H Pre Calculus

3D Vectors, Lines, Planes

Name: KEY

30

1. Find the positive unit normal to plane $5x-2y+z+2=0$

$$\vec{n} = \frac{5i-2j+k}{-\sqrt{30}}$$

$\frac{25}{5}$ / $30$

+2

2. If $A=(0,1,5)$, $B(-1,2,3)$, $C(-2,1,0)$, $D(5,1,5)$. Find angle between $\vec{AB}$ and $\vec{CD}$

A 0 1 5

$\vec{AB} = (-1,1,-2)$

$\angle (-2,1,0)$

$\vec{CD} = 7;0,5)$

method +3 (-2)

$$\cos\theta = \frac{(-1,1,-2)\cdot(7,0,5)}{\sqrt{6}\sqrt{74}} = \frac{-7-10}{\sqrt{444}} = \frac{-17}{\sqrt{444}}$$

49 / 25 / 74

$\theta = 143.783$

+2

3 decimals

3. Find the positive unit normal of the plane through $(3,2,1)$, $(4,-1,2)$, $(5,6,7)$

$(3,2,1)$ $(3,2,1)$

$(1,-3,1)$ $(2,4,6)$

$$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & -3 & 1 \\ 2 & 4 & 6 \end{vmatrix}$$

$$\vec{n} = i\begin{vmatrix} -3 & 1 \\ 4 & 6 \end{vmatrix} - j\begin{vmatrix} 1 & 1 \\ 2 & 6 \end{vmatrix} + k\begin{vmatrix} 1 & -3 \\ 2 & 4 \end{vmatrix}$$

$$\vec{n} = i(-18-4) - j(6-2) + k(4+6)$$

$$\vec{n} = -22i - 4j + 10k$$

$-22x-4y+10z+64=0$

$$\frac{-22i-4j+10k}{-\sqrt{600}} = \frac{-11i-2j+5k}{-\sqrt{150}}$$

$\sqrt{150} = 5\sqrt{6}$

25 · 6

or

$$\frac{11i+2j-5k}{\sqrt{150}}$$

4. Find the vector equation of the plane with parametric equations $x = 2+s+t$, $y = 1-s+t$, $z = -1+2s-t$

$\vec{r} = (2,1,-1) + s(1,-1,2) + t(1,1,-1)$

or

$\vec{r} = (2+s+t)\hat{i} + (1-s+t)\hat{j} + (-1+2s-t)\hat{k}$

5. Find the rectangular equation of the plane through $(3,0,0), (0,-2,0), (0,0,4)$

$$\frac{x}{3} + \frac{y}{-2} + \frac{z}{4} = 1$$

$$4x - 6y + 3z = 12$$

6. $A = (-1,3,2)$ and $B = (4,7,5)$. Find point C where C is on $\overleftrightarrow{AB}$ and between A and B such that C is $\frac{2}{3}$ of the way from A to B.


$\frac{2}{3}$ of 1 unit = 2

$\vec{AB} = (5,4,3)$

$\vec{r} = (-1,3,2) + t(5,4,3)$

$\vec{r} = (-1,3,2) + \frac{2}{3}(5,4,3)$

$\vec{r} = \left(-1+\frac{10}{3}, 3+\frac{8}{3}, 2+2\right) = \left(-\frac{3}{3}+\frac{10}{3}, \frac{9}{3}+\frac{8}{3}, 4\right)$

$\left(\frac{7}{3}, \frac{17}{3}, 4\right)$