9.4 Sequences

p739 11-20 Determine whether the sequence converges or diverges. If it converges, find the limit.

12. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots, \frac{1}{2^n}, \ldots$

14. $\{3n-1\}$

16. $\left\{\frac{2n-1}{n+1}\right\}$

18. $\{(1.5)^n\}$

Find $\lim\limits_{n\to\infty} a_n$ for the following

1. $a_n = \dfrac{\frac{1}{n+1}}{\frac{1}{n}}$

2. $a_n = \dfrac{(n+1)!}{n!}$

3. $a_n = \dfrac{n!}{(n+1)!}$

4. $a_n = \dfrac{n}{n+1} x$

5. $a_n = \left(\dfrac{3n+1}{2-3n}\right) x$

6. $a_n = \left(\frac{4}{3}\right)^n$

7. $a_n = \dfrac{(n+1)(n-4)}{(n+2)(n+5)}$

8. $a_n = 352(.92)^n$

9. $a_n = \frac{1}{n} - \frac{1}{n+2}$

10. $a_n = \frac{1}{4^n}$

11. $a_n = \left(1+\frac{1}{n}\right)^n$

12. $a_n = \dfrac{(2n+1)!}{(2n-1)!}$

13. $a_n = \dfrac{n!\,x}{(n+1)!}$

14. $a_n = \dfrac{(n+1)!\,x}{n!}$

15. $a_n = \frac{1}{n!}$

16. $a_n = \dfrac{\frac{(n+1)^2+1}{(n+1)!}}{\frac{n^2+1}{n!}}$

17. $a_n = \dfrac{\frac{1}{(n+1)!}}{\frac{1}{n!}}$

18. $a_n = \dfrac{\frac{(n+1)!}{3^{n+1}}}{\frac{n!}{3^n}}$

19. $a_n = \dfrac{3^n}{n!}$

20. $a_n = \dfrac{(n+1)(10/9)^{n+1}}{n(10/9)^n}$

21. $\dfrac{\frac{n+1}{4^{n+1}}}{\frac{n}{4^n}} = a_n$

22. $a_n = \dfrac{\frac{e^{n+1}}{(n+1)!}}{\frac{e^n}{n!}}$

9.4 Sequences solutions.

9. $\lim_{n\to\infty} \frac{1}{n}\left(\frac{n+2}{n+2}\right) - \frac{n}{n}\left(\frac{1}{n+2}\right)$

$= \lim_{n\to\infty} \frac{n+2-n}{n(n+2)}$

$= \lim_{n\to\infty} \frac{2}{n(n+2)}$

$= 0$, converges

10. $\lim_{n\to\infty} \left(\frac{1}{4}\right)^n = 0$ converges

11. $\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e \approx 2.718281828$

12. $\lim_{n\to\infty} \frac{(2n+1)(2n)(2n-1)!}{(2n-1)!}$

$\lim_{n\to\infty} (2n+1)(2n) = \infty$ diverges

13. $\lim_{n\to\infty} \frac{n!}{(n+1)n!} \cdot x$

$\lim_{n\to\infty} \frac{1}{n+1} \cdot x$

$0 \cdot x = 0$

converges

14. $\lim_{n\to\infty} \frac{(n+1)n! \, x}{n!}$

$\lim_{n\to\infty} (n+1)x$

$\infty \cdot x$

∞

diverge

15. $0 \leq \lim_{n\to\infty} \frac{1}{n!} \leq \lim_{n\to\infty} \frac{1}{n}$

$0 \leq \lim_{n\to\infty} \frac{1}{n!} \leq 0$

0, ~~diverges~~ converges

16. $\lim_{n\to\infty} \frac{(n+1)^2+1}{(n+1)!} \cdot \frac{n!}{n^2+1}$

$\lim_{n\to\infty} \frac{(n^2+2n+2)\,n!}{(n+1)\cdot n! \cdot (n^2+1)}$

$\lim_{n\to\infty} \frac{n^2+2n+2}{(n+1)(n^2+1)} = 0$

convergent

17. $\lim_{n\to\infty} \frac{1}{(n+1)!} \cdot \frac{n!}{1}$

$\lim_{n\to\infty} \frac{n!}{(n+1)\,n!} =$

$\lim_{n\to\infty} \frac{1}{n+1} = 0$

converged.

18. $\lim_{n\to\infty} \frac{(n+1)!}{3^{n+1}} \cdot \frac{3^n}{n!}$

$\lim_{n\to\infty} \frac{(n+1)\,n! \cdot 3^n}{3^n \cdot 3^1 \cdot n!}$

$\lim_{n\to\infty} \frac{n+1}{3} = \infty$

diverged.

19. $\lim_{n\to\infty} \frac{3^n}{n!}$ use calc $= 0$

$n=69$, $\frac{3^{69}}{69!} = 5\times10^{-66}$

20. $\lim_{n\to\infty} \frac{(n+1)(10/9)^1(10/9)^n}{n\,(10/9)^n}$

$\lim_{n\to\infty} \frac{10(n+1)}{9n} = \frac{10}{9}$

converges

21. $\lim_{n\to\infty} \frac{n+1}{4^{n+1}} \cdot \frac{4^n}{n}$

$\lim_{n\to\infty} \frac{(n+1)\,4^n}{4^n \cdot 4^1 \cdot n}$

$\lim_{n\to\infty} \frac{n+1}{4n} = \frac{1}{4}$

Converges

22. $\lim_{n\to\infty} \frac{e^{n+1}}{(n+1)!} \cdot \frac{n!}{e^n}$

$= \lim_{n\to\infty} \frac{e^1 \cdot e^n \cdot n!}{(n+1)\,n!\,e^n} \Rightarrow \lim_{n\to\infty} \frac{e}{n+1} = e \cdot 0 = 0$

Converges