \cos x)}{2} + C$ $\qquad \ln\left|\frac{x}{x+1}\right| + C$ $\qquad \frac{2}{5}\ln\left|\frac{x-4}{x+1}\right| + C.$

4. Use $u = 5x - 2$ to write an integral equivalent to $\int_0^3 x^2\sqrt{5x-2}\,dx$

$\frac{1}{125}\int_{-2}^{13} (u^2+4u+4)u^{1/2}\,du$

5. Determine whether each integral converges or diverges

a) $\int_1^{\infty} \frac{dx}{x^2}$ conv. b) $\int_{-\infty}^{0} x e^x\,dx$ conv. c) $\int_0^2 \frac{dx}{(x-1)^2}$ div.

6. Evaluate $\lim_{x\to 0^+} x^{\sin x}$ $\quad 1$

7. Write the first four non-zero terms of the power series representation for

a) $\frac{1}{1+5x}$ b) $\frac{x}{1-2x}$

$1 - 5x + 25x^2 - 125x^3 \ldots$ $\qquad x + 2x^2 + 4x^3 + 8x^4 \ldots$

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8. Write the first four non-zero terms of the MacLaurin series for $\sin x^2$ $x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} \ldots$

9. Let $P(x) = 5 - 2(x-1) + 4(x-1)^2 - 3(x-1)^3 + (x-1)^4$ be the fourth degree Taylor polynomial for the function f about x = 1. Assume f has derivatives of all orders for all real numbers.

   a) Find f(1), f'''(1) $f(1) = 5,\ f'''(1) = -18$
   b) Write the second-degree Taylor polynomial for g if g(x) = f'(x) and use it to approximate g(1.2) $g(x) \approx -2 + 8(x-1) - 9(x-1)^2,\ g'(1.2) \approx -.76$
   c) Write the third-degree Taylor polynomial for h, if $h(x) = \int_1^x f(x)\,dt$ about x = 1.

   $h(x) \approx 5 + 5(x-1) - (x-1)^2 + \frac{4}{3}(x-1)^3$

10. What is the maximum possible error in approximating $\cos \pi$ using

    $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$? $\approx .235$

11. Determine whether the series is convergent or divergent Justify your answer.

    a) $\sum_{n=1}^{\infty} \frac{2}{n\sqrt{n}}$ b) $\sum_{n=1}^{\infty} \frac{2n}{n+1}$ c) $\sum_{n=1}^{\infty} \left(\frac{\pi}{3}\right)^n$ d) $\sum_{n=1}^{\infty} \frac{4^n}{n!}$ C; ratio

    C; $p > 1$ D, n-th term test   D; Geo $|r| > 1$

12. Determine the radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{2^n x^n}{n^2}$ $r = \frac{1}{2},\ -\frac{1}{2} \le x \le \frac{1}{2}$

13. Solve the differential equations:

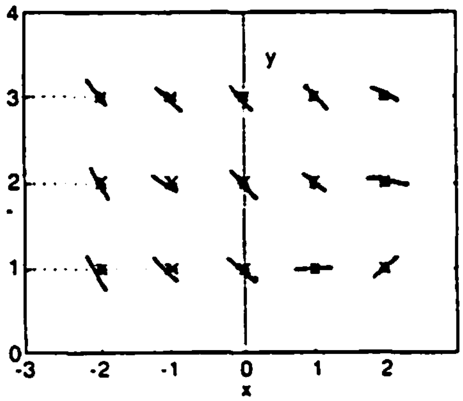
    a) $\frac{dy}{dx} = xy$ for y, if y(4) = e.   b) $\frac{dy}{dx} = \frac{x^2 - 1}{3y}$ when y(0) = 2

14. Given $\frac{dy}{dx} = \frac{3x}{y}$ and f(1) = 2, use Euler's method with $\Delta x = 1$ to approximate f(3).

13. (a) $y = e^{\frac{x^2}{2} - 7}$ (b) $9y^2 = 2x^3 - 6x + 36$

14. $y(2) = 3.5$
    $y(3) = 5.214$

5. Construct the slope field for $\frac{dy}{dx} = x - y$ on the axis provided.



16. A population of gray wolves is modeled by the differential equation $\frac{dP}{dt} = \frac{P(500-P)}{640}$.

a) Solve the differential equation if at t = 0, P = 40 wolves.
b) When will the population consist of 230 wolves?

$P = \frac{500}{1+11.5e^{-.781t}}$; $t = 2.92$

17. The rate at which a student learns vocabulary words is given by the equation $\frac{dw}{dt} = (75 - w)$.

a) Solve the differential equation if at t = 0, 10 words are learned. $w = 75 - 65e^{-t}$
b) What is the maximum number of words learned? 75 words

18. Sales of widgets grow at a rate proportional to the amount present.

a) Set up a differential equation to model this problem. $\frac{dy}{dt} = ky$
b) Solve the differential equation if at t = 0, there are 20 widgets sold and after 3 days 500 widgets are sold.

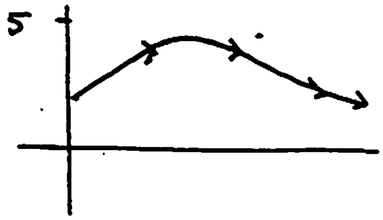
$y = 20e^{\frac{\ln 25}{3}t}$ or $y = 20 \cdot 25^{t/3}$

19. Find the area of one loop of $r = 2 \sin 3\phi$. $\pi/3$ or 1.047

19. A particle moves in the xy plane so that its position at any time t, $0 \le t \le 2\pi$, is given by $x(t) = t^2$ and $y(t) = -2\cos(2t) + 3$

a) Sketch the path of the particle in the xy plane. Indicate the direction of the particle along the path.
b) Find the velocity and acceleration vectors of the particle.
c) Find the speed of the particle at $t = \pi$

(a)



(b) $v(t) = (2t, 4\sin(2t))$ or $2t\vec{i} + 4\sin 2t\,\vec{j}$

$a(t) = (2, 8\cos 2t)$ or $2\vec{i} + 8\cos(2t)\vec{j}$

(c) ≈ 6.283